Automated Reasoning:

Essays in Honor of Woody Bledsoe

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Kluwer Academic Publishers Group

Dordrecht, The Netherlands

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Preface

These essays have been written to honor W. W. Bledsoe, a scientist who has contributed to such diverse fields as mathematics, systems analysis, pattern recognition, biology, artificial intelligence, and automated reasoning. The first essay provides a sketch of his life, emphasizing his scientific contributions. The diversity of the fields to which Bledsoe has contributed is reflected in the range of the other essays, which are original scientific contributions by some of his many friends and colleagues. Bledsoe is a founding father of the field of automated reasoning, and a majority of the essays are on that topic. These essays are collected together here not only to acknowledge Bledsoe's manifold and substantial scientific contributions but also to express our appreciation for the great care and energy that he has devoted to nurturing many of the scientists working in those scientific fields he has helped found.

Robert S. Boyer
Austin

February, 1991

Acknowledgements

Thanks to Larry Wos, editor of the *Journal of Automated Reasoning*, and Derek Middleton and Martin Scrivener, Kluwer Academic editors, for supporting the idea of initiating this collection of essays. Thanks to A. Michael Ballantyne and Michael Spivak, for help with LaTeX, especially in identifying many formatting problems and providing fixes.

Thanks to the following individuals for help in obtaining financial support of the Bledsoe Symposium (planned for November 15 and 16, 1991); S. Kamal Abdali, from the National Science Foundation (NSF Grant CCR-9102868); Alfred Dale, from the Computer Sciences Department of the University of Texas at Austin; Alfred King, from the King Foundation; Vesko Marinov, from Ricoh; Grant Dove, from MCC; Donald Good, from Computational Logic; and Neil Iscoe, from EDS.

We are grateful for research support under National Science Foundation Grant MIP-9017499.

The views and conclusions expressed in this volume are those of the authors and should not be interpreted as necessarily representing the official policies, either expressed or implied, of the National Science Foundation, the United States government, or any of the corporate sponsors of the Bledsoe Symposium.

R. S. B.

Chapter 1

A Biographical Sketch of W. W. Bledsoe

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1.1 Introduction

W. W. Bledsoe is a major figure in the evolution of the new scientific field *artificial intelligence* and one of the founding fathers of the related scientific field *automated reasoning*. Into the following biographical sketch of Bledsoe we weave personal, educational, historical, social, and scientific commentary. At the time we write, Bledsoe is an active contributor to science and education at the University of Texas at Austin. We hope that our fondness for Bledsoe, whom we have known well for twenty-three years, has not clouded our assessment of his many achievements. We are certain that we have failed to treat adequately many aspects of Bledsoe's life prior to our first meeting him in 1966, and sadly fear that lack of space and lack of investigative effort cause us to omit quite a few interesting aspects of his career since then. We hope, however, that this short sketch of Bledsoe will please his friends and perhaps provide some useful information for a future biographer or historian of science.

1.2 Early Days

Woodrow Wilson ("Woody") Bledsoe was born November 12, 1921, in Maysville, Oklahoma. He was named for the then recent president of the United States, whose policies and idealism his parents greatly admired. At the time of Woody's birth, the Bledsoe clan had swelled to a considerable size. His father Thomas brought six children with him when he married his second wife, Eva Matthews, and together they had six more children. Woody was Eva's fourth child. The others were Pickens, Mary Ellen, Sula, Charles, and Tom.

It was not an easy childhood. Thomas Bledsoe had owned a plantation in Alabama, but a hired hand upset some turpentine one evening, a fire broke out, and the farm was lost. After that, it was moving from place to place, sharecropping. Eventually, the family settled in Oklahoma. Like so many other farmers in Oklahoma at that time, Thomas found that trying to wrest a living from the land was exhausting and hardly profitable.

The children, however, were happy living in the country and scarcely aware that they lived in poverty. Most unfortunately for the family, Thomas died in 1934. It was a blow from which he never recovered, says Woody, who was only twelve at the time. The Bledsoes were obliged to move into town (Lexington, Oklahoma), and the children were forced to confront their poverty. Now they knew they were, indeed, very poor.

"We scrambled," says Woody, who mowed lawns and delivered papers and did any honorable work to keep body and soul together. His mother was managing a small dress shop, but there were few women who could afford a new dress in those days, and sales were few. Woody worked for about a dollar a day. At one time he worked for a restaurant that provided employees free pies, and Woody once found himself covered with boils after eating several pecan pies in a single day.

The Depression. Hard times all around. Elsewhere in Oklahoma, people were packing up and fleeing the Dust Bowl. Everything seemed to conspire against even the most hardworking folks. Famous folksinger Woody Guthrie, also from Oklahoma, must have had kids like Woody Bledsoe in mind when he sang his now classic songs about the grit and gumption of the poor people in Oklahoma then. Woody Bledsoe might have found the going rough, but he did not cave in to adversity, and discovered that even in poverty there are joys to be savored in life. He did very well in school, and distinguished himself in mathematics, especially under the guidance of teachers Ellen Sherman and Lois Peyton. He also proved to be extremely adept at tennis, which he continues to play enthusiastically. In his high school years, Woody ran away from home several times, catching rides by hitchhiking or riding freight trains throughout Texas and Oklahoma—a common enough method of transportation during the Depression. In his senior year, to spare his mother some of the burden of supporting so many children, he left home again. He traveled to Calliham in South Texas, arriving unannounced at the door of an acquaintance (Lon Claunch), with whom he spent a year, graduating from Calliham High School in 1939.

The next year held poverty and uncertainty. Woody moved back to Oklahoma and spent most of the next year washing dishes in a restaurant. Hoping to break free from drudgery, working part time to support himself, he entered the University of Oklahoma, but the effort to support himself proved too much even for Woody's apparently boundless energy, and he dropped out to join the army on May 27, 1940.

Life in the army was not much of an improvement, at least in terms of personal fulfillment. As Bledsoe puts it, "Soldiers of the old army were shunned by civilians," and he felt intensely lonely. World War II, however, changed all that, when so many of the young men in the nation received a summons from the government to join the fight. Now it was "okay," even admirable, to be in the armed forces. Bledsoe attended Officers Candidacy School in 1942, was commissioned in the Corps of Engineers, and eventually achieved the rank of captain.

