Doubly-efficient zkSNARKs without trusted setup

Riad S. Wahby*, Ioanna Tzialla°,
abhi shelat†, Justin Thaler‡, and Michael Walfish°

*Stanford University
°New York University
†Northeastern University
‡Georgetown University

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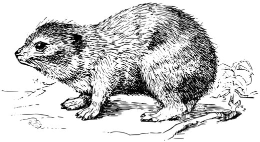

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Succinct It's short...

Non-interactive ...and it can be written down...

(Publicly verifiable) ...so that anyone can check it.

zkSNARKs: Costs and desiderata

Proof size

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Prover ($\mathcal{P}$) time

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Cryptographic assumptions

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Trusted setup?

Our contributions

➔ We design and implement *Hyrax*, a zkSNARK
for "parallel" arithmetic circuit satisfiability:
for $\mathcal{V}$'s input x, $\exists w : \mathcal{C}(x, w) = 1$ (and $\mathcal{P}$ knows w)

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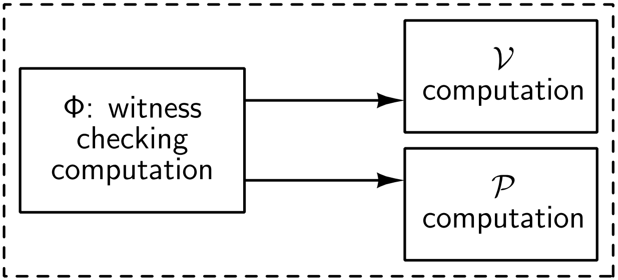
Hyrax is one useful point in a large tradeoff space

Roadmap

1. General-purpose ZK proof systems
2. Hyrax at a high level
3. Evaluation

General-purpose ZK proof systems for NP

On input x, $\mathcal{P}$ convinces $\mathcal{V}$ that $\Phi(x, w) = 1$
(for a witness w that $\mathcal{P}$ knows)



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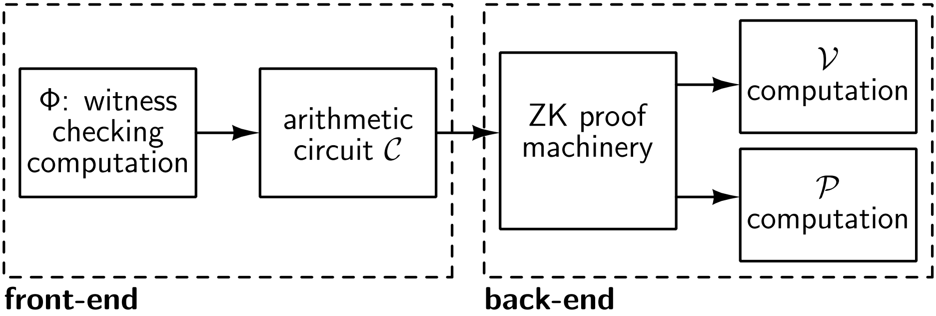

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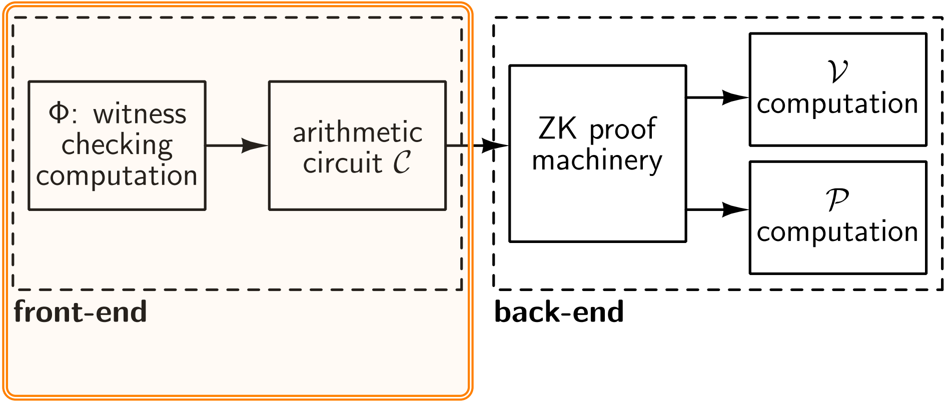

