

## Image formation

Monday March 21

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UT-Austin

## Announcements

- Reminder: Pset 3 due March 30
- Midterms: pick up today

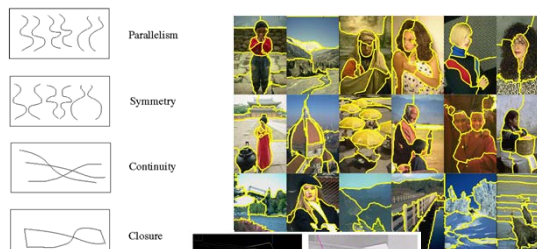
## Recap: Features and filters



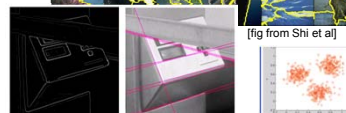
Transforming and describing images; textures, colors, edges

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## Recap: Grouping & fitting



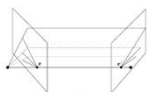
Clustering, segmentation, fitting; what parts belong together?



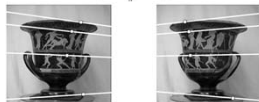
[fig from Shi et al]

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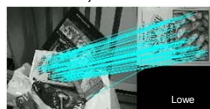
## Multiple views



Multi-view geometry, matching, invariant features, stereo vision



Hartley and Zisserman



Lowe



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## Plan

- Today:
  - Local feature matching btwn views (wrap-up)
  - Image formation, geometry of a single view
- Wednesday: Multiple views and epipolar geometry
- Monday: Approaches for stereo correspondence

## Previously

### Local invariant features

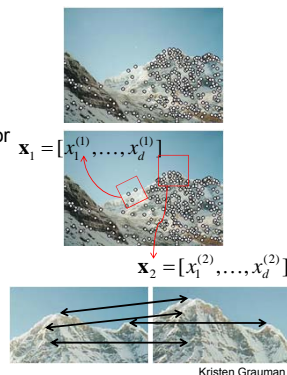
- Detection of interest points
  - Harris corner detection
  - Scale invariant blob detection: LoG
- Description of local patches
  - SIFT : Histograms of oriented gradients
- Matching descriptors

## Local features: main components

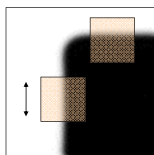
1) Detection: Identify the interest points

2) Description: Extract vector feature descriptor surrounding each interest point.

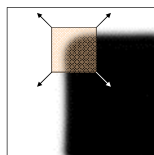
3) Matching: Determine correspondence between descriptors in two views



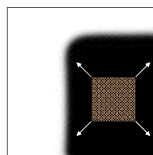
### Recall: Corners as distinctive interest points



"edge":  
 $\lambda_1 \gg \lambda_2$   
 $\lambda_2 \gg \lambda_1$



"corner":  
 $\lambda_1$  and  $\lambda_2$  are large,  
 $\lambda_1 \sim \lambda_2$



"flat" region  
 $\lambda_1$  and  $\lambda_2$  are small;

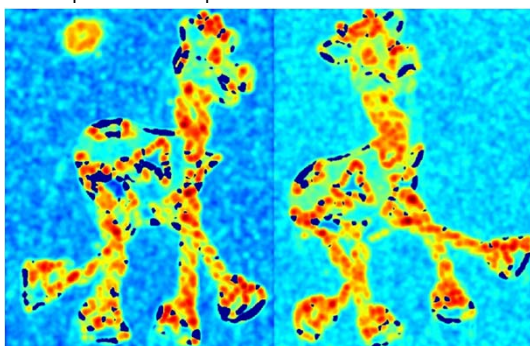
One way to score the cornerness: 
$$f = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$