In July, 1943, one of his buddies, Richard Norgaard, in the Corps of Engineers persuaded him to come home with him on leave to Salt Lake City. As it happened, Richard had a pretty sister, Virginia, and as both recall, it was love at first sight. Inspired by her beauty, the usually circumspect Bledsoe was even moved to write a poem in her praise. They corresponded, Bledsoe called her by telephone to propose marriage, and on January 29, 1944, the couple was married in her home.

Eventually, four children were born to them: Margaret, Greg, Pamela, and Lance. Margaret died shortly after birth, in a dysentery epidemic during World War II, while Bledsoe was in Europe. Incredibly, fifty-six of the ninety-three babies born in the hospital during the ten days of Virginia's confinement died because of a shortage of penicillin and skilled medical help due to the war effort. Many years later, in 1985, Greg was killed in an automobile accident. Greg's death was the most severe blow Woody and Virgina have ever sustained. Greg left three children. Pamela, who has a daughter, is a traffic consultant. Lance, who has four children, has followed his father's footsteps into the computer industry.

Woody and Virginia have shared their joys and struggles together for nearly fifty years now, and both attribute much of their spiritual strength and growth to their commitment to the Church of Jesus Christ of the Latter Day Saints (Mormons). Woody was not a Mormon when he married Virginia, but he sat by her side in church many Sundays for eighteen years, read *The Book of Mormon*, and so was eventually converted, on September 21, 1961. He remains very active in the church to this day; he has served as a bishop twice (altogether about four years) and once as a counselor of the stake presidency. (In terms of Roman Catholic organization, the position of Mormon bishop is roughly equivalent to head of a parish and a stake presidency is roughly equivalent to head of a small diocese.)

In August 1944, Bledsoe participated in the war in Europe as a member of Patton's Third Army, in the Army Corps of Engineers. Bledsoe received the Bronze Star medal for his heroic activities in arranging the transportation of troops across the Rhine in March, 1945. At that time, all the Rhine bridges except the one at Remagen had been destroyed by the retreating German army. Patton's Third Army decided to cross the Rhine by boats near Frankfurt rather than suffer the delay of waiting for bridge construction. Therefore the Army Corps of Engineers hauled naval landing craft (designed for beach landings) by truck across Europe to ferry troops across the Rhine. Bledsoe, by then an Army captain, recalls that there was only light enemy fire during the crossing; his main contribution was figuring out how to get the very large landing craft down narrow roads and actually into the water. His first "research" was experimenting with techniques for launching the craft from trucks into the Mosel River; initial experiments were disastrous, but ingenuity prevailed. The simple idea of backing the trucks into the water, floating off the boats, and hauling the trucks back out with tractors turned out to be the key. While hauling trucks down to the Rhine, Bledsoe temporarily confused some allied forces about the location of German artillery by dynamiting away a railroad bridge that created an overhead blockage against the passing of the landing craft. Near the Rhine, Bledsoe took his orders from Patton's headquarters in Kaiserslautern, Germany. (Kaiserslautern is now a major center of work in artificial intelligence, under the direction of Bledsoe's longtime colleague Michael Richter. At the Tenth Conference on Automated Deduction, held in Kaiserslautern in 1990, Bledsoe delivered the banquet address in the nearby Hambacher Schloß, site of one of the earliest German democratic movements.) Bledsoe remained in Europe until October, 1945.

Upon returning to the United States, Bledsoe enrolled as an undergraduate at the University of Utah in Salt Lake City, Utah. After considering electrical engineering and physics, Bledsoe chose to major in mathematics. Perhaps the decisive point in Bledsoe's decision to become a mathematician was his discovery on his own of a proof of the nontrivial theorem that every real-valued continuous function defined on the reals has a Riemann integral over any closed interval. Bledsoe proved this theorem in a course taught by Ferdinand Biesele in the Socratic or Moore style, wherein the students find and present proofs in class rather than listen to lectures. As a mathematics professor, Bledsoe himself would emphasize to his students the importance of finding for themselves proofs of hard theorems. His research achievements have been to a significant extent in the area of automating proof discovery.

1.3 Berkeley

After graduation from Utah, Bledsoe pursued a Ph. D. in mathematics at the University of California at Berkeley under a full fellowship. Berkeley was then (and is still) a dominant force in American mathematics. Among the many renowned faculty members there from whom Bledsoe took classes were the logician A. Tarski and the topologist J. Kelley. Bledsoe did his Ph. D. thesis under the analyst A. P. Morse, about whom we briefly digress.

1.3.1 A. P. Morse

Bledsoe's advisor A. P. Morse believed in the viability and desirability of being absolutely formal in the statement of mathematical theorems. In his book on set theory [58], Morse formally develops in about a hundred pages a flexible syntax akin to ordinary mathematical notation, predicate logic, set theory (including ordinal and cardinal arithmetic), and elementary topology. Remarkably, each theorem is stated in the formal logic. Considering the relatively elementary character of the theorems established in *Principia Mathematica*, we suspect that Morse's book stands as the most mathematically advanced but totally formal treatise on mathematics ever published. Bledsoe's work was heavily influenced by Morse's work in several ways, and thus we elaborate on what we regard as some of Morse's contributions to formal reasoning.

- Morse provides a logic in which mathematics can be conveniently expressed via an extremely flexible and extensible syntax and via a very sturdy and powerful definition principle. (Most formal logics, e.g., that of [61], are intended to be *studied* rather than *used* by people. As a result, their grammars tend to be unnatural because they do not support common notatation. Worse, from the practical point of view, these logics fail to have principles of definition, especially for the definition of concepts introducing new kinds of quantification such as the set builder notation or the definite integral. Rather, definition is regarded as an extralogical, abbreviational, inessential, and extraneous activity.)
- Whenever possible, Morse formulates theorems that are mechanical in spirit, like the equations for ordinary algebraic operations. Morse goes out of his way, for example, to define his concepts in such a way that theorems can be stated as equations, without hypotheses.

Thus it is not surprising that it was a student of Morse (namely Bledsoe) who produced the first powerful prover for set theory, a prover that incorporated a rich conceptual and definitional facility and that derived much of its power from equational set theory theorems.

After he graduated from Berkeley, Bledsoe continued an extensive collaboration and correspondence with A. P. Morse. In 1981, during a meeting of the

American Mathematical Society in San Francisco, Bledsoe organized a dinner held in honor of A. P. Morse; attendees included not only many of Morse's Ph. D. students but also many distinguished members of the mathematics department at the University of California at Berkeley, including A. Tarski and J. Kelley.

