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|#

EVENT: Start with the initial **nqthm** theory.

;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;; mutex-atomic.ev ;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;

;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;; com.ev ;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;

;*sequence and finite set utilities

;;;The ith entry in l.

DEFINITION:

nth(l, i)
= if listp(l)

```

then if  $i = 1$  then  $\text{car}(l)$ 
      else  $\text{nth}(\text{cdr}(l), i - 1)$  endif
elseif  $l \in \mathbf{N}$ 
then if  $i = 1$  then  $l$ 
      else f endif
else f endif

```

EVENT: Disable nth.

```
;;;update ith entry of l to be k
```

```

DEFINITION:
move( $l, i, k$ )
= if  $i = 0$  then  $l$ 
  elseif  $l \simeq \text{nil}$ 
  then if  $i = 1$  then  $k$ 
    else  $l$  endif
  elseif  $i = 1$  then  $\text{cons}(k, \text{cdr}(l))$ 
  else  $\text{cons}(\text{car}(l), \text{move}(\text{cdr}(l), i - 1, k))$  endif

```

EVENT: Disable move.

```
DEFINITION:  $\text{at}(l, i, k) = (\text{nth}(l, i) = k)$ 
```

EVENT: Disable at.

```

DEFINITION:
length( $l$ )
= if  $\text{listp}(l)$  then  $1 + \text{length}(\text{cdr}(l))$ 
  else ZERO endif

```

EVENT: Disable length.

```
;;;The nth entry in l is in the list i.
```

```
DEFINITION:  $\text{union-at-n}(l, n, i) = (\text{nth}(l, n) \in i)$ 
```

EVENT: Disable union-at-n.

```
;;;Any entry in l is in the list i.
```

```

DEFINITION:
all-union( $l, n, i$ )
= if  $n \simeq 0$  then t
  else  $\text{union-at-n}(l, n, i) \wedge \text{all-union}(l, n - 1, i)$  endif

```

EVENT: Disable all-union.

;;;There exists an entry in l which belongs to
;;;the list i, moreover when exists, some such
;;;j is returned.

DEFINITION:

exist-union(l, n, i)
= **if** $n \simeq 0$ **then f**
 elseif union-at-n(l, n, i) **then** n
 else exist-union($l, n - 1, i$) **endif**

EVENT: Disable exist-union.

;;;n is in the intersection of l8-12 and g34.

DEFINITION:

intersect-8-12-3-4-at-n(n, l, g)
= (union-at-n($l, n, '(8 9 10 11 12)$) \wedge union-at-n($g, n, '(3 4)$))

EVENT: Disable intersect-8-12-3-4-at-n.

;;;There exists n in the intersection of l8-12 and g34.

DEFINITION:

exist-intersect-8-12-3-4(n, l, g)
= **if** $n \simeq 0$ **then f**
 elseif intersect-8-12-3-4-at-n(n, l, g) **then** n
 else exist-intersect-8-12-3-4($n - 1, l, g$) **endif**

EVENT: Disable exist-intersect-8-12-3-4.

;*Flag invariant.

DEFINITION:

lg-1-at-n(n, l, g)
= ((at($l, n, 0$) \wedge at($g, n, 0$))
 \vee (at($l, n, 1$) \wedge at($g, n, 0$))
 \vee (at($l, n, 2$) \wedge at($g, n, 0$))
 \vee (at($l, n, 3$) \wedge at($g, n, 1$))
 \vee (at($l, n, 4$) \wedge at($g, n, 1$)))

EVENT: Disable lg-1-at-n.

DEFINITION:

$$\begin{aligned} \text{lg-2-at-n}(n, l, g) &= ((\text{at}(l, n, 5) \wedge \text{at}(g, n, 3)) \\ &\quad \vee (\text{at}(l, n, 6) \wedge \text{at}(g, n, 3)) \\ &\quad \vee (\text{at}(l, n, 7) \wedge \text{at}(g, n, 2)) \\ &\quad \vee (\text{at}(l, n, 8) \wedge \text{at}(g, n, 3)) \\ &\quad \vee (\text{at}(l, n, 8) \wedge \text{at}(g, n, 2))) \end{aligned}$$

EVENT: Disable lg-2-at-n.

DEFINITION:

$$\begin{aligned} \text{lg-3-at-n}(n, l, g) &= ((\text{at}(l, n, 9) \wedge \text{at}(g, n, 4)) \\ &\quad \vee (\text{at}(l, n, 10) \wedge \text{at}(g, n, 4)) \\ &\quad \vee (\text{at}(l, n, 11) \wedge \text{at}(g, n, 4)) \\ &\quad \vee (\text{at}(l, n, 12) \wedge \text{at}(g, n, 4))) \end{aligned}$$

EVENT: Disable lg-3-at-n.

DEFINITION:

$$\begin{aligned} \text{lg-at-n}(n, l, g) &= (\text{lg-1-at-n}(n, l, g) \wedge \text{lg-2-at-n}(n, l, g) \wedge \text{lg-3-at-n}(n, l, g)) \end{aligned}$$

EVENT: Disable lg-at-n.

DEFINITION:

$$\begin{aligned} \text{lg}(n, l, g) &= \text{if } n \simeq 0 \text{ then } t \\ &\quad \text{else } \text{lg-at-n}(n, l, g) \wedge \text{lg}(n - 1, l, g) \text{ endif} \end{aligned}$$

EVENT: Disable lg.