### Harris Detector: Steps

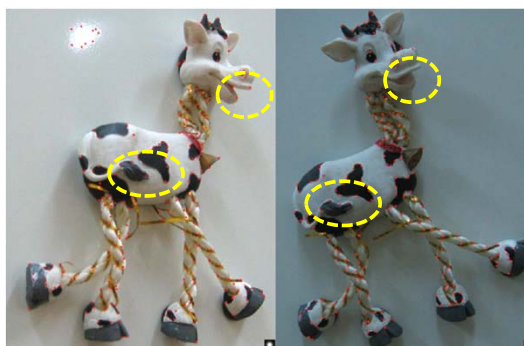


### Harris Detector: Steps

Compute corner response  $f$



### Harris Detector: Steps



### Blob detection in 2D: scale selection

Laplacian-of-Gaussian = "blob" detector  $\nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2}$

filter scales

img1 img2 img3

### Blob detection in 2D

We define the *characteristic scale* as the scale that produces peak of Laplacian response

characteristic scale

Slide credit: Lana Lazebni

### Example

Original image at 1/4 the size

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### Scale invariant interest points

Interest points are local maxima in both position and scale.

Squared filter response maps

scale

⇒ List of  $(x, y, \sigma)$

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### Scale-space blob detector: Example

original image      scale-space maxima of  $(\nabla_{norm}^2 L)^2$

T. Lindeberg. Feature detection with automatic scale selection. IJCV 1998.

### Local features: main components

- 1) Detection: Identify the interest points
- 2) Description: Extract vector feature descriptor surrounding each interest point.  $\mathbf{x}_1 = [x_1^{(1)}, \dots, x_d^{(1)}]$
- 3) Matching: Determine correspondence between descriptors in two views.  $\mathbf{x}_2 = [x_1^{(2)}, \dots, x_d^{(2)}]$

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### SIFT descriptor [Lowe 2004]

- Use histograms to bin pixels within sub-patches according to their orientation.

0  $2\pi$

*Why subpatches?  
Why does SIFT have some illumination invariance?*

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### Making descriptor rotation invariant

- Rotate patch according to its dominant gradient orientation
- This puts the patches into a canonical orientation.

Image from Matthew Brown

### SIFT descriptor [Lowe 2004]

- Extraordinarily robust matching technique
  - Can handle changes in viewpoint
    - Up to about 60 degree out of plane rotation
  - Can handle significant changes in illumination
    - Sometimes even day vs. night (below)
  - Fast and efficient—can run in real time
  - Lots of code available
    - [http://people.csail.mit.edu/abertladyack/wiki/index.php/known\\_Implementations\\_of\\_SIFT](http://people.csail.mit.edu/abertladyack/wiki/index.php/known_Implementations_of_SIFT)

Steven Seitz

### SIFT properties

- Invariant to
  - Scale
  - Rotation
- Partially invariant to
  - Illumination changes
  - Camera viewpoint
  - Occlusion, clutter

### Local features: main components

- 1) Detection: Identify the interest points
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### Matching local features

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### Matching local features

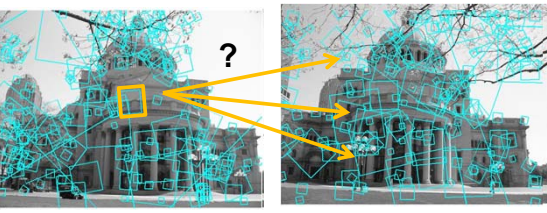


Image 1                      Image 2

To generate **candidate matches**, find patches that have the most similar appearance (e.g., lowest SSD)

Simplest approach: compare them all, take the closest (or closest k, or within a thresholded distance)

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### Ambiguous matches

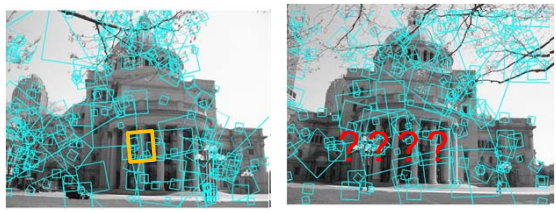


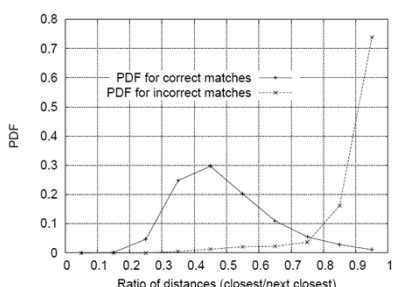
Image 1                      Image 2

At what SSD value do we have a good match?  
To add robustness to matching, can consider **ratio** :  
distance to best match / distance to second best match  
If low, first match looks good.  
If high, could be ambiguous match.