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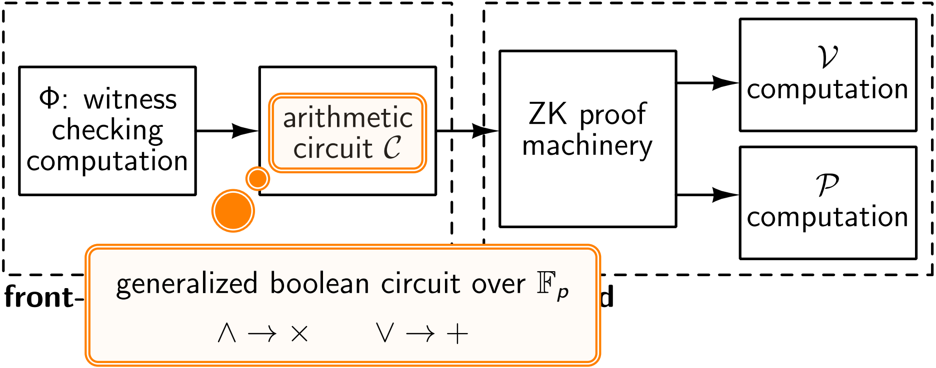

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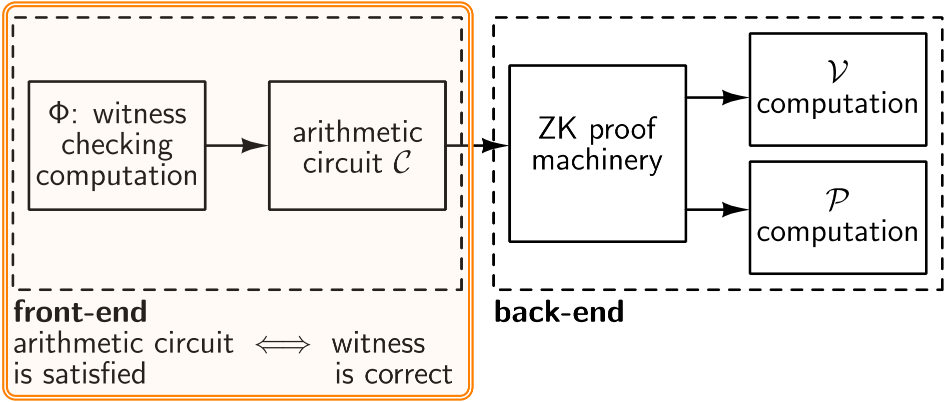

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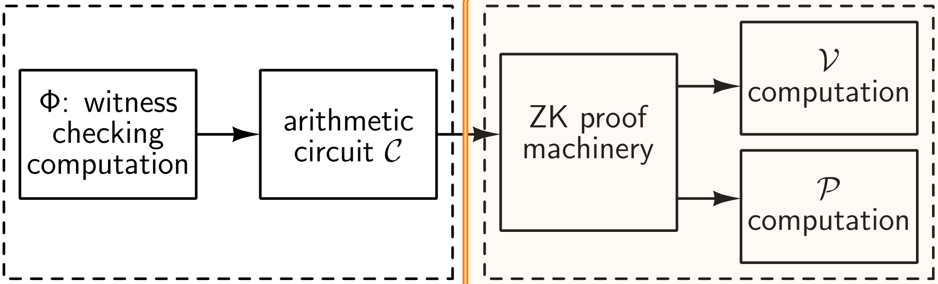

front-end
arithmetic circuit is satisfied $\iff$ witness is correct

back-end
valid proof $\iff$ arithmetic circuit is satisfied

Existing systems use a wide range of proof machinery

Linear PCPs [IKO07,Gro09,Gro10,BG12,Lip12,BCIOP13,GGPR13,...]

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Short PCPs [Kil94,Mic00,BS08,BCN16,RRR16,BBC+17,BBHR17,...]

- libSTARK [BBHR18]

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High-level idea: Replace each of $\mathcal{P}$'s messages in the IP with a *commitment* to the message; $\mathcal{V}$ runs checks "under the commitments."

Cryptographic commitments

Sender computes $C \leftarrow \mathsf{Com}(m)$, sends to *receiver*.
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We also require a *linear homomorphism*, $\odot$:
given $C_0 \leftarrow \text{Com}(m_0), C_1 \leftarrow \text{Com}(m_1)$, we have

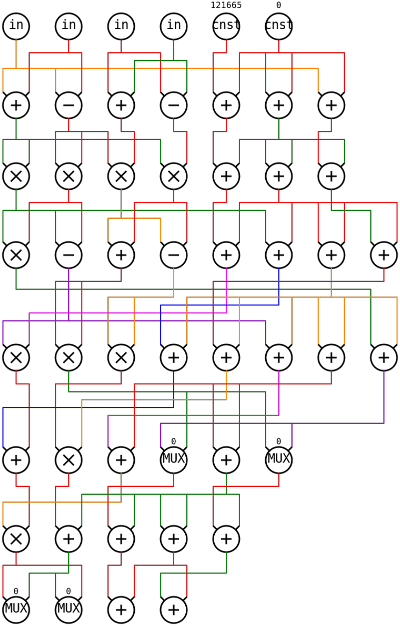
$$C_0 \odot C_1 \triangleq \text{Com}(m_0 + m_1)$$

$$C_1^k \triangleq C_1 \odot \cdots \odot C_1 = \text{Com}(k \cdot m_1)$$

The Pedersen commitment has this property.

