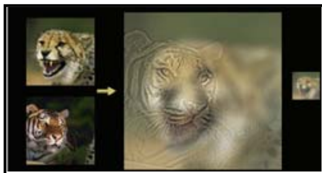
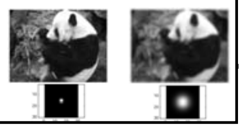


Linear Filters

Monday, Jan 24

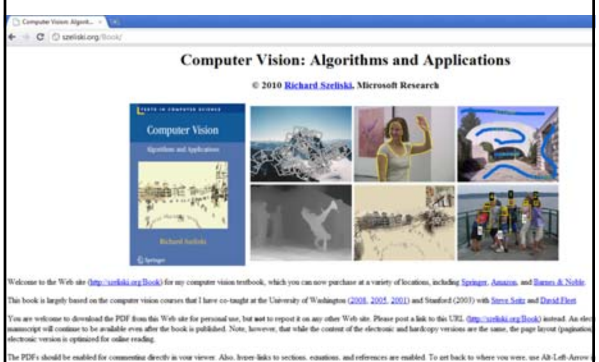
Prof. Kristen Grauman

UT-Austin



Announcements

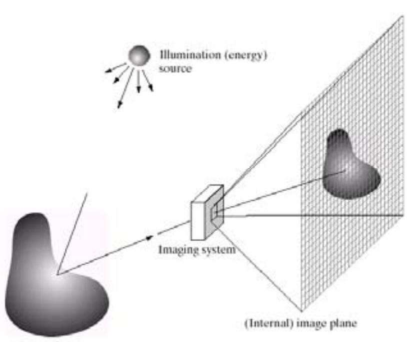
- **Office hours** Mon-Thurs 5-6 pm
  - Mon: Yong Jae, PAI 5.33
  - Tues/Thurs: Shalini, PAI 5.33
  - Wed: Me, ACES 3.446
- cv-spring2011@cs.utexas.edu for assignment questions outside of office hours
- **Pset 0** due Friday Jan 28. Drop box in PAI 5.38. Attach cover page with name and CS 376



Plan for today

- Image noise
- Linear filters
  - Examples: smoothing filters
- Convolution / correlation

Image Formation



Slide credit: Derek Hoiem

Digital camera

A digital camera replaces film with a sensor array

- Each cell in the array is light-sensitive diode that converts photons to electrons
- http://electronics.howstuffworks.com/digital-camera.htm

Slide by Steve Seitz

Digital images

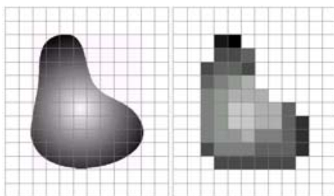
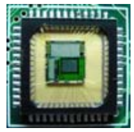


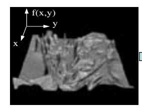
FIGURE 2.17 (a) Continuous image projected onto a sensor array. (b) Result of image sampling and quantization.



Slide credit: Derek Hoiem

Digital images

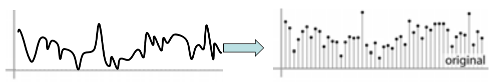
- **Sample** the 2D space on a regular grid
- **Quantize** each sample (round to nearest integer)
- Image thus represented as a matrix of integer values.



j →, i ↓

62	79	23	119	120	105	4	0
10	10	9	62	12	78	34	0
10	58	197	46	46	0	0	48
176	135	5	188	191	68	0	49
2	1	1	29	26	37	0	77
0	89	144	147	187	102	62	208
255	252	0	166	123	62	0	31
166	63	127	17	1	0	99	30