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### Matching SIFT Descriptors

- Nearest neighbor (Euclidean distance)
- Threshold ratio of nearest to 2<sup>nd</sup> nearest descriptor



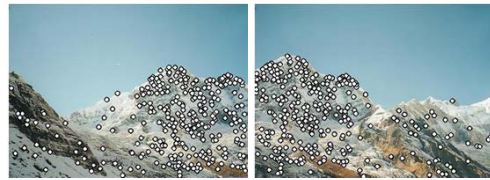
Derek Hoiem                      Lowe IJCV 2004

### Recap: robust feature-based alignment



Source: L. Lazebnik


### Recap: robust feature-based alignment



- Extract features

Source: L. Lazebnik

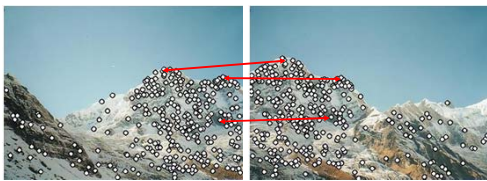
### Recap: robust feature-based alignment



- Extract features
- Compute *putative matches*

Source: L. Lazebnik

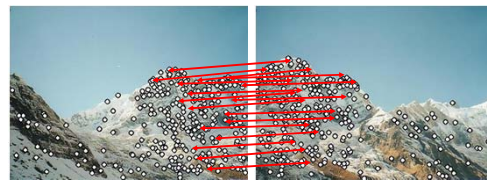
### Recap: robust feature-based alignment



- Extract features
- Compute *putative matches*
- Loop:
  - Hypothesize transformation  $T$  (small group of putative matches that are related by  $T$ )

Source: L. Lazebnik

### Recap: robust feature-based alignment



- Extract features
- Compute *putative matches*
- Loop:
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  - Verify transformation (search for other matches consistent with  $T$ )

Source: L. Lazebnik

### Recap: robust feature-based alignment



- Extract features
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Source: L. Lazebnik

### Applications of local invariant features

- Wide baseline stereo
- Motion tracking
- Panoramas
- Mobile robot navigation
- 3D reconstruction
- Recognition
- ...

### Automatic mosaicing



<http://www.cs.ubc.ca/~mbrown/autostitch/autostitch.html>

### Wide baseline stereo



[Image from T. Tuytelaars ECCV 2006 tutorial]

### Recognition of specific objects, scenes

Schmid and Mohr 1997

Sivic and Zisserman, 2003

Rothganger et al. 2003

Lowe 2002

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### Plan

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  - Image formation, geometry of a single view
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### Image formation

- How are objects in the world captured in an image?

### Physical parameters of image formation

- Geometric
  - Type of projection
  - Camera pose
- Optical
  - Sensor's lens type
  - focal length, field of view, aperture
- Photometric
  - Type, direction, intensity of light reaching sensor
  - Surfaces' reflectance properties

### Image formation

object

film

- Let's design a camera
  - Idea 1: put a piece of film in front of an object
  - Do we get a reasonable image?

Slide by Steve Seitz

### Pinhole camera

object

barrier

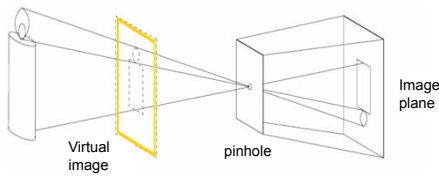
film

- Add a barrier to block off most of the rays
  - This reduces blurring
  - The opening is known as the **aperture**
  - How does this transform the image?