1.4 Mathematical Contributions

Bledsoe regards mathematics as his first and greatest academic love, over and above pattern recognition, artificial intelligence, systems analysis, and automated reasoning. Bledsoe has made a number of contributions to the mathematical literature, including these publications in regular mathematics journals: [5, 6, 7, 9, 17, 27, 30, 32].

We now briefly review Bledsoe's mathematical work. Our review is literal in part because the authors of this biography lack the mathematical background to assess the work properly. Why then do we review it at all? We review it in order to emphasize the point that Bledsoe is a "real mathematician." We believe this point deserves emphasis as a lesson to potential contributors to automated reasoning. Most of the most important contributors to automated reasoning had already become serious mathematicians before beginning research on automated reasoning, e.g., Martin Davis, Hao Wang, Bledsoe, Don Knuth, Wu Wen-Tsün, and Larry Wos. We suspect that this is because substantial mathematical ability is required to contribute to this field.

As a graduate student, before finishing his Ph. D. thesis, Bledsoe published two papers [5, 6]. The first of these [5] presents the following charming result:

> In the following ρ metrizes S and ρ' metrizes S'. We agree that a function f is *neighborly at the point* x if and only if for each $\epsilon > 0$ there exists an open sphere α of S such that $\rho(x, y) + \rho'(f(x), f(y)) \leq \epsilon$ whenever $y \in \alpha$. We also agree that a function is *neighborly* if and only if it is neighborly at each point of S. Obviously every continuous function is neighborly.
>
> It is well known that if g is a function on S to S' and if f is such a sequence of continuous functions that $\lim_{n\to\infty} \rho'(f_n(x), g(x)) = 0$ for each x in S, then the points of discontinuity of g form a set of first ρ category. It is the principal purpose of the present note to show that this same conclusion can be drawn when the approximating functions are merely restricted to being neighborly.

Bledsoe's Ph. D. dissertation, written under the supervision of A. P. Morse, is reported in [9]. The topic was "product measures." Concerning the general problem of product measures, Bledsoe remarks in [30]

> It has been known for many years that the product of two regular borel measures on compact hausdorff topological spaces may not be borel in the product topology. The problem of defining a new product measure that extends the classical product measure and carries over this borel property has been approached in different ways ...

Among the approaches is that presented in [9]. In summarizing this work, Bledsoe and Morse remark

> One of our aims is to free topological Fubini theory from the usual restraints of local compactness. In this connection we shall regard Radon measures as very special indeed. Another of our aims is to associate with any two measures μ and ν such a product measure ϕ that in the event μ and ν *happen* to be topological measures of a rather general sort, then our topology-free measure ϕ will actually be a similar measure under which the Borel subsets of the product space are measurable. We shall reach both these goals even though an obvious obstacle in our path to the second is the fact that the open sets of a topological product are not always contained in the σ-field generated by the open rectangles. The product measure we construct in order to overcome this difficulty allows more summable functions and greater freedom of action than the product measures considered heretofore. ...
>
> Suppose μ is an (outer) measure defined for all the subsets of a topological space $\mathcal{S}$. Suppose that $\mu(\mathcal{S}) < \infty$ and that each open set is not only measurable but equal in measure to the upper bound of the measure of closed subsets. Suppose further that from each covering of $\mathcal{S}$ by open sets a countable subfamily can be extracted which covers almost all of $\mathcal{S}$. If ν is another measure like μ, then our associated product measure bears to the topological product space the same relation as μ does to $\mathcal{S}$, and at the same time satisfies the Fubini equality for summable functions. The novel feature of our product measure is that we require to be of measure zero each set whose characteristic function integrates iteratively in both orders to zero.

In some cases, Bledsoe and Morse adopt a language, style, and formality that is quintessentially Morsean in its formality, precision, and succinctness, e.g., the following definition of Mspr:

> Mspr $\mu\nu\mathcal{F}$ = E$\phi \in$ Msr rct rlm μ rlm ν [$\mu \in$ Measure, $\nu \in$ Measure, $\mathcal{F} \in$ bsc ϕ, and, for each $a \in \mathcal{F}$,
>
> $$\int\int \mathrm{Cr}(x,y)A\mu\, dx\, v\, dy = \phi(A) = \int\int \mathrm{Cr}(x,y)A\nu\, dy\, \mu\, dx]$$

Bledsoe continued to pursue his investigations on product measures in the 70's. In [30] he and Wilks extend his Ph. D. dissertation work [9] by considering the case in which one of the two measure spaces is borel and the other is constrained by a separation of variables condition.

In [17] Bledsoe, writing again with Morse, generalizes two well-known results from the theory of metric spaces to topological measure theory. The principal result concerns the construction of a measure from a nonnegative set function.

Also, a short note by Bledsoe appears in the Monthly [27] proving that

> Every finite set $\{z_1, z_2, \ldots, z_n\}$ of complex numbers has a subset S such that
>
> $$\left|\sum_{z \in S} z\right| \geq \frac{1}{\pi} \sum_{j=1}^{n} |z_j| .$$

The two well-known mathematicians R. H. Bing and R. D. Mauldin collaborated with Bledsoe in [32] on a collection of set theory theorems with connections to the continuum hypothesis.

1.5 Sandia

After finishing his Ph. D. in mathematics at the University of California at Berkeley in 1953, Bledsoe decided to pursue research rather than take an academic post at Michigan or Virginia. He decided to work in the mathematics department at Sandia Corporation in Albuquerque, New Mexico, which did work almost exclusively on weapons for the Atomic Energy Commission (now the Department of Energy). Bledsoe was attracted both by the desire to focus his mathematical talent on questions in physics and by the desire to be involved in the then rapidly developing field of nuclear weapons. (The first hydrogen bomb was exploded shortly before Bledsoe joined Sandia.) Also influential in his decision to join Sandia was the fact that then, as now, the salary of mathematicians in industry was often an integer multiple of salaries in academia. Albuquerque became the Bledsoes' favorite town. At Sandia, Bledsoe worked in "systems analysis" in the mathematics department. His work there is almost entirely classified. Systems analysis is a term used to express engineering concerned with the interactions of components in a "system" that is itself composed of many other systems, e.g., a telephone system or a modern battlefield. (After Bledsoe left Sandia, an official there told him, in an attempt to entice him to return, that he was the best systems analyst that Sandia had ever had.) Concerning systems analysis, Bledsoe later remarks in [15]:

> In analyzing a system, we often are searching for a set of characteristics which will make the system most optimum in some

> sense. Frequently, this boils down to determining the "best" values of a set of parameters, and we find ourselves doing a *parameter study* in which we vary parametric values according to some policy while we search for an optimal value of a response function. Also, we often try to determine tradeoffs between the parameters.
>
> The system may include some random elements, such as a weather or enemy action, and often the response function we are trying to optimize is itself a random variable.
>
> Many such systems arise in the *war games* studied in weapons systems analysis. There the parameters varied are such things as aircraft velocity, yields of offensive weapons, yields and types of defensive weapons, spacing of vehicles, warning time, etc.