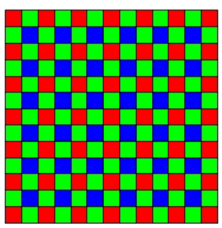
2D



1D

Adapted from S. Seitz

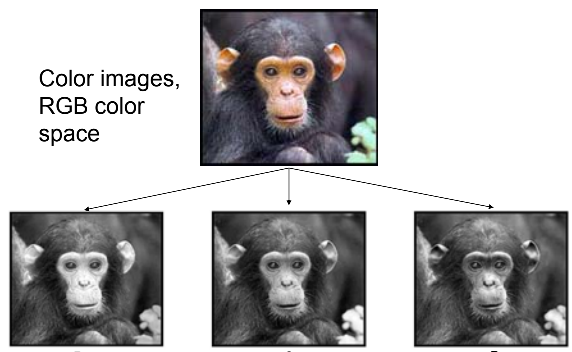
Digital color images



Bayer filter

Digital color images

Color images, RGB color space



Images in Matlab

- Images represented as a matrix
- Suppose we have a NxM RGB image called "im"
  - im(1,1,1) = top-left pixel value in R-channel
  - im(y, x, b) = y pixels down, x pixels to right in the b[th] channel
  - im(N, M, 3) = bottom-right pixel in B-channel
- imread(filename) returns a uint8 image (values 0 to 255)
  - Convert to double format (values 0 to 1) with im2double

row ↓, column →, R (G, B channels stacked behind)

0.92	0.93	0.94	0.97	0.62	0.37	0.85	0.97	0.93	0.92	0.99
0.95	0.89	0.82	0.89	0.56	0.31	0.75	0.92	0.81	0.95	0.91
0.89	0.72	0.51	0.55	0.51	0.42	0.57	0.41	0.49	0.91	0.92
0.96	0.95	0.88	0.94	0.56	0.46	0.91	0.87	0.90	0.97	0.95
0.71	0.81	0.81	0.87	0.57	0.37	0.80	0.88	0.89	0.79	0.85
0.49	0.62	0.60	0.58	0.50	0.60	0.58	0.50	0.61	0.45	0.33
0.86	0.84	0.74	0.58	0.51	0.39	0.73	0.92	0.91	0.49	0.74
0.96	0.67	0.54	0.85	0.48	0.37	0.88	0.90	0.94	0.82	0.93
0.69	0.49	0.56	0.66	0.43	0.42	0.77	0.73	0.71	0.90	0.99
0.79	0.73	0.90	0.67	0.33	0.61	0.69	0.79	0.73	0.93	0.97
0.91	0.94	0.89	0.49	0.41	0.78	0.78	0.77	0.89	0.99	0.93

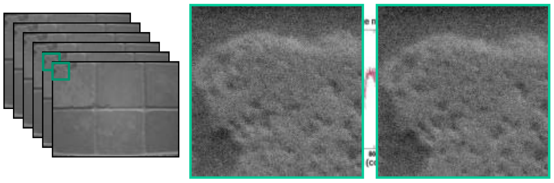
Slide credit: Derek Hoiem

Image filtering

- Compute a function of the local neighborhood at each pixel in the image
  - Function specified by a "filter" or mask saying how to combine values from neighbors.
- Uses of filtering:
  - Enhance an image (denoise, resize, etc)
  - Extract information (texture, edges, etc)
  - Detect patterns (template matching)

Adapted from Derek Hoiem

Motivation: noise reduction



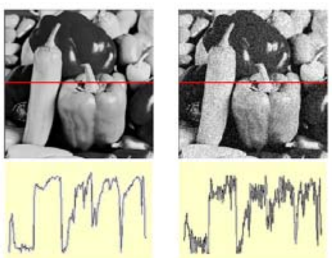
- Even multiple images of the **same static scene** will not be identical.

Common types of noise

- **Salt and pepper noise**: random occurrences of black and white pixels
- **Impulse noise:** random occurrences of white pixels
- **Gaussian noise**: variations in intensity drawn from a Gaussian normal distribution

Source: S. Seitz

Gaussian noise



$$f(x,y) = \overbrace{\hat{f}(x,y)}^{\text{Ideal Image}} + \overbrace{\eta(x,y)}^{\text{Noise process}}$$

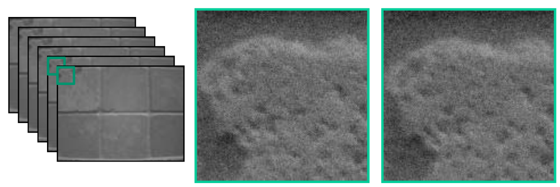
Gaussian i.i.d. ("white") noise: $\eta(x,y) \sim \mathcal{N}(\mu,\sigma)$

```
>> noise = randn(size(im)).*sigma;
>> output = im + noise;
```

What is impact of the sigma?