Slide by Steve Seitz

### Pinhole camera

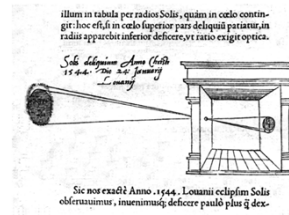
- Pinhole camera is a simple model to approximate imaging process, perspective **projection**.



If we treat pinhole as a point, only one ray from any given point can enter the camera.

Fig from Forsyth and Ponce

### Camera obscura



In Latin, means 'dark room'

"Reinerus Gemma-Frisius, observed an eclipse of the sun at Louvain on January 24, 1544, and later he used this illustration of the event in his book *De Radio Astronomica et Geometrica*, 1545. It is thought to be the first published illustration of a camera obscura..."  
Hammond, John H., *The Camera Obscura, A Chronicle*

[http://www.acmi.net.au/AIC/CAMERA\\_OBSCURA.html](http://www.acmi.net.au/AIC/CAMERA_OBSCURA.html)

### Camera obscura



An attraction in the late 19<sup>th</sup> century

<http://brightbytes.com/cosite/collection2.html>  
Adapted from R. Duraiswami

### Camera obscura at home

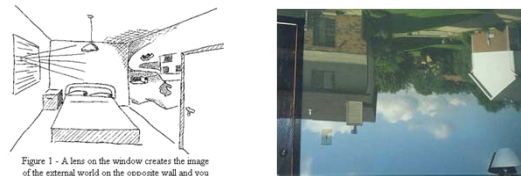


Figure 1 - A lens on the window creates the image of the external world on the opposite wall and you can see it every morning, when you wake up.

Sketch from [http://www.funsci.com/fun3\\_en/sky/sky.htm](http://www.funsci.com/fun3_en/sky/sky.htm)

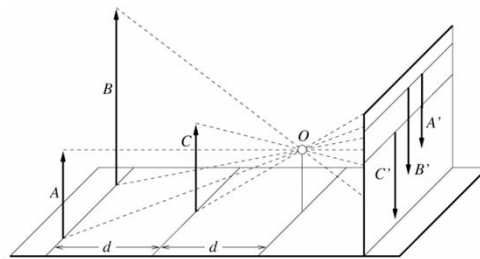
[http://blog.makezine.com/archive/2006/02/how\\_to\\_room\\_sized\\_camera\\_obscura.html](http://blog.makezine.com/archive/2006/02/how_to_room_sized_camera_obscura.html)