GKR08: IP for arithmetic circuit evaluation (non-ZK)

Witness checker must be expressed as a *layered* AC.



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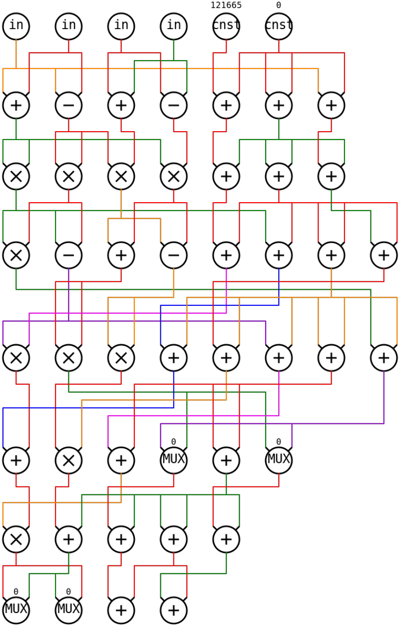

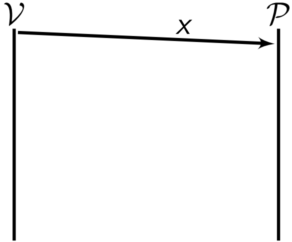

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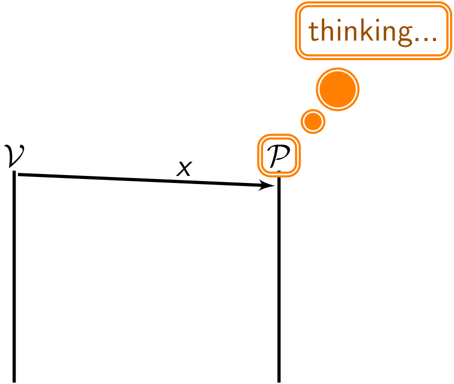

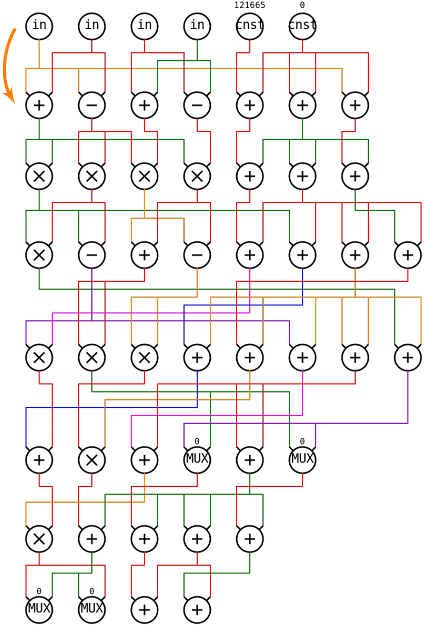

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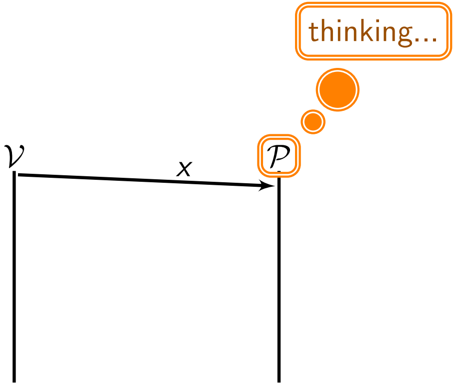

