Linearization of a two-dimensional search

Let the integer pair (x_i, y_i) be denoted by P_i. Let of a finite number of such pairs all x_i be distinct and let all y_i be distinct. Let the pairs be partially ordered by $P_i < P_j$, defined by

$$P_i < P_j \;=\; x_i < x_j \wedge y_i < y_j$$

P_j is a minimal element means $\neg(\underline{E}\, i :: P_i < P_j)$, and it is requested to find the minimal elements. We rewrite the condition of minimality for P_j

$$\begin{aligned} & \neg(\underline{E}\, i :: P_i < P_j) \\ = \; & \neg(\underline{E}\, i : x_i < x_j : y_i < y_j) \\ = \; & (\underline{A}\, i : x_i < x_j : y_i \geq y_j) \end{aligned}$$

Under the assumption that the pairs have been numbered in the order of increasing x, the above reduces to

$$(\underline{A}\, i : i < j : y_i \geq y_j) \text{ or } (\underline{A}\, i : i < j : y_i > y_j) .$$

In other words: scanning the y's in the order of increasing x yields a new minimal pair each time we encounter a y smaller than we have encountered so far.

Since sorting can be done in $N \cdot \log N$ steps, we have an $N \cdot \log N$ algorithm. The solution is worth noting because I could not achieve that result —to my great regret— without destroying the symmetry between the x's and the y's. The above was triggered by a reconsideration of the longest upsequence problem

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