Addendum to EWD 969

(To be inserted after "(End of Proof 5.6)" at EWD 969 - 16.)

<u>Remark</u> Theorem 5.6 can be extended. Let $f.Y$ be the strongest solution of

$$X: [p.(X,Y) \equiv X]$$

i.e. our formal knowledge about f is captured by

(20') $\quad [p.(f.Y, Y) \equiv f.Y]$

(21') $\quad [p.(X,Y) \Rightarrow X] \Rightarrow [f.Y \Rightarrow X]$.

Then, analogous to the conclusion $[g.P \equiv Q]$, Theorem 5.6 allows us to conclude $[f.Q \equiv P]$, i.e. (P,Q) is a solution of

$$(X,Y): [(f.Y, g.X) \equiv (X,Y)] \quad .$$

We shall now show that (P,Q) is the strongest solution of the above equation, i.e. we shall show

$$[(f.Y, g.X) \equiv (X,Y)] \Rightarrow [(P,Q) \Rightarrow (X,Y)] \quad .$$

To this end we observe for any X, Y satisfying the antecedent

$\quad [(P,Q) \Rightarrow (X,Y)]$
$\Leftarrow \quad \{(19)\}$
$\quad [p.(X,Y) \Rightarrow X] \wedge [q.(X,Y) \Rightarrow Y]$
$= \quad$ { the antecedent equivales $[f.Y \equiv X] \wedge [g.X \equiv Y]$ }
$\quad [p.(f.Y) \Rightarrow f.Y] \wedge [q.(X, g.X) \Rightarrow g.X]$
$= \quad$ { (20) and (20') }
$\quad$ true .

The next theorem will show that above f and g are monotonic. The theorem of Knaster-Tarski is therefore applicable and we have therefore that (P,Q) is also the strongest solution of

$$(X,Y):\ [(f.Y,\ g.X) \Rightarrow (X,Y)] \quad .$$

Note how in the above we have proved the weaker implication, i.e. the one with the stronger antecedent: the middle step in the above proof makes essential use of the equivalence. (End of Remark.)

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