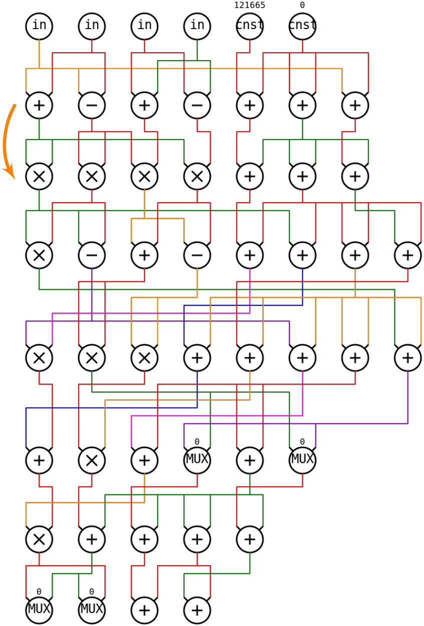

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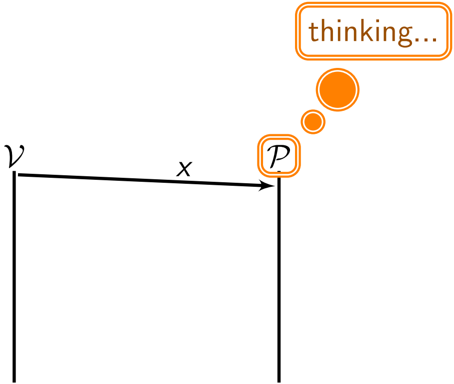

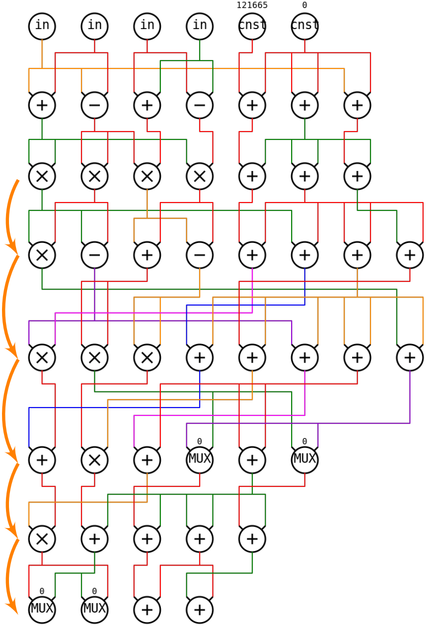

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1. $\mathcal{V}$ sends inputs
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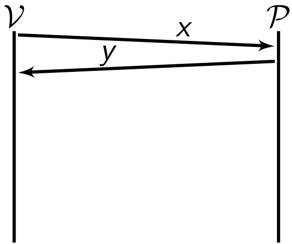

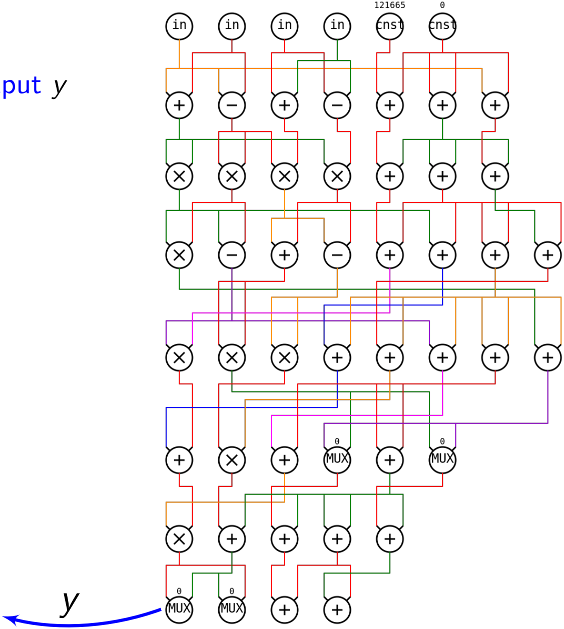

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1. $\mathcal{V}$ sends inputs
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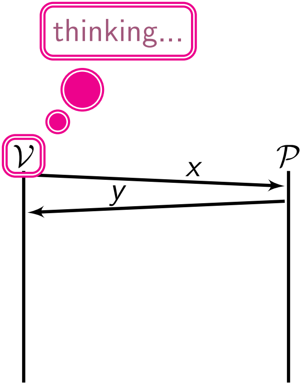
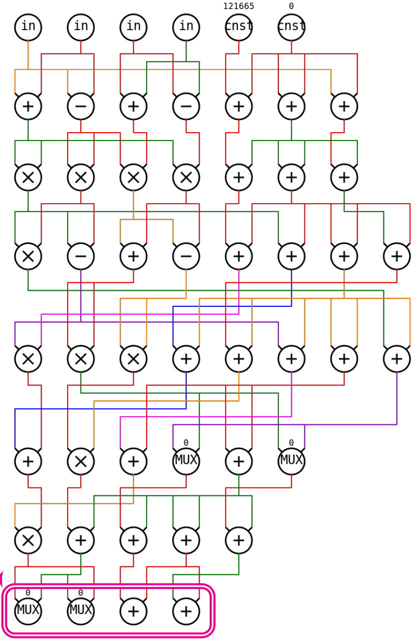
3. $\mathcal{V}$ constructs polynomial relating y to last layer's input wires





