On a problem posed by M.R. Khalil (for the record)

For some positive integers N and L it is asked to generate -print- in lexical order the integer sequences $f(i: 0 \leq i < N)$ satisfying

(0) $(\underline{A}\, i: 0 \leq i < N: 0 \leq f.i < L)$

(1) f is ascending, i.e. $(\underline{A}\, i,j: 0 \leq i < j < N: f.i \leq f.j)$

An f-sequence containing n different values $(1 \leq n \leq L)$ can be represented by two sequences of length n, v (of values) and fr (of frequencies) satisfying

(2) $(\underline{A}\, i: 1 \leq i \leq n: 0 \leq v.i < L)$

(3) v is ascending

(4) $(\underline{A}\, i: 1 \leq i \leq n: 1 \leq fr.i)$

(5) $(\underline{S}\, i: 1 \leq i \leq n: fr.i) = N$

and coupled to f by

(6) $(\underline{A}\, i: 1 \leq i \leq n: (\underline{E}\, j :: (\underline{A}\, k: j \leq k < j + fr.i : f.k = v.i)))$.

Since $n \leq \min(N, L)$, n is at most N and may be much smaller. From given v- and fr-sequences, the corresponding f-sequence is generated by the following (trivial) block

```
"print f": |[ var i : int; i:=0
          ; do i≠n → |[ var j: int; j:=0; i:= i+1
                      ; do j≠fr.i → print(v.i); j:=j+1 od ]|
            od
          ]|
```

The program is

```
|[ var n: int; var v, fr: array of int
; n:=1 ; v.1 := 0 ; fr.1 := N
; do n≠0 →
    |[ var tv, tfr: int ; "print f"
    ; if v.n < L-1 → tfr := 1
      [] v.n = L-1 → tfr := fr.n + 1 ; n := n-1
      fi {n>0 ⇒ v.n < L-1 ; (Si: 1≤i≤n: fr.i) + tfr = N+1}
    ; if n=0 → skip {successor construction aborted}
      [] n>0 → tv := v.n + 1 {tv < L}
              ; if fr.n > 1 → fr.n := fr.n-1 ; n := n+1
                [] fr.n = 1 → skip
                fi ; v.n := tv ; fr.n := tfr
    fi
    ]|
  od
]|
```

The point of the (v,fr)-representation is that the lexical successor can be constructed without repetition. The point of the above program is the separate successive determinations of the "top frequency" and the "top value".

Nuenen, 15 August 1985

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