### Perspective effects



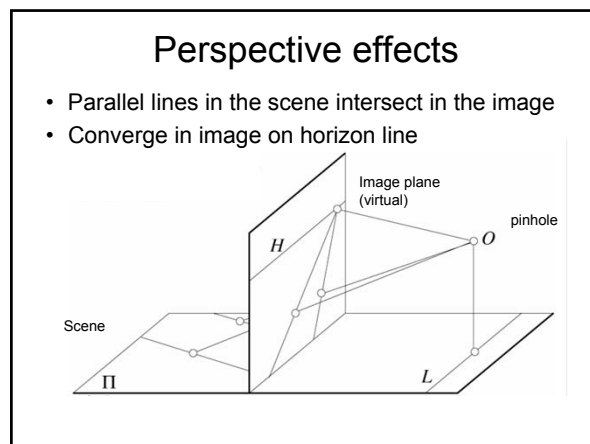
### Perspective effects

- Far away objects appear smaller

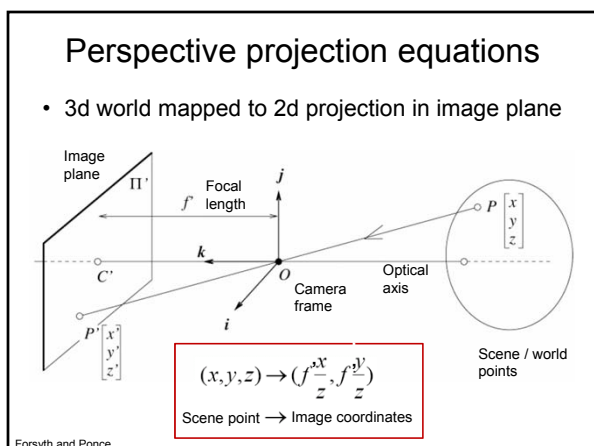
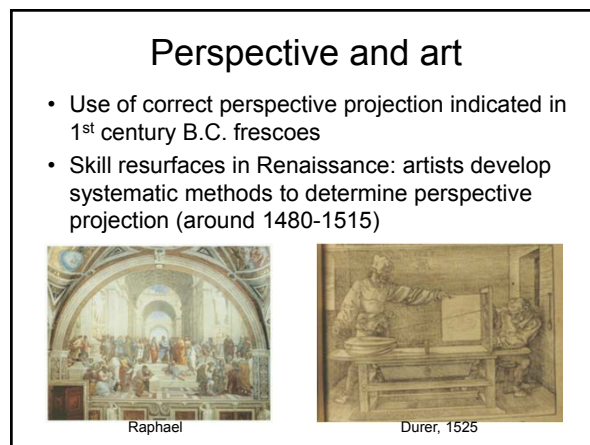


Forsyth and Ponce





- ### Projection properties
- Many-to-one: any points along same ray map to same point in image
  - Points → points
  - Lines → lines (collinearity preserved)
  - Distances and angles are **not** preserved
  - Degenerate cases:
    - Line through focal point projects to a point.
    - Plane through focal point projects to line
    - Plane perpendicular to image plane projects to part of the image.



### Homogeneous coordinates

Is this a linear transformation?

- no—division by z is nonlinear

Trick: add one more coordinate:

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene coordinates

Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Slide by Steve Seitz

### Perspective Projection Matrix

- Projection is a matrix multiplication using homogeneous coordinates:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f' & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z/f' \\ 1 \end{bmatrix} \Rightarrow (f' \frac{x}{z}, f' \frac{y}{z})$$

divide by the third coordinate to convert back to non-homogeneous coordinates

Slide by Steve Seitz

### Weak perspective

- Approximation: treat magnification as constant
- Assumes scene depth  $\ll$  average distance to camera

World points:  $x' = f \frac{x}{z} \approx \frac{f}{z_0} x$   
 $y' = f \frac{y}{z} \approx \frac{f}{z_0} y$

### Orthographic projection

- Given camera at **constant** distance from scene
- World points projected along rays parallel to optical axis

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow (x, y)$$

### Physical parameters of image formation

- Geometric
  - Type of projection
  - Camera pose
- Optical
  - Sensor's lens type
  - focal length, field of view, aperture
- Photometric
  - Type, direction, intensity of light reaching sensor
  - Surfaces' reflectance properties

### Pinhole size / aperture

How does the size of the aperture affect the image we'd get?

Larger  
↓  
Smaller

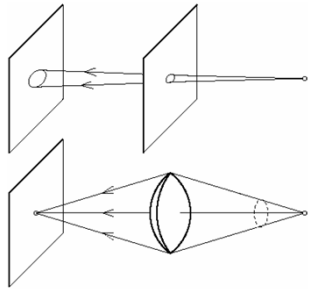
Fig. 5.98 The pinhole camera. Note the variation in image clarity as the hole diameter decreases. [Photos courtesy Dr. N. Joni, UNESCO.]

### Adding a lens

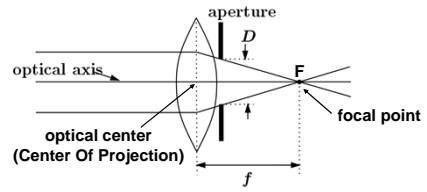
- A lens focuses light onto the film
  - Rays passing through the center are not deviated
  - All parallel rays converge to one point on a plane located at the focal length  $f$

