Suppose they differ!

The following quotation is from the chapter on Logic (!), p.26, "Concepts in Discrete Mathematics" by Sartaj Sahni. (1985)

> **Example 1.15:** The utility of proof methods M1 - M6 is not limited to wffs. We may show that the identity and zero elements of Boolean algebra are unique by providing a proof by contradiction. Suppose that the identity element is not unique. Then, there exist two distinct identity elements 1_1 and 1_2. From postulate 4, it follows that $a \cdot 1_1 = a$ and $a \cdot 1_2 = a$ for every element a in K. Substituting 1_2 for a in the first equality and 1_1 for a in the second, we obtain $1_2 \cdot 1_1 = 1_2$ and $1_1 \cdot 1_2 = 1_1$. But, $1_2 \cdot 1_1 = 1_1 \cdot 1_2$ (postulate 2). So, $1_1 = 1_2$ which contradicts the assumption that 1_1 and 1_2 are different. The uniqueness of the zero element now follows by duality. □

Compare the above with

$$
\begin{array}{ll}
 & (x \text{ and } y \text{ are both identity element}) \\
= & \quad \{\text{postulate 4}\} \\
 & (\underline{A}\, a :: a \cdot x = a) \wedge (\underline{A}\, b :: b \cdot y = b) \\
\Rightarrow & \quad \{\text{instantiate with } a := y \text{ and } b := x\} \\
 & y \cdot x = y \wedge x \cdot y = x \\
\Rightarrow & \quad \{\text{Leibniz (i.e. substituting equals for equals)}\} \\
 & x = y \equiv x \cdot y = y \cdot x \\
= & \quad \{\text{postulate 2}\} \\
 & x = y
\end{array}
$$

and you see that Sahni's assumption of non-uniqueness is entirely spurious: it is only used to construct a contradiction after $1_1 = 1_2$ has already been established! In short: Mr. Sahni's mathematical fingernails are dirty.

Mr. Sahni is by no means the only one with this specific dirt under his fingernails. On the contrary, it can be observed so frequently that we can only conclude that the average mathematician has not been taught to care.

There is little else to say: the obvious nonsense is too well known. I only wanted to give my students a glaring example of it; hence this little note. (To their credit I should mention that they lent me a copy as prime example of a terrible book.)

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