The one unclassified document that Bledsoe produced concerning his systems analysis work at Sandia is entitled "Program for Computing Probabilities of Fallout from a Large-Scale Thermonuclear Attack" [10]. The report presents a computer program for computing the distribution of fallout from nuclear weapons explosions. The program allows for variations in longitude, latitude, wind velocity, megatonage, and for such probabilities as one weapon destroying another. As a reminder of the difficulty of using early computers, we cannot resist mentioning a few archaic operating instructions necessary in those days:

- 1. Put the magnetic drum in a "four interlace" condition.
- 3. Put all weapons cards in READ hopper.
- 8. Start at 40100 (if MS1 is in "ON" position, machine halts with PAK 40104, so hit COMPUTE button).

An extensively commented program listing of about 1400 lines of code is included, with all of the instructions given in octal, and with no evidence of an assembler much less a compiler having been available for coding the computer, an ERA 1103.

By 1957, Bledsoe had become the head of the mathematics department at Sandia and remained so until his departure in 1960. He collaborated there frequently with his good friend Joe Weihe.

While working for Sandia, Bledsoe witnessed several atomic bomb explosions in Nevada. He also traveled to the South Pacific and witnessed a hydrogen bomb explosion (from the island of Eniwetock).

During his work for Sandia, Bledsoe recalls encounters with the extraordinary mathematicians S. Ulam and John von Neumann. Ulam showed him a hand drawing of the first hydrogen bomb design, for which the basic calculations were done on a hand calculator. The calculations could have been refined: the first hydrogen bomb explosion, the "Mike shot," was twice as powerful as expected, almost destroying some observers.

Once, when giving a lecture at Lawrence Livermore Labs on the possibility of using very small atomic "shells" in artillery, a person in the audience asked whether the calculation was "linear" in a certain sense. Bledsoe replied "No," at which point Herb York, the head of Livermore Labs said: "I agree with John von Neumann, I too believe it is linear." This was Bledsoe's introduction to von Neumann. Indeed, it was not "linear," but von Neumann's remark was close to true, something Bledsoe subsequently emphasized.

Toward the end of his stay at Sandia, Bledsoe and a very close friend, the biologist and genius polymath Iben Browning, began work on using computers to do recognition, first of characters and later of faces. Turning to work on what has become known as artificial intelligence was a revolutionary event in Bledsoe's intellectual life. Many years later, in his presidential address to the American Association of Artificial Intelligence in 1985, "I Had a Dream" [42], Bledsoe recalls that after he had his first dream about artificial intelligence:

> When I awoke from this daydream, ... I decided then and there to quit my job and set about spending the rest of my life helping bring this dream to reality.

Bledsoe and Browning's initial contribution to artificial intelligence was presented in the paper "Pattern Recognition and Reading by Machine" [11]. Although the principal topic at hand was the recognition of characters, the entire topic of pattern recognition was clearly imagined. Distinguishing the planned work from previous work by others, Bledsoe and Browning remark:

> All of these approaches prove upon inspection to center upon analysis of the specific characteristics of patterns into parts, followed by a synthesis of the whole from the parts. In these studies, pattern recognition of the whole, that is, Gestalt recognition, was chosen as a more fruitful avenue of approach and as a satisfactory problem for the initial phases of the over-all study.

In simplified form, the Bledsoe-Browning algorithm for recognizing characters is based upon the following idea. We imagine that we wish to recognize single characters that are presented as bit patterns in a fixed rectangular array of bits (i.e., pixels). Initially, we learn some characters. Later we wish to recognize some. When presented with a character to recognize, we will independently compute a "score" for the possibility that the character might be an A, B, or C, etc., and choose the possibility with the highest score. How do we score? Even before we have begun the learning process, we randomly divide the rectangular array into *pairs* of pixels, $p_1, p_2, p_3 \ldots$ During the learning phase, whenever we are asked to learn a given pattern as an instance of a specific letter, say A, we note for each pixel pair p_i the *state* of that pair: (0,0), (0,1), (1,0), or (1,1), depending on the states (on or off) of the individual pixels. Later when we are trying to recognize a character, we score one point towards the possibility that the character is an A for each i such that the p_ith pixel pair has the same state as it did when learning at least one instance of A. Clearly, the algorithm can be generalized for tuples other than pairs, and in fact this method is sometimes known as the "n-tuple method" instead of the "Bledsoe-Browning method."

An analysis of Bledsoe and Browning's method may be found in Ullman's [66]. Ullman presents results of tests he performed showing the Bledsoe-Browning method obtaining recognition rates of about 93% on hand-printed numerals and of about 95% on multifont printed numerals, using n-tuples with $n = 14$, at the cost of about 42 million bits of storage. Related work by Ullman on the Bledsoe and Browning n-tuple method may be found in [64, 65]. The latter reduces fourfold what Ullman terms the "storage gluttony of the Bledsoe and Browning system."

Besides the basic n-tuple method, the paper [11] describes several additional techniques that were employed to increase the accuracy of character recognition. The most successful of these techniques was to finalize the identity of characters using context: "using a vocabulary of words of the length in question, add in their proper order the letter scores of each word in the vocabulary to obtain a total score for each word."

Several themes that were to influence the remainder of Bledsoe's work in artificial intelligence can already be seen here:

- Reliance on partial information for making informed guesses: heuristics.
- Reliance on a variety of partial techniques, working together, to solve a problem, rather than seeking a single "uniform" strategy for solving a problem.
- Asking the question "how can we make a machine do something like what people do?"—e.g., using contextual information.

1.6 Panoramic Research, Inc.

While working at Sandia, Bledsoe, Iben Browning, and Lloyd Lockingen received encouragement from Department of Defense sources, promising financial support to set up a small research firm to pursue topics related to pattern recognition. The three left Sandia in 1960 and established the company Panoramic Research in Palo Alto, California; Panoramic grew to a staff of about 10 people. Bledsoe was president of Panoramic from 1963 to 1965, before his departure for the University of Texas at Austin in January 1966.