Fig: M. Hebert

Motivation: noise reduction



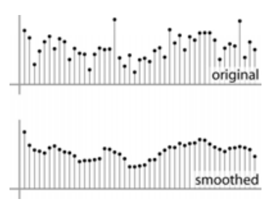
- Even multiple images of the same static scene will not be identical.
- How could we reduce the noise, i.e., give an estimate of the true intensities?
- **What if there's only one image?**

First attempt at a solution

- Let's replace each pixel with an average of all the values in its neighborhood
- Assumptions:
  - Expect pixels to be like their neighbors
  - Expect noise processes to be independent from pixel to pixel

First attempt at a solution

- Let's replace each pixel with an average of all the values in its neighborhood
- Moving average in 1D:

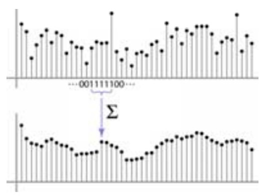


Source: S. Marschner

Weighted Moving Average

Can add weights to our moving average

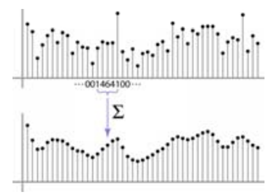
Weights [1, 1, 1, 1, 1] / 5



Source: S. Marschner

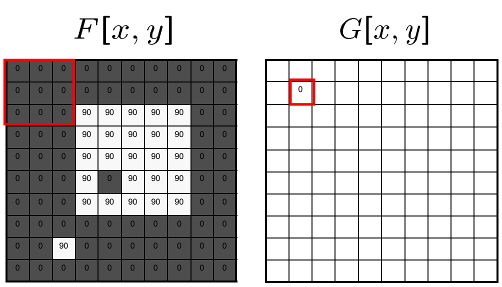
Weighted Moving Average

Non-uniform weights [1, 4, 6, 4, 1] / 16



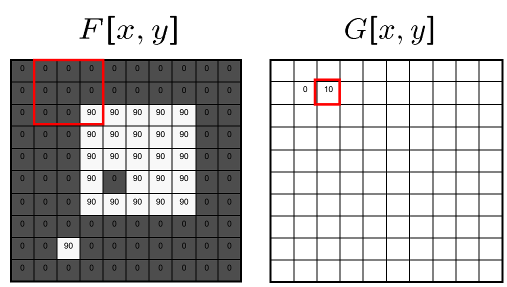
Source: S. Marschner

Moving Average In 2D



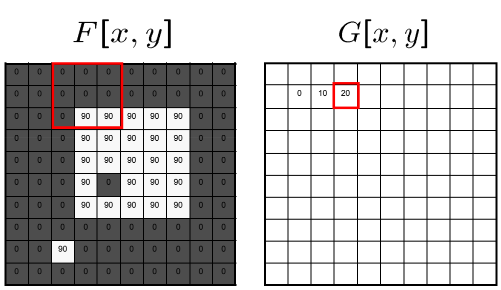
Source: S. Seitz

Moving Average In 2D



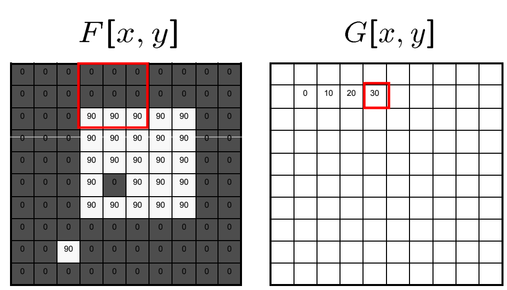
Source: S. Seitz

Moving Average In 2D



Source: S. Seitz

Moving Average In 2D



Source: S. Seitz

Moving Average In 2D

$F[x,y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x,y]$

	0	10	20	30	30				

Source: S. Seitz

Moving Average In 2D

$F[x,y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x,y]$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

Source: S. Seitz

Correlation filtering

Say the averaging window size is 2k+1 x 2k+1:

$$G[i,j] = \underbrace{\frac{1}{(2k+1)^2}}_{\text{Attribute uniform weight to each pixel}} \underbrace{\sum_{u=-k}^{k}\sum_{v=-k}^{k} F[i+u, j+v]}_{\text{Loop over all pixels in neighborhood around image pixel } F[i,j]}$$

Attribute uniform weight to each pixel

Loop over all pixels in neighborhood around image pixel F[i,j]

Now generalize to allow **different weights** depending on neighboring pixel's relative position:

$$G[i,j] = \sum_{u=-k}^{k}\sum_{v=-k}^{k} \underbrace{H[u,v]}_{\text{Non-uniform weights}} F[i+u, j+v]$$

Non-uniform weights

Correlation filtering

$$G[i,j] = \sum_{u=-k}^{k}\sum_{v=-k}^{k} H[u,v]F[i+u, j+v]$$

This is called **cross-correlation**, denoted $G = H \otimes F$

Filtering an image: replace each pixel with a linear combination of its neighbors.