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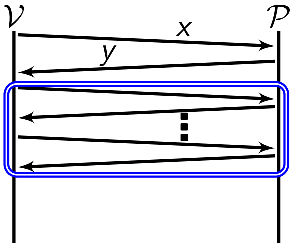
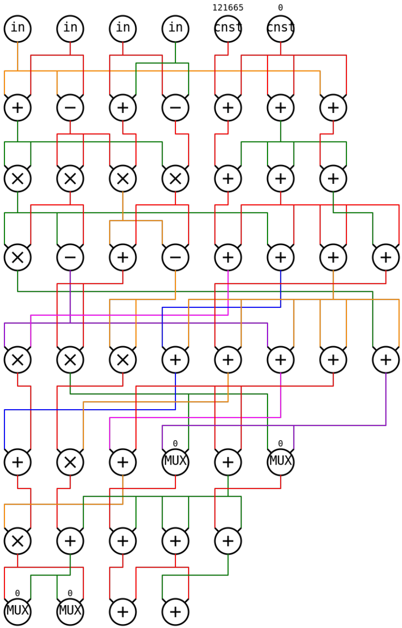


sum-check [LFKN90]



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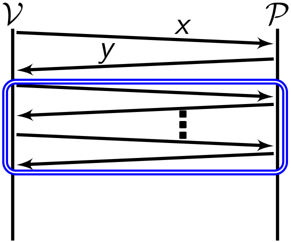

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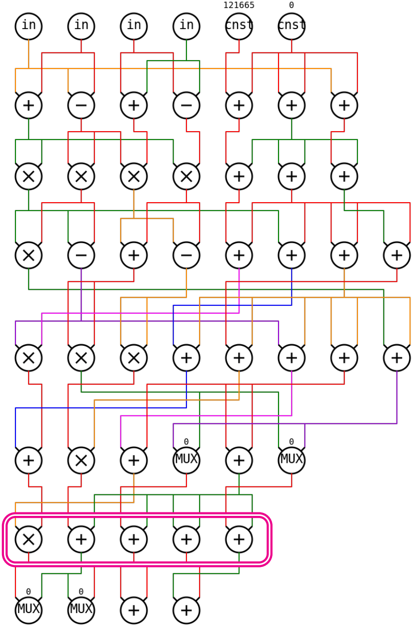

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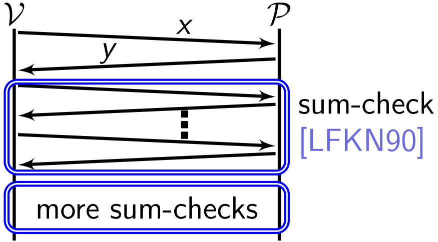

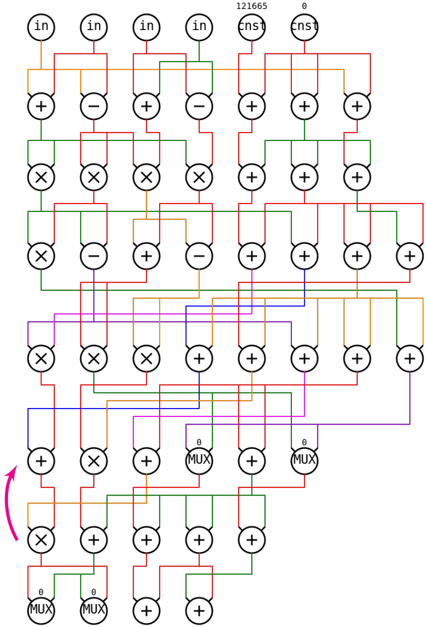

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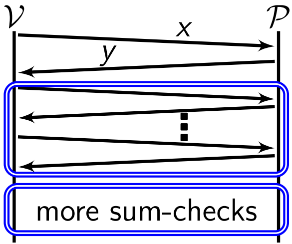

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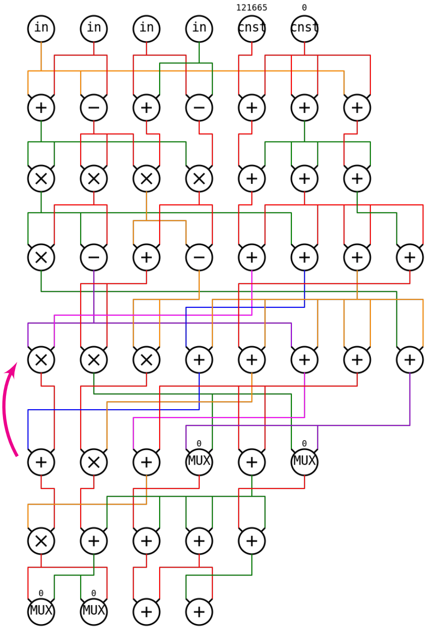