Panoramic was a very early company to focus upon research in the general area of artificial intelligence, including pattern recognition, facial recognition, genetics, and neural nets. Frequent visitors to Panoramic included Marvin Minsky, Raj Reddy, John McCarthy, Hans Bremermann, Seymour Papert, and Danny Bobrow, many of whom were founding fathers of artificial intelligence.

While at Panoramic, Bledsoe contributed a short note [13] that describes how the Heisenberg uncertainty principle can be used to show limits on the speed of a single von Neumann computer. For example,

> Thus a computer with a billion bits of storage can have an access time of no less than 10^{-18} seconds, unless materials are used whose density is greater than 20 grams/cm^3.

Perhaps Bledsoe's major research project at Panoramic was on computer facial recognition. Working with Helen Chan, Charles Bisson, and others, Bledsoe produced a system that was remarkably successful at recognizing faces in photographs. The system worked as follows.

A. Database Setup. A human operator examines a photograph and abstracts from it certain facial features, all of which are distances between fixed points on the face, for example the distance from the top to the bottom of an ear. The entry of this data was facilitated by the use of a very early analog-to-digital data entry device known as the Grafacon or Rand Tablet. About forty photographs per hour could be processed in this way by a relatively new operator. Before storing the distances in the database, the distances were mechanically normalized to compensate for head position, e.g., rotation and tilt.

B. Identification. Given a new photograph that we wish to match against some of the photographs already entered into the database, we compute a "pseudo distance" between the new photograph and each of the previously entered photographs. The distance is computed by summing the differences of similar features, first dividing each difference by a standard deviation of measurement for that feature. Photograph pairs that are "close enough" are then presented to a human, who then finally decides which photographs really match.

In experiments involving a database of several thousand different photographs, the computer was able to reduce the number of photographs that needed to be considered by the human to one percent of the photographs in the database. Although one of the authors of this note (RSB) has seen a report on this excellent project, Bledsoe was not permitted to publish the work because of the sensitivity of the photographs used in the experiment! Many reports on this project were produced, including [18, 19, 22, 24].

Even after Bledsoe left Panoramic, he continued to collaborate on the topic of facial recognition with Peter Hart at Stanford Research Institute, as reported in [26]. In that report are described experiments in facial recognition using (a) slight modifications of the technique mentioned above, (b) using modifications of those techniques based on Bayesian decision theory, and (c) using humans to do the entire job of photograph identification. It was concluded that both methods (a) and (b) worked approximately as well as one another and both worked considerably better than (c), but that (b) performed somewhat better on high quality photographs than (a).

While at Panoramic, Bledsoe also made some mathematical analyses of genetic mutation [15, 14] and addressed a National Academy of Sciences meeting on the topic of "convergence rates in pseudo-evolution" in Austin in 1962. He was one of the first to study the evolution of genetic structures by computer. (To this day, Bledsoe is not yet aware of any reasonable computer simulation of genetic mutations that would predict the evolution of entire new species.) Through his work on genetics, Bledsoe also began working with Wilson Stone, a renowned biologist at the University of Texas at Austin.

Another research topic pursued at Panoramic was an algorithm that an aircraft could use to fly low over mountainous terrain so as to avoid detection [20, 21].

After several years of the constant press of fundraising and management incumbent on the president of a small research firm, Bledsoe decided to pursue an academic career. He was strongly encouraged to join the University of Texas by Wilson Stone, who offered him *carte blanche* choice of department and salary.

1.7 The University of Texas

Bledsoe's many contributions to automated reasoning all were made after his move to the University of Texas. We discuss these contributions in a subsequent section.

1.7.1 Teaching

From the beginning of his tenure at Texas, Bledsoe has been a teacher of considerable inspirational influence. In 1966, in the first course at the University of Texas on artificial intelligence, a Lisp compiler for the CDC 6600 was produced by students (including James B. Morris) as a class project. Especially memorable to one of the authors (RSB) was a class that Bledsoe taught in the fall of 1967 in the Moore style, but based on Morse's book on set theory [58]. Among those attending the class were Forrest Baskett (later professor of computer science at Stanford, who conceived the Sun workstation); James B. Morris (later professor of computer science at Purdue); Robert Anderson (later professor of computer science at the University of Houston); Dallas Lankford (later professor of mathematics at the Louisiana Tech University) and Robert S. Boyer (later professor of computer science and mathematics at the University of Texas at Austin). Of these five, four went on to do Ph. D.'s in automated reasoning under Bledsoe.

Mark Moriconi, a former Bledsoe student and now director of the Computer Science Laboratory of SRI International, observes the following about Bledsoe's influence on his students (private communication):

> Bledsoe is very focused, a stickler for detail, and always works from concrete examples. His approach to problem solving strongly influenced the way that I work on all kinds of things, both professionally and in everyday life, and I am sure that it has influenced other students as well. Moreover, he instilled high standards for quality in his students, always by example and not by fiat. Without question, Bledsoe has been an effective role model for his students, which after all is one of the most important, but difficult to meet, responsibilities of a professor.

Bledsoe's Ph. D. Students (to date)

John Wade Ulrich	1968, Computer Sciences
Stephen Charles Darden	1969, Computer Sciences
Charles Edward Wilks	1969, Mathematics
James Bertram Morris	1969, Computer Sciences
Robert Brockett Anderson	1970, Mathematics
Robert Stephen Boyer	1971, Mathematics
Dallas Lankford	1972, Mathematics
Vesko Genov Marinov	1973, Computer Sciences
Mark Steven Moriconi	1977, Computer Sciences
John Threecivelous Minor	1979, Computer Sciences
William Mabry Tyson	1981, Computer Sciences
Tie Cheng Wang	1986, Computer Sciences
Larry Marvin Hines	1988, Computer Sciences
Don Simon	1990, Computer Sciences

1.7.2 Visits to Other Universities

While a professor at the University of Texas, Bledsoe has traveled extensively. He spent the 1970-71 academic year at the Artificial Intelligence Laboratory of the Massachusetts Institute of Technology, as a guest of Marvin Minsky. In 1973 Bledsoe visited Bernard Meltzer, J Moore, and the authors at the Metamathematics Unit at Edinburgh University. (While in Edinburgh he toured the Highlands in Moore's taxi and played golf at St. Andrews.) In 1978 Bledsoe spent a semester at Carnegie-Mellon University, where he began a collaboration with Doug Lenat that continued later at MCC.

