A proof by Rutger M. Dijkstra and me.

Theorem. Consider the grid points, i.e. the points (x,y) with integer coordinates x and y, and let each grid point be painted with one of C distinct colours. For each X and Y, X distinct vertical grid lines and Y distinct horizontal grid lines exist such that their $X \cdot Y$ points of intersection have all the same colour.

Proof. Consider, for some k, the "strip" of points (x,y) with $0 \leq x < k$. In this strip the number of distinct possible colour patterns for a row is bounded (by C^k, to be precise). The number of rows in the strip being unbounded, at least one colour pattern occurs therefore in at least Y distinct rows. By choosing k larger than $C \cdot (X-1)$ we ensure that, in each and hence in this colour pattern, at least one colour occurs at least X times.

Acknowledgement. After having proved the theorem for $X = Y = C = 2$ —that was the problem as originally posed— my younger son, Rutger, removed during our discussion of the generalizations the last case analysis from the argument.

Plataanstraat 5
5671 AL NUENEN
the Netherlands

23 November 1980
prof. dr. Edsger W. Dijkstra
Burroughs Research Fellow