#|

|#

; William R. Bevier

EVENT: Start with the initial **nqthm** theory.

DEFINITION:
delete (x, l)
= **if** listp (l)
  then if x = car (l) **then** cdr (l)
    else cons (car (l), delete (x, cdr (l))) **endif**
  else l **endif**

DEFINITION:
bagdiff (x, y)
= **if** listp (y)
  then if car (y) $\in x$ **then** bagdiff (delete (car (y), x), cdr (y))
    else bagdiff (x, cdr (y)) **endif**
  else x **endif**

DEFINITION:

bagint $(x,\ y)$
$=$ **if** listp (x)
then if car $(x) \in y$
then cons (car (x), bagint (cdr (x), delete (car (x), y)))
else bagint (cdr (x), y) **endif**
else nil endif

DEFINITION:
occurrences $(x,\ l)$
$=$ **if** listp (l)
then if $x =$ car (l) **then** $1 +$ occurrences $(x$, cdr $(l))$
else occurrences $(x$, cdr $(l))$ **endif**
else 0 **endif**

DEFINITION:
subbagp $(x,\ y)$
$=$ **if** listp (x)
then if car $(x) \in y$ **then** subbagp (cdr (x), delete (car (x), y))
else f endif
else t endif

THEOREM: listp-delete
listp (delete $(x,\ l)$)
$=$ **if** listp (l) **then** $(x \neq$ car $(l)) \vee$ listp (cdr (l))
else f endif

EVENT: Disable listp-delete.

THEOREM: delete-non-member
$(x \notin y) \rightarrow$ (delete $(x,\ y) = y$)

THEOREM: delete-delete
delete (y, delete $(x,\ z)$) $=$ delete (x, delete $(y,\ z)$)

THEOREM: equal-occurrences-zero
(occurrences $(x,\ l) = 0$) $= (x \notin l)$

THEOREM: member-non-list
$(\neg$ listp $(l)) \rightarrow (x \notin l)$

THEOREM: member-delete
($x \in$ delete $(y,\ l)$)
$=$ **if** $x \in l$
then if $x = y$ **then** $1 <$ occurrences $(x,\ l)$
else t endif
else f endif

THEOREM: member-delete-implies-membership
$(x \in \text{delete}(y, l)) \rightarrow (x \in l)$

THEOREM: occurrences-delete
$$\begin{aligned}&\text{occurrences}(x, \text{delete}(y, l))\\ = \quad & \textbf{if } x = y\\ & \textbf{then if } x \in l \textbf{ then } \text{occurrences}(x, l) - 1\\ & \qquad \textbf{else } \texttt{0} \textbf{ endif}\\ & \textbf{else } \text{occurrences}(x, l) \textbf{ endif}\end{aligned}$$

THEOREM: member-bagdiff
$(x \in \text{bagdiff}(a, b)) = (\text{occurrences}(x, b) < \text{occurrences}(x, a))$

THEOREM: bagdiff-delete
$\text{bagdiff}(\text{delete}(e, x), y) = \text{delete}(e, \text{bagdiff}(x, y))$

THEOREM: subbagp-delete
$\text{subbagp}(x, \text{delete}(u, y)) \rightarrow \text{subbagp}(x, y)$

THEOREM: subbagp-cdr1
$\text{subbagp}(x, y) \rightarrow \text{subbagp}(\text{cdr}(x), y)$

THEOREM: subbagp-cdr2
$\text{subbagp}(x, \text{cdr}(y)) \rightarrow \text{subbagp}(x, y)$

THEOREM: subbagp-bagint1
$\text{subbagp}(\text{bagint}(x, y), x)$

THEOREM: subbagp-bagint2
$\text{subbagp}(\text{bagint}(x, y), y)$

THEOREM: occurrences-bagint
$$\begin{aligned}&\text{occurrences}(x, \text{bagint}(a, b))\\ = \quad & \textbf{if } \text{occurrences}(x, a) < \text{occurrences}(x, b) \textbf{ then } \text{occurrences}(x, a)\\ & \textbf{else } \text{occurrences}(x, b) \textbf{ endif}\end{aligned}$$

THEOREM: occurrences-bagdiff
$\text{occurrences}(x, \text{bagdiff}(a, b)) = (\text{occurrences}(x, a) - \text{occurrences}(x, b))$

THEOREM: member-bagint
$(x \in \text{bagint}(a, b)) = ((x \in a) \wedge (x \in b))$

EVENT: Let us define the theory *bags* to consist of the following events: occurrences-bagint, bagdiff-delete, occurrences-bagdiff, member-bagint, member-bagdiff, subbagp-bagint2, subbagp-bagint1, subbagp-cdr2, subbagp-cdr1, subbagp-delete.

EVENT: Make the library `"bags"` and compile it.

Index