1.7.3 Administration

Despite having received assurances from the University of Texas that he would not be asked to serve as chairman of the mathematics department, Bledsoe was nevertheless lured into being chairman twice, 1967-69 and 1973-75. The first period of Bledsoe's chairmanship involved considerable administrative tension because the department was sharply divided between the "third floor" group, which included the eighty-year old topologist R. L. Moore, and the rest of the department. (The third floor group was so named because it occupied the entire third floor of the mathematics building. The group had in common a dedication to the Moore method of teaching, which opposed the use of lectures and textbooks.) Eventually the remainder of the mathematics department and Dean John Silber (who was himself fired not much later) determined finally to retire Moore, who had taught at the University since 1920.

1.8 IJCAI, CADE, and AAAI

Bledsoe has been exceptionally active in the building of organizations and conferences for the fields of artificial intelligence and automated reasoning. For example, he was the general chairman of the International Joint Conference on Artificial Intelligence at MIT in 1977 and a member of the IJCAI board of trustees from 1978 to 1983. He hosted the Conference on Automated Deduction in Austin in 1979. He has served numerous times on program committees for IJCAI, AAAI, and CADE.

In 1983, Bledsoe was named president-elect of the American Association of Artificial Intelligence. In 1984, he helped organize the annual AAAI meeting in Austin, where he was installed as president of AAAI. In an address to the 1984 AAAI meeting, outgoing president John McCarthy expressed dismay at the extremely large size of the meeting, over four thousand; in less than twenty years the field had grown from a handful of practitioners to a giant academic and commercial enterprise. Sociologists of academia may note that organizing such major conferences is remarkably frustrating and time consuming. Bledsoe vowed at the end of the conference that he would never again run another conference at the University of Texas because of the bureaucratic ensnarlments he encountered in attempting to provide space for the many computer manufacturers who wished to advertize their wares at the conference, a surprising vow from a man well known for his commitment to positive thinking.

In his 1985 presidential address to the American Association of Artificial Intelligence [42], we find no technical monologue but rather an extremely frank and personal expression of Bledsoe's vision for artificial intelligence and his faith in its future. He opens the address by remarking:

> Twenty-five years ago I had a dream, a *daydream*, if you will. A dream shared with many of you. I dreamed of a special kind of computer, which had eyes and ears and arms and legs, in addition to its "brain."

In the address, Bledsoe is clear that the dream has not yet been fulfilled:

> The twenty-five years have not been totally kind to my dream: Shakey[1] liked shaking more than running and thinking, and was laid aside for a season; language translation sputtered, died, and was resurrected; facial recognition was pushed back on the researchers stack; automatic provers showed signs of growing pains, which disheartened the fainthearted; no machine stepped forward to try the Turing test; robot arms were duplicating block castles instead of playing squash; etc., etc., etc.; many AI researchers lost faith and dropped out.

Yet Bledsoe is adamant that progress is being made:

> First, let me express my annoyance with some of the distracted individuals who criticize AI researchers for not "jumping to infinity" in one leap. Somehow, to them it is OK to work step by step on the dream of obtaining controlled thermonuclear energy or a cure for cancer or a cure for the common cold, but no such step-by-step process is allowed for those trying (partially) to duplicate the intelligent behavior of human beings. To these cynics, a natural language system that converses with us in a limited form of English is not a legitimate step toward passing the Turing test. ...Indeed, almost all of our AI accomplishments have been of the *partial* kind: natural language processors that handle a *subset* of English (or French, etc.); systems that recognize and synthesize *limited* forms of speech; character recognition machines that read only typewritten characters; expert systems that perform a variety of tasks (but not all that a human can); theorem provers that can prove difficult theorems in a *particular* area of mathematics or that can handle the inferencing needed for elementary expert systems, including monotonic reasoning; programs that play expert-level chess; programs that exhibit an elementary level of learning and reasoning by analogy. And the list goes on.

In this presidential address, Bledsoe also describes the characteristics of a good researcher, enumerates the important research topics in AI, and expresses considerable enthusiasm for the prospects of progress in AI.

1.9 MCC

In late 1983, after an intensive consideration of alternative sites, and with considerable financial incentives from the University of Texas, Admiral Bobby Inman (former assistant director of the CIA and former director of the NSA)

[1] A robot developed at SRI in the early 70's.

opened up in Austin the Microelectronics and Computer Technology Corporation. MCC, as it is generally known, was backed by commitments of research funding by over twenty American computer companies to do advanced research. Such unprecedented collaboration was endorsed by a congressional act explicitly waiving anti-trust prohibitions against collaborative efforts on research. The consortium was developed in part as a response to a fear that the Japanese computer industry was on the road to surpassing American dominance in computing.

In early 1984, Bledsoe indicated to Inman a general interest in supporting the goals of MCC, even to the extent of spending some time at MCC. Inman quickly offered Bledsoe the position of vice-president for artificial intelligence, a multi-million dollar per year budget, and the charter to set up a first class AI research center. Bledsoe did set up such a center and directed it for three years.

Shortly after Bledsoe took leave from the University of Texas to work at MCC, the Austin Chamber of Commerce's glossy publication *Austin Magazine* did a story on artificial intelligence. On the cover is a handsome photograph of Bledsoe, with the headline "Machines Who Think—Austin scientists are at the forefront of research to create a new breed of computers that think like humans." In the photograph, Bledsoe is leaning on a computer terminal, whose display reads "I Think, Therefore I Am." A subhead reads "MCC's Woodrow Bledsoe and friend."

During this period, perhaps the most visible artificial intelligence project established at MCC was the CYC project of Doug Lenat. This ambitious project has as its objective the encoding into a computer of all the basic principles and fundamental facts of knowledge necessary for reasoning, perhaps the most ambitious scientific undertaking ever.

Bledsoe left MCC in 1987 to return to the joys of teaching and doing research in automated reasoning.

1.10 Return to the University of Texas

Upon returning to the University of Texas at Austin, Bledsoe was appointed to the "million dollar" Peter O'Donnell, Jr. Chair in Computing Systems, named after a philanthropist extremely generous in endowing major chairs in science and engineering at the University of Texas.