Slide by Steve Seitz

### Pinhole vs. lens

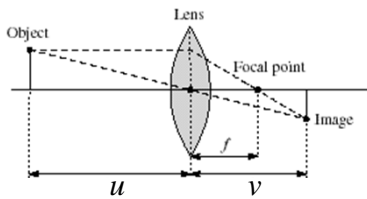


### Cameras with lenses



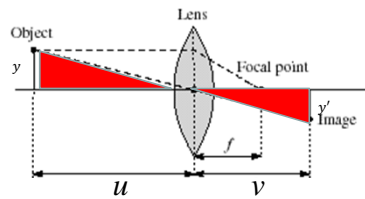
- A lens focuses parallel rays onto a single focal point
- Gather more light, while keeping focus; make pinhole perspective projection practical

### Thin lens equation



- How to relate distance of object from optical center ( $u$ ) to the distance at which it will be in focus ( $v$ ), given focal length  $f$ ?

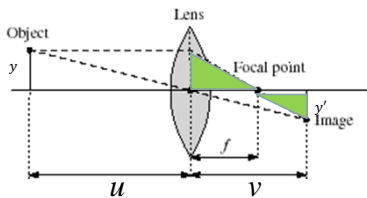
### Thin lens equation



$$\frac{y'}{y} = \frac{v}{u}$$

- How to relate distance of object from optical center ( $u$ ) to the distance at which it will be in focus ( $v$ ), given focal length  $f$ ?

### Thin lens equation

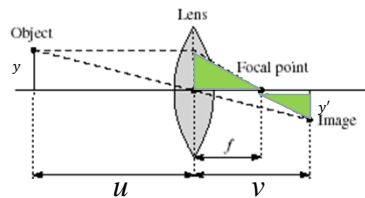


$$\frac{y'}{y} = \frac{v}{u}$$

$$\frac{y'}{y} = \frac{(v - f)}{f}$$

- How to relate distance of object from optical center ( $u$ ) to the distance at which it will be in focus ( $v$ ), given focal length  $f$ ?

### Thin lens equation



$$\frac{y'}{y} = \frac{v}{u}$$

$$\frac{y'}{y} = \frac{(v - f)}{f}$$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

- Any object point satisfying this equation is in focus

### Focus and depth of field

Image credit: cambridgeincolour.com

### Focus and depth of field

- Depth of field: distance between image planes where blur is tolerable

Thin lens: scene points at distinct depths come in focus at different image planes.  
(Real camera lens systems have greater depth of field.)

Shapiro and Stockman

### Focus and depth of field

- How does the aperture affect the depth of field?

- A smaller aperture increases the range in which the object is approximately in focus

Flower images from Wikipedia [http://en.wikipedia.org/wiki/Depth\\_of\\_field](http://en.wikipedia.org/wiki/Depth_of_field) Slide from S. Seitz

### Field of view

- Angular measure of portion of 3d space seen by the camera

Images from [http://en.wikipedia.org/wiki/Angle\\_of\\_view](http://en.wikipedia.org/wiki/Angle_of_view)

### Field of view depends on focal length

- As  $f$  gets smaller, image becomes more *wide angle*
  - more world points project onto the finite image plane
- As  $f$  gets larger, image becomes more *telescopic*
  - smaller part of the world projects onto the finite image plane

from R. Duraiswami

### Field of view depends on focal length

Size of field of view governed by size of the camera retina:

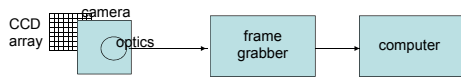
$$\phi = \tan^{-1}\left(\frac{d}{2f}\right)$$

Smaller FOV = larger Focal Length

Slide by A. Efros

## Digital cameras

- Film → sensor array
- Often an array of charge coupled devices
- Each CCD is light sensitive diode that converts photons (light energy) to electrons



## Summary

- Image formation affected by geometry, photometry, and optics.
- Projection equations express how world points mapped to 2d image.
  - Homogenous coordinates allow linear system for projection equations.
- Lenses make pinhole model practical.
- Parameters (focal length, aperture, lens diameter,...) affect image obtained.

## Next time

- Geometry of two views

