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|#

EVENT: Start with the library "mlp" using the compiled version.

```
; macc.bm
; Circuit is similar to acc, but uses multiplication instead of
; addition, i.e. it's a multiplying accumulator. It's expressed in CSXA form,
; which is the form we've currently settled on.
; NOTE that it has to be initialized with 1 in order to function right! See
; prod0 for what happens with 0-initialization...
;
;;; DEFINITION OF CIRCUIT:
#|
(setq sysd '(sy-macc ( x)
(Ymacc S Times x Ymacc2)
```

```

(Ymacc2 R 1 Ymacc)
))

(setq macc '(
; BM DEFINITIONS and A2 LEMMAS, generated by BMSYSD:
|#
; comb_times.bm: Times combinational element.
; U7-DONE

; no character function def since BM already knows about Times..

; Everything below generated by:      (bmcomb 'times '() '(x y))

DEFINITION:
s-times (x, y)
=  if empty(x) then E
   else a(s-times(p(x), p(y)), l(x) * l(y)) endif

;; A2-Begin-S-TIMES

THEOREM: a2-empty-s-times
empty(s-times(x, y)) = empty(x)

THEOREM: a2-e-s-times
(s-times(x, y) = E) = empty(x)

THEOREM: a2-lp-s-times
len(s-times(x, y)) = len(x)

THEOREM: a2-lpe-s-times
eqlen(s-times(x, y), x)

THEOREM: a2-ic-s-times
(len(x) = len(y))
→ (s-times(i(c_x, x), i(c_y, y)) = i(c_x * c_y, s-times(x, y)))

THEOREM: a2-lc-s-times
(¬ empty(x)) → (l(s-times(x, y)) = (l(x) * l(y)))

THEOREM: a2-pc-s-times
p(s-times(x, y)) = s-times(p(x), p(y))

THEOREM: a2-hc-s-times
((¬ empty(x)) ∧ (len(x) = len(y)))
→ (h(s-times(x, y)) = (h(x) * h(y)))

```

THEOREM: a2-bc-s-times  
 $(\text{len}(x) = \text{len}(y)) \rightarrow (\text{b}(\text{s-times}(x, y)) = \text{s-times}(\text{b}(x), \text{b}(y)))$

THEOREM: a2-bnc-s-times  
 $(\text{len}(x) = \text{len}(y)) \rightarrow (\text{bn}(n, \text{s-times}(x, y)) = \text{s-times}(\text{bn}(n, x), \text{bn}(n, y)))$

;; A2-End-S-TIMES

; eof:comb\_times.bm

DEFINITION:  
topor-sy-macc(*ln*)  
= **if** *ln* = 'ymacc **then** 1  
   **elseif** *ln* = 'ymacc2 **then** 0  
   **else** 0 **endif**

DEFINITION:  
sy-macc(*ln*, *x*)  
= **if** *ln* = 'ymacc **then** s-times(*x*, sy-macc('ymacc2, *x*))  
   **elseif** *ln* = 'ymacc2  
   **then if** empty(*x*) **then** E  
      **else** i(1, sy-macc('ymacc, p(*x*))) **endif**  
   **else** sfix(*x*) **endif**

;; A2-Begin-SY-MACC

THEOREM: a2-empty-sy-macc  
empty(sy-macc(*ln*, *x*)) = empty(*x*)

THEOREM: a2-e-sy-macc  
(sy-macc(*ln*, *x*) = E) = empty(*x*)

THEOREM: a2-lp-sy-macc  
len(sy-macc(*ln*, *x*)) = len(*x*)

THEOREM: a2-lpe-sy-macc  
eqlen(sy-macc(*ln*, *x*), *x*)

THEOREM: a2-pc-sy-macc  
p(sy-macc(*ln*, *x*)) = sy-macc(*ln*, p(*x*))

;; A2-End-SY-MACC

;;; SPEC definition:

DEFINITION:

```
numer-macc(x)
=  if empty(x) then 1
   else numer-macc(p(x)) * l(x) endif
```

; this is the standard extension from last-char-fun to MLP-string-fun.

DEFINITION:

```
spec-macc(x)
=  if empty(x) then E
   else a(spec-macc(p(x)), numer-macc(x)) endif
```

;;; Circuit CORRECTNESS:

; Macc-correct-ax is a "predicative correctness statement", i.e. what we would  
; do if we didn't have functional equality as a specification method, but  
; instead used a purely axiomatic approach.

THEOREM: macc-correct-ax

$$(\neg \text{empty}(x)) \rightarrow (l(\text{sy-macc}('ymacc, x)) = \text{numer-macc}(x))$$

; to go to a functional equality once we have the "last" (ax) statement is  
; a trivial induction, if we start out with an P-L split which is unnatural  
; for BM, so we force it w/ a USE hint of A-p-l-split  
; We really would like to use it as a one-time rewrite, but it's a looping  
; rule, so we can't. Instead we have to use it in USE hints, which in case  
; of induction, makes things more complex than they should.

THEOREM: a-p-l-split

$$\begin{aligned} &(\neg \text{empty}(x)) \\ &\rightarrow (\text{sy-macc}('ymacc, x) \\ &\quad = a(\text{p}(\text{sy-macc}('ymacc, x)), l(\text{sy-macc}('ymacc, x)))) \end{aligned}$$

THEOREM: macc-correct

$$\text{sy-macc}('ymacc, x) = \text{spec-macc}(x)$$

; eof: macc.bm

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