Under the philanthropy of the computer scientist and entrepreneur Ed Fredkin, some prizes in the area of artificial intelligence have been established. Bledsoe chaired and initially formed one such prize committee, now known as the Automatic Theorem Proving Prize Committee of the American Mathematical Society. Under Bledsoe's chairmanship, the committee's first prizes were awarded at a special session in the 1983 Denver meeting of the American Mathematical Society, which Bledsoe organized. The first "milestone" prize of the committee was given to Hao Wang, and the first "current" prize was awarded to L. Wos and S. Winker. A second milestone prize was later awarded to J. A. Robinson. Bledsoe left the committee in 1985. In 1991 Bledsoe was awarded the third "milestone" prize, by a committee then chaired by the Harvard mathematician D. Mumford. In characteristic bashfulness and modesty, Bledsoe's first remark after learning of the award was that it should have been awarded to someone else. The award was presented by John McCarthy at a special session on automated reasoning at the 1991 meeting of the American Mathematical Society in San Francisco. The citation for the prize reads thus:

> The Milestone Award of 1991 goes to Woodrow W. Bledsoe, who has been a central figure in Automatic Theorem Proving, inspiring and guiding this field for over twenty years. His broad view of the subject, using resolution and non-resolution techniques, his deep study of theorem proving in analysis and with inequalities, and his work on interactive theorem provers distinguish him as a major innovator in the field.

Bledsoe received the 1991 Distinguished Service Award of the International Joint Conferences on Artificial Intelligence.

1.11 Automated Reasoning

We now review Bledsoe's contributions to automated reasoning. Bledsoe's receipt of the "milestone" award for contributions to automatic theorem proving is a clear indication than his contributions in automated reasoning have been truly major. What are the respects in which Bledsoe has contributed to automated reasoning?

- Wrote, with J. Morris, one of the first, and one of the best-ever "proof-checking" programs, which was used to check a substantial part of A. P. Morse's [58].
- Invented a remarkably easy-to-use method for doing completeness proofs of resolution strategies and showed the completeness of a new strategy for linear resolution.
- First emphasized the importance of building a prover with many different kinds of heuristic components, reflecting the diversity of reasoning methods employed by mathematicians. Among the components incorporated together in Bledsoe provers have been:
  - Ordinary resolution
  - Natural deduction

- Special arithmetic decision procedures
- Rewriting procedures
- Inductive procedures
- Special heuristics for analysis
- Use of example generation techniques
- Use of analogical reasoning
- Use of type inference

- Demonstrated that by using such a variety of techniques, proofs for moderately deep theorems in analysis could be automatically discovered.
- Virtually singlehandedly identified the subfield of "non-resolution" theorem proving with several comprehensive survey articles [34, 47].

Extremely characteristic of Bledsoe is his "try it out and see" attitude toward ideas in Artificial Intelligence. To quote further from his AAAI address

> The principle I want to make is this: when you have what looks like a good idea, give it your best shot, waste a little money to get some early feedback. Don't take forever to study the problem, because that is even more expensive (and less exciting).

An excellent summary of Bledsoe's work may be found in [41]. This paper gives an overview of techniques used in Bledsoe's prover, including reduction, induction, resolution, natural deduction, the limit heuristic, variable restrictions and elimination, and shielding. Bledsoe also enumerates theorems in analysis proved entirely automatically by his prover, including:

- The limit of sums and products are the sums and products of the limits.
- The sums, products, and compositions of continuous functions are continuous.
- Differentiable and uniformly continuous functions are continuous.
- If a function f is continuous on a compact set, then it is uniformly continuous on f, and $f[S]$ is compact.
- The Bolzano-Weierstrass Theorem.
- The Intermediate Value Theorem

1.11.1 Mechanical Proof Checking

How did Bledsoe happen to shift from the study of pattern recognition and evolution to automated reasoning? Both while working at Panoramic and after moving to the University of Texas, Bledsoe continued to consult at Sandia. In the summer of 1966, Bledsoe began consulting with Sandia employees E. Gilbert, D. Morrison, and J. Weihe on a mechanical proof-checking program for the Morse logic [58]. Weihe, like Bledsoe, had done his Ph. D. under A. P. Morse, as had T. McMinn, who also consulted on this proof-checking project, and had written the trenchant foreward to [58]. This remarkable proof-checker project is described in [25]. Among the rules of inference of the Morse logic supported by the checker were:

- detachment
- variable instantiation
- schematic (second order) instantiation
- change of bound variable
- universalization

The proof-checker included quite a few additional, derived rules of inference, including tautology checking [25]. Over the course of several years, Bledsoe and Morris developed this checker to the point that they were able to use it to check a substantial percentage of the theorems in Morse's book. This experience convinced him that it was, from the mathematical point of view, not only feasible but even easy to proof check almost any part of mathematics if the user of the proof-checking system were sufficiently patient to provide enough intermediate steps. But Bledsoe became both bored by the tedium of providing steps (especially since at this time the technology was to submit long trays of IBM punch cards and wait hours for a listing of a "run") and excited about the possibility of getting a computer to take much larger inference steps.

1.11.2 Resolution—Yeas and Nays

Like most of the early workers in automated reasoning, Bledsoe became fascinated by the infinite improvement over previous techniques represented by the resolution technique of J. A. Robinson, which had been developed at Argonne National Laboratory in the summer of 1963. Working with his Ph. D. student Robert Anderson, Bledsoe made important mathematical contributions to the theory of resolution, including a new "linear format" and a technique for proving completeness [28]. The completeness technique, known as the excess literal technique, is surprisingly easy to apply. The idea is simply this:

when trying to prove the (ground) completeness of a new resolution strategy, do the induction "on" the "excess literal" measure. The excess literal measure of a set of clauses is the numeric difference between the number of occurrences of literals in the set of clauses and the number of clauses in the set. This method has been used successfully by many people, including one of the authors of this note (RSB) in his Ph. D. thesis under Bledsoe, which introduced the "locking" restriction of resolution and proved its completeness [48, 51]. Other Ph. D. students of Bledsoe who made serious contributions to the resolution literature include Robert Anderson [1, 2], Dallas Lankford [55], James B. Morris [56, 57], and T. C. Wang [46].

One of the first extensions Bledsoe made to his proof-checker was a resolution theorem-prover coded by his student James B. Morris. But despite the fascination that Bledsoe has always felt for the simplicity and power of the resolution approach to theorem-proving, his practical experience with using it to prove difficult theorems was and remains largely a disappointment to him. As he remarks in his [34], "by the early 70's there was emerging a belief that resolution type systems could never really 'hack' it, could not prove really hard mathematical theorems, without some extensive changes in philosophy."