The filter "**kernel**" or "**mask**" *H*[*u*,*v*] is the prescription for the weights in the linear combination.

Averaging filter

- What values belong in the kernel *H* for the moving average example?

$F[x,y]$ $\otimes$ $H[u,v]$ $G[x,y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$\frac{1}{9}$

1	1	1
1	?	1
1	1	1

"box filter"

	0	10	20	30	30				

$$G = H \otimes F$$

Smoothing by averaging

depicts box filter:
white = high value, black = low value

original

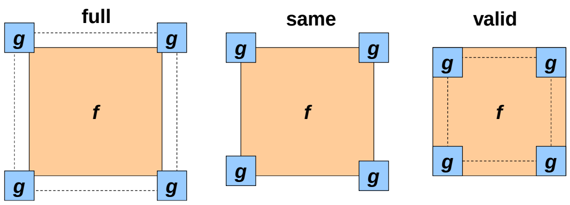
filtered

What if the filter size was 5 x 5 instead of 3 x 3?

Boundary issues

What is the size of the output?

- MATLAB: output size / "shape" options
  - *shape* = 'full': output size is sum of sizes of f and g
  - *shape* = 'same': output size is same as f
  - *shape* = 'valid': output size is difference of sizes of f and g



Source: S. Lazebnik

Boundary issues

What about near the edge?

- the filter window falls off the edge of the image
- need to extrapolate
- methods:
  - – clip filter (black)
  - – wrap around
  - – copy edge
  - – reflect across edge

Source: S. Marschner

Boundary issues

What about near the edge?

- the filter window falls off the edge of the image
- need to extrapolate
- methods (MATLAB):
  - – clip filter (black): imfilter(f, g, 0)
  - – wrap around: imfilter(f, g, 'circular')
  - – copy edge: imfilter(f, g, 'replicate')
  - – reflect across edge: imfilter(f, g, 'symmetric')

Source: S. Marschner

Gaussian filter

- What if we want nearest neighboring pixels to have the most influence on the output?

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$F[x, y]$

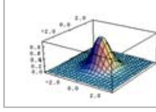
$\frac{1}{16}$

1	2	1
2	4	2
1	2	1

$H[u, v]$

This kernel is an approximation of a 2d Gaussian function:

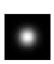
$$h(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{\sigma^2}}$$



- Removes high-frequency components from the image ("low-pass filter").

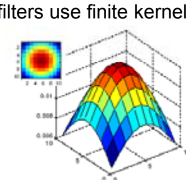
Source: S. Seitz

Smoothing with a Gaussian

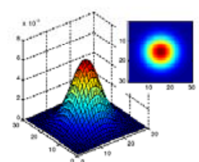


Gaussian filters

- What parameters matter here?
- **Size** of kernel or mask
  - – Note, Gaussian function has infinite support, but discrete filters use finite kernels



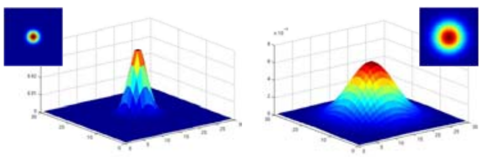
σ = 5 with 10 x 10 kernel



σ = 5 with 30 x 30 kernel

Gaussian filters

- What parameters matter here?
- **Variance** of Gaussian: determines extent of smoothing

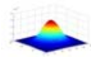


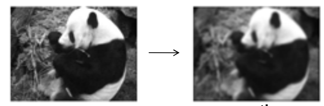
σ = 2 with 30 x 30 kernel

σ = 5 with 30 x 30 kernel

Matlab

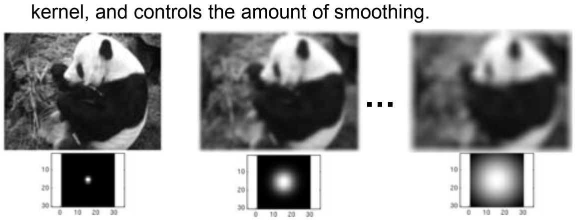
```
>> hsize = 10;
>> sigma = 5;
>> h = fspecial('gaussian' hsize, sigma);

>> mesh(h);

>> imagesc(h);

>> outim = imfilter(im, h); % correlation
>> imshow(outim);
```





Smoothing with a Gaussian

Parameter σ is the "scale" / "width" / "spread" of the Gaussian kernel, and controls the amount of smoothing.