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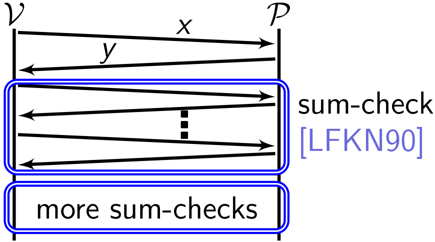

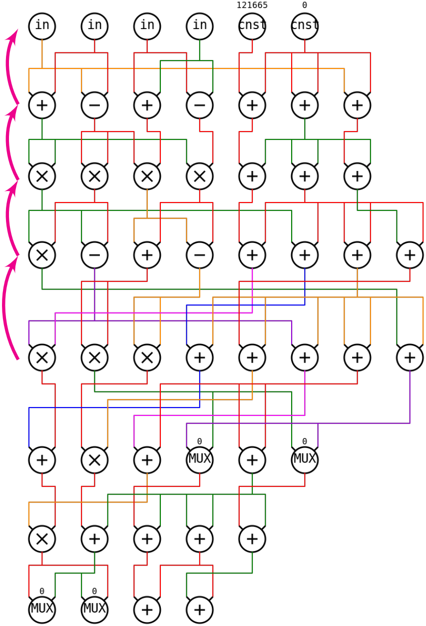

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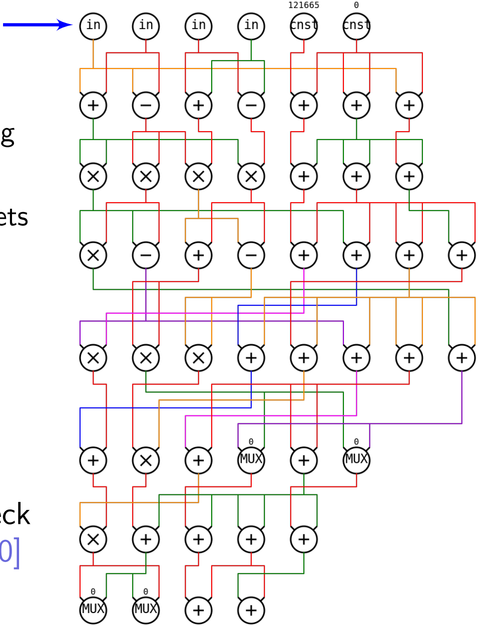

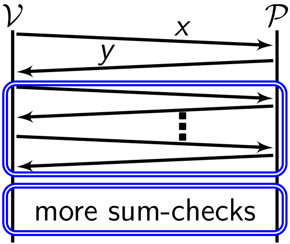

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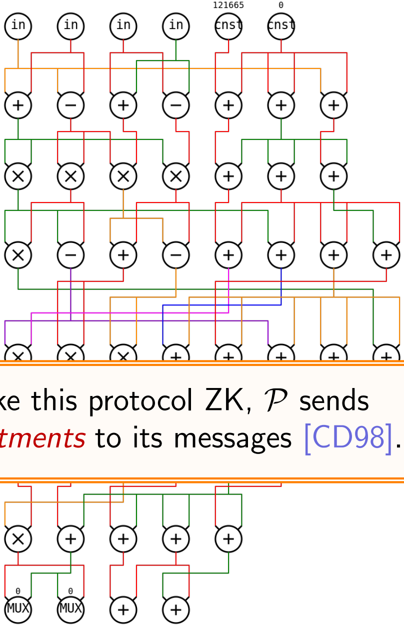
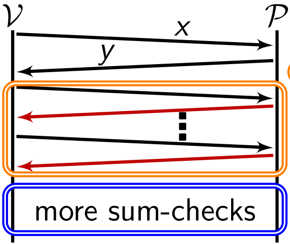


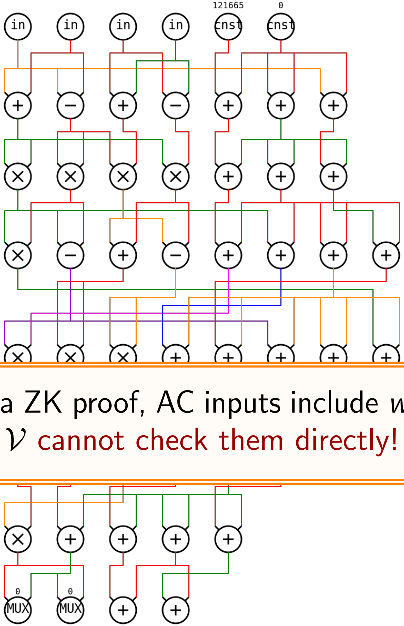
To make this protocol ZK, $\mathcal{P}$ sends *commitments* to its messages [CD98].