1.11.3 PROVER

In order to develop a theorem prover that was actually capable of proving deep theorems using a feasible amount of computing resources, Bledsoe began the construction and evolution of his most well-known theorem-proving system, which has been known as the "UT Interactive Theorem Prover," or simply as "PROVER." The initial substantial components of this prover, in addition to a resolution component, are described in [29]:

- A splitting routine for proving separable subgoals completely independently.
- A simplification (rewriting, normalizing, or demodulating) routine together with about thirty built-in, set-theoretic rewrite rules.
- An induction routine for setting up base cases and induction steps.
- An equality substitution routine.
- A governing scheme called CYCLE to decide which of the above routines to call in which order.

This prover used resolution as a "black box," but only for short periods of run time. That is, the resolution component could not call the reducer or other routines and did not contribute to finding a proof except by succeeding entirely by itself on very simple subgoals. With this first prover, Bledsoe was able to prove some nontrivial set theory theorems that he found impossible to get solely by resolution, including the theorem that every natural integer is an ordinal.

With a strong background in "analysis," it was natural for Bledsoe to move from set theory to elementary theorems about limits and continuous functions. But to achieve the goal of obtaining a satisfactory number of mechanical proofs in this area, Bledsoe found it necessary to make quite a number of additional changes to his prover.

First, he abandoned resolution altogether, replacing it with a natural deduction theorem-proving program called IMPLY. Unlike the resolution routine that he had been using, IMPLY provided a natural environment for embedding some of his other heuristics and the reducer.

Second, intertwined with IMPLY was not only the rewriter, but a new typing facility, which associated with terms some expressions that were (conservative) characterizations of sets over which the terms could range. As an example, a variable x might be known to range over the interval $[a, b]$. Explicit association of types with certain expressions, together with simple operations for taking set unions and intersections, made it possible to build into the deduction machinery set-theoretic reasoning rules that would be difficult to implement efficiently in a pure resolution setting.

Third, he added a mechanism for solving linear inequalities. Although it is possible to express as clauses appropriate axioms for linear order and polynomials, it appears vastly more efficient to embed such axioms in a decision procedure or even a partial decision procedure rather than to permit such axioms to "slosh around" in a general clause database.

Bledsoe also added to IMPLY the "limit heuristic," a very simple trick for proving certain inequalities. This trick "uses up" a hypothesis inequality by factoring it out of the desired conclusion.

These additions are first described in [31], which was written at M. I. T. Further development of the IMPLY prover is described in [33]. This IMPLY prover of Bledsoe's was influential in the design of the prover of Boyer and Moore [49, 50].

Probably the most significant subsequent improvement to IMPLY is the mechanism reported in [36] for instantiating set variables. Here Bledsoe describes a mechanism for constructing sets with specified properties, and uses it to prove such theorems as "if a set B contains an open neighborhood of each of its points, then B is open" and The Intermediate Value Theorem.

1.11.4 Work With A. Michael Ballantyne

Perhaps the colleague who has worked most on automated reasoning with Bledsoe is Mike Ballantyne. Ballantyne was clearly among the most creative of the many Bledsoe graduate students; Mike also consulted at MCC under Bledsoe. Two Ballantyne-Bledsoe projects are especially worth mentioning.

Perhaps the high point of success with the IMPLY prover is their work on nonstandard analysis [3]. In that paper, some very interesting theorems in intermediate analysis are proved, including the Bolzano-Weierstrass theorem and the theorem that the image of a continuous function on a compact set is compact. Although these theorems were proved by the IMPLY prover, with its natural deduction and rewriting facilities, what is most remarkable about this work is that the axiomatization used for the reals is based upon the non-standard analysis of A. Robinson [60], with its infinitesimals and infinite integers. It is a remarkable fact that proofs of some theorems in intermediate analysis are much simpler in a nonstandard setting than in the usual ϵ and δ approach.

The usefulness of examples in finding proofs has been well known at least since the earliest days of geometry. A very few efforts at using examples in automated reasoning have been successful, notably the early geometry work of Gelernter. In [4], Ballantyne and Bledsoe demonstrate the value of using examples in the area of analysis, especially topology. In that article is described a system, named GRAPHER, that constructs counterexamples as an aid to eliminating false subgoals arising in proofs. The program considers finite families of finite sets together with considerable knowledge of elementary topology. Among many, many decidedly nontrivial inferences, the program found an example of a subspace of a normal space that is not normal.

1.11.5 Work With Larry Hines

Another long time colleague and student of Bledsoe is Larry Hines. The most well-known result of Bledsoe and Hines is the variable elimination method, which is a technique for proving, in a resolution setting, theorems involving inequalities. Although the basic ideas of variable elimination are easily stated [38], a considerable amount of deep thinking has gone into proving that this technique is "complete" [43]. The completeness work was done with Rob Shostak and Ken Kunen.

1.11.6 Program Verification and Natural Language

In 1974 Bledsoe began to contribute to the field of *mechanical program verification.* The IMPLY prover that he developed was incorporated into a verification system for Pascal [52] and was later utilized in the verification system for Gypsy [53, 54, 63], which has been used to verify several significant computing systems. See also [35].

In 1990, Bledsoe's student Don Simon finished a Ph. D. thesis [62] about a program that could proof check number theory proofs directly from a textbook ([67]) typeset in LaTeX.

1.11.7 The Future of Automated Reasoning

In Bledsoe's view, some of the earliest work on automated reasoning, such as that of Newell, Simon, and Shaw [59], started a trend in the direction of employing "people-oriented" reasoning techniques, but this trend was "overshadowed by the emergence of other techniques emphasizing speed, and thus the field took a turn away from people methods." In Bledsoe's view, it is "time to make a significant change in direction" to pursue more automated reasoning studies along the lines of "people methods." According to Bledsoe's [45], the following are crucial elements of mathematical thought that an automated reasoning system should incorporate, all analogous to the way that people reason:

- analogy
- generation and use of examples
- use of counterexamples to prune the search
- conjecturing of lemmas and subgoals
- intelligent fetching of useful lemmas and definitions from a large knowledge base
- an agenda mechanism for controlling the search
- mathematical pattern recognition
- learning
- planning and abstraction
- higher-level and meta reasoning

Since his return to the University of Texas in 1987, Bledsoe has primarily focused on "reasoning by analogy," which he believes is perhaps the most important key to advances in automated reasoning.

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