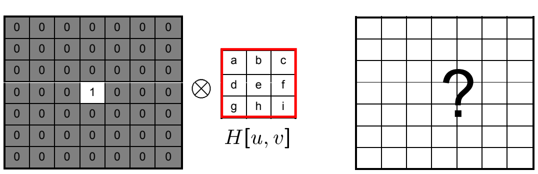
```
for sigma=1:3:10
   h = fspecial('gaussian', fsize, sigma);
   out = imfilter(im, h);
   imshow(out);
   pause;
end
```

Properties of smoothing filters

- Smoothing
  - Values positive
  - Sum to 1 → constant regions same as input
  - Amount of smoothing proportional to mask size
  - Remove "high-frequency" components; "low-pass" filter

Filtering an impulse signal

What is the result of filtering the impulse signal (image) *F* with the arbitrary kernel *H*?



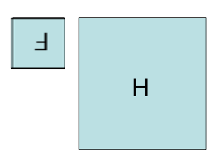
Convolution

- Convolution:
  - Flip the filter in both dimensions (bottom to top, right to left)
  - Then apply cross-correlation

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v] F[i-u, j-v]$$

$$G = H \star F$$

Notation for convolution operator



Convolution vs. correlation

Convolution

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u,j-v]$$

$$G = H \star F$$

Cross-correlation

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i+u,j+v]$$

$$G = H \otimes F$$

For a Gaussian or box filter, how will the outputs differ?
If the input is an impulse signal, how will the outputs differ?

Predict the outputs using correlation filtering

$$* \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = ?$$

$$* \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = ?$$

$$* \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = ?$$

Practice with linear filters

Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

?

Source: D. Lowe

Practice with linear filters

Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Filtered (no change)

Source: D. Lowe

Practice with linear filters

Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

?

Source: D. Lowe

Practice with linear filters

Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Shifted left by 1 pixel with correlation

Source: D. Lowe

Practice with linear filters

Original

1/9 ×

1	1	1
1	1	1
1	1	1

?

Source: D. Lowe

Practice with linear filters

Original

1/9 ×

1	1	1
1	1	1
1	1	1

Blur (with a box filter)

Source: D. Lowe

Practice with linear filters

Original

0	0	0
0	2	0
0	0	0

- 1/9 ×

1	1	1
1	1	1
1	1	1

?

Source: D. Lowe

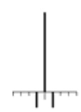
Practice with linear filters

Original

0	0	0
0	2	0
0	0	0

- 1/9 ×

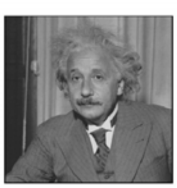
1	1	1
1	1	1
1	1	1



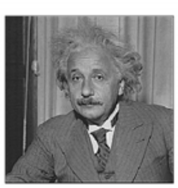
Sharpening filter:
accentuates differences
with local average

Source: D. Lowe

Filtering examples: sharpening



before



after

Properties of convolution

- **Shift invariant:**
  - Operator behaves the same everywhere, i.e. the value of the output depends on the pattern in the image neighborhood, not the position of the neighborhood.
- **Superposition:**
  - h * (f1 + f2) = (h * f1) + (h * f2)

Properties of convolution

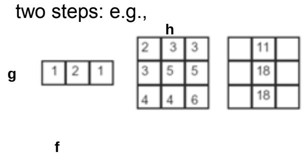
- Commutative:
  f * g = g * f
- Associative
  (f * g) * h = f * (g * h)
- Distributes over addition
  f * (g + h) = (f * g) + (f * h)
- Scalars factor out
  kf * g = f * kg = k(f * g)
- Identity:
  unit impulse e = […, 0, 0, 1, 0, 0, …]. f * e = f

Separability

- In some cases, filter is separable, and we can factor into two steps:
  - Convolve all rows
  - Convolve all columns