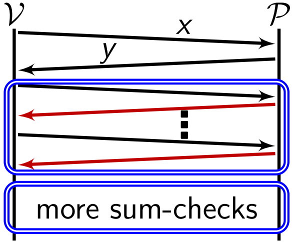
sum-check [LFKN90]

GKR08: IP for arithmetic circuit evaluation (with ZK)

1. $\mathcal{V}$ sends inputs
2. $\mathcal{P}$ evaluates, returns output y
3. $\mathcal{V}$ constructs polynomial relating y to last layer's input wires
4. $\mathcal{V}$ engages $\mathcal{P}$ in a sum-check, gets claim about second-last layer
5. $\mathcal{V}$ iterates, gets claim about inputs, which it can check



In a ZK proof, AC inputs include w, so $\mathcal{V}$ cannot check them directly!



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Hyrax uses a new polynomial commitment scheme tailored to *multilinear** polynomials like $\widetilde{m}$

*multivariate, linear in each variable

A polynomial commitment for $\widetilde{m}$

$$\widetilde{m}(r) \triangleq L \cdot T \cdot R^T$$

$\mathcal{V}$ can compute L and R from r, and

$$T \triangleq \begin{bmatrix} w_0 & w_\ell & \cdots & w_{\ell^2-\ell} \\ w_1 & w_{\ell+1} & \cdots & w_{\ell^2-\ell+1} \\ \vdots & \vdots & \ddots & \vdots \\ w_{\ell-1} & w_{2\cdot\ell-1} & \cdots & w_{\ell^2-1} \end{bmatrix}$$

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✗ Proof size and $\mathcal{V}$ time are both O($|w|$)!

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Better: $\mathcal{P}$ sends a *multi-commitment* to each row:

$T_0 = \mathrm{Com}(w_0, w_\ell, \ldots, w_{\ell^2-\ell})$ [*Gro*09]

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Pedersen commitments: vector-wise homomorphism.

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1. $\mathcal{V}$ uses homomorphism to compute $\mathrm{Com}(L \cdot T)$.
2. $\mathcal{P}$ sends a commitment to an evaluation of $\widetilde{m}(r)$
3. $\mathcal{P}$ uses a *dot-product argument* to convince $\mathcal{V}$ that $\mathrm{Com}(\widetilde{m}(r))$ is consistent with R and $\mathrm{Com}(L \cdot T)$.

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Details and refinements (see paper)

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Gir^{++} IP: Giraffe [WJBsTWW17] plus a tweak [CFS17]
➔ reduces proof size

Roadmap

1. General-purpose ZK proof systems
2. Hyrax at a high level
3. Evaluation

Evaluation overview

Baselines:

- ◄ BCCGP-sqrt [BCCGP16]—re-implemented
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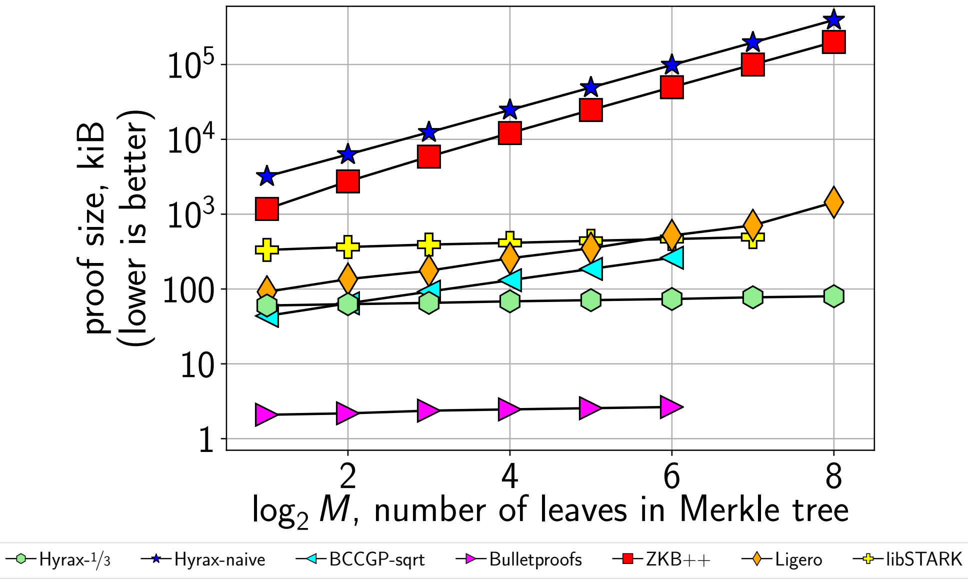
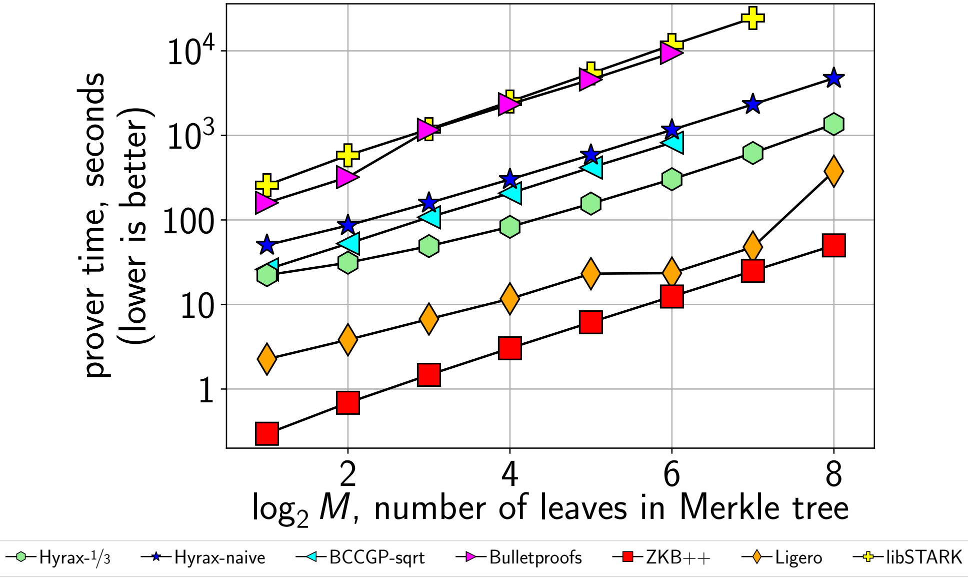
Benchmark: SHA-256 Merkle tree,
varying number of leaves

Proof size

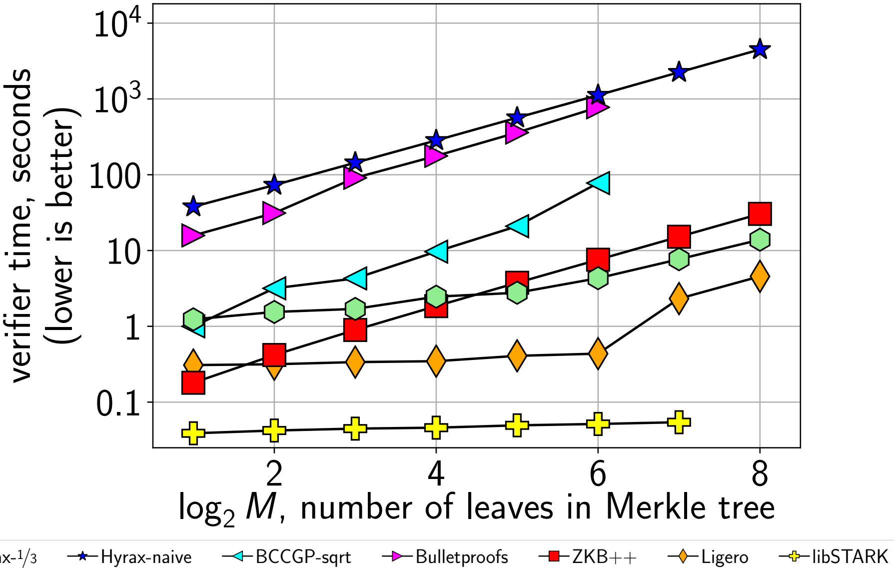


proof size, kiB
(lower is better)
10^5
10^4
10^3
100
10
1
2
4
6
8
$\log_2 M$, number of leaves in Merkle tree
Hyrax-1/3
Hyrax-naive
BCCGP-sqrt
Bulletproofs
ZKB++
Ligero
libSTARK

$\mathcal{P}$ time



$\mathcal{V}$ time



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`https://hyrax.crypto.fyi`

`https://github.com/hyraxZK`