Separability

- In some cases, filter is separable, and we can factor into two steps: e.g.,



What is the computational complexity advantage for a separable filter of size k x k, in terms of number of operations per output pixel?

f * (g * h) = (f * g) * h

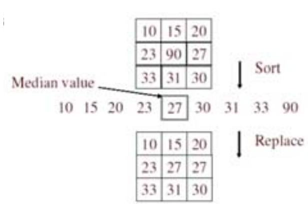
Effect of smoothing filters

5x5

Additive Gaussian noise

Salt and pepper noise

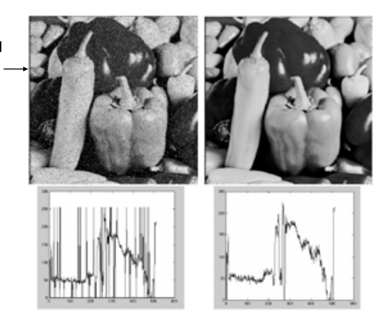
Median filter



- No new pixel values introduced
- Removes spikes: good for impulse, salt & pepper noise
- Non-linear filter

Median filter

Salt and pepper noise



Median filtered

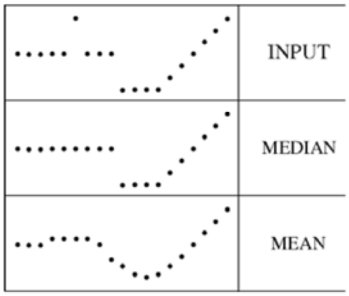
Plots of a row of the image

Matlab: output im = medfilt2(im, [h w]);

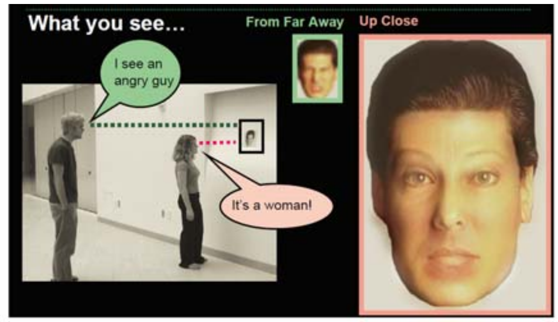
Source: M. Hebert

Median filter

- Median filter is edge preserving



Filtering application: Hybrid Images

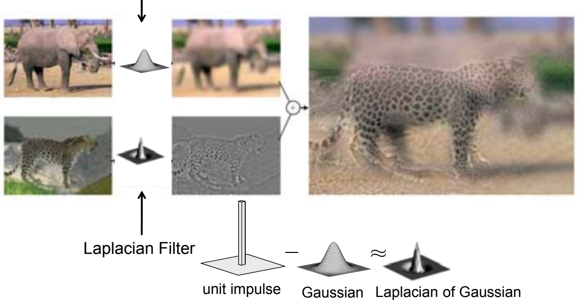


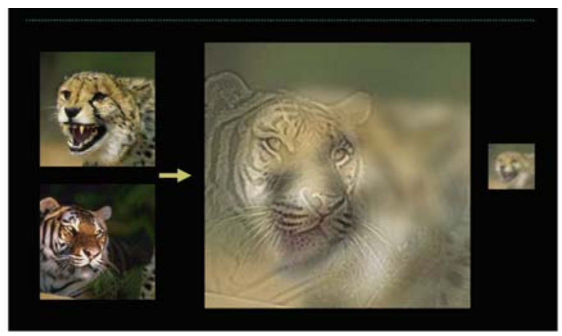
Aude Oliva & Antonio Torralba & Philippe G Schyns, SIGGRAPH 2006

Application: Hybrid Images

Gaussian Filter

A. Oliva, A. Torralba, P.G. Schyns, "Hybrid Images," SIGGRAPH 2006





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Summary

- Image "noise"
- Linear filters and convolution useful for
  - Enhancing images (smoothing, removing noise)
    - Box filter
    - Gaussian filter
    - Impact of scale / width of smoothing filter
  - Detecting features (next time)
- Separable filters more efficient
- Median filter: a non-linear filter, edge-preserving

Coming up

- **Wednesday:**
  - Filtering part 2: filtering for features
- **Friday:**
  - Pset 0 is due via turnin, 11:59 PM