;*The set {1...n}.

DEFINITION:

$$\begin{aligned} \text{nset}(n) &= \text{if } n \simeq 0 \text{ then nil} \\ &\quad \text{else } \text{cons}(n, \text{nset}(n - 1)) \text{ endif} \end{aligned}$$

EVENT: Disable nset.

;;n belongs to nset.

THEOREM: n-in-nset

$$(n \neq 0) \rightarrow (n \in \text{nset}(n))$$

;;;Any element in nset is a number.

THEOREM: nset-number

$$(k \in \text{nset}(n)) \rightarrow (k \in \mathbf{N})$$

;;;If a nonzero number plus one belongs to nset,
;;;then so does the nonzero number itself.

THEOREM: add1-nset

$$((k \neq 0) \wedge ((1 + k) \in \text{nset}(n))) \rightarrow (k \in \text{nset}(n))$$

;;;Any list has its length at least nonzero.

THEOREM: list-ln

$$\text{listp}(l) \rightarrow (\text{length}(l) \neq 0)$$

;;;(move l k i) is again a list if l is a list.

THEOREM: move-is-list

$$\text{listp}(l) \rightarrow \text{listp}(\text{move}(l, k, i))$$

EVENT: Enable length.

;;;(move l k i) has i as its kth entry.
;;;(enable length) is critical to prove this lemma.

THEOREM: move-nth

$$(\text{listp}(l) \wedge (k \in \text{nset}(\text{length}(l)))) \rightarrow (\text{nth}(\text{move}(l, k, i), k) = i)$$

THEOREM: zero-not-member-nset

$$0 \notin \text{nset}(n)$$

;;;Lists l and (move l k i) have the same length.

THEOREM: move-unchange-length

$$(\text{listp}(l) \wedge (k \in \text{nset}(\text{length}(l)))) \\ \rightarrow (\text{length}(\text{move}(l, k, i)) = \text{length}(l))$$

;;;Lists l and (move l k i) have the same entries
;;;except kth one.

THEOREM: move-unchange-other-than-nth

$$(\text{listp}(l) \wedge (k \in \text{nset}(\text{length}(l))) \wedge (j \neq k)) \\ \rightarrow (\text{nth}(\text{move}(l, k, i), j) = \text{nth}(l, j))$$

THEOREM: member-ex-union
 $\text{exist-union}(l, n, i) \rightarrow (\text{exist-union}(l, n, i) \in \text{nset}(n))$
 ;;;(exist-union l n i) is a number.

THEOREM: number-ex-union
 $\text{exist-union}(l, n, i) \rightarrow (\text{exist-union}(l, n, i) \in \mathbf{N})$
 ;;;(exist-intersect-8-12-3-4 n l g) belongs to nset.

THEOREM: member-intersect
 $\text{exist-intersect-8-12-3-4}(n, l, g)$
 $\rightarrow (\text{exist-intersect-8-12-3-4}(n, l, g) \in \text{nset}(n))$
 ;;;(exist-intersect-8-12-3-4 n l g) is a number.

THEOREM: number-intersect
 $\text{exist-intersect-8-12-3-4}(n, l, g) \rightarrow (\text{exist-intersect-8-12-3-4}(n, l, g) \in \mathbf{N})$
 ;;;any member of nset is nonzero.

THEOREM: k-not-0
 $(k \in \text{nset}(n)) \rightarrow (k \neq 0)$
 ;*lemmas for a0

;;;If j's entry in l is between 8..12 then
 ;;;(exist-union l n '(8 9 10 11 12)) holds.

THEOREM: j-ex-l8-12
 $((j \in \text{nset}(n)) \wedge \text{union-at-n}(l, j, '(8 9 10 11 12)))$
 $\rightarrow \text{exist-union}(l, n, '(8 9 10 11 12))$

;;;Witness of (exist-union lp n '(8 9 10 11 12))
 ;;;has in lp its entry between 8...12.

THEOREM: ex-lp8-12-in-lp8-12
 $\text{exist-union}(lp, n, '(8 9 10 11 12))$
 $\rightarrow \text{union-at-n}(lp,$
 $\quad \text{exist-union}(lp, n, '(8 9 10 11 12)),$
 $\quad '(8 9 10 11 12))$

;;;If (not (exist-union l n '(8 9 10 11 12)))
 ;;;holds, then (not (exist-union g n '(4))) by lg.

THEOREM: ex-if4
 $((\neg \text{exist-union}(l, n, '(8 9 10 11 12))) \wedge \text{lg}(n, l, g))$
 $\rightarrow (\neg \text{exist-union}(g, n, '(4)))$

;;;If (not (exist-union g n '(1))) holds,
 ;;; then there is no entry either 3 or 4.

THEOREM: l34-empty
 $((j \in \text{nset}(n)) \wedge \text{lg}(n, l, g) \wedge (\neg \text{exist-union}(g, n, '(1))))$
 $\rightarrow (\neg \text{union-at-n}(l, j, '(3\ 4)))$

;;;If j's entry in lp is 4, then (certainly)
 ;;;it is either 3 or 4.

THEOREM: lp4-then-un34
 $\text{at}(lp, j, 4) \rightarrow \text{union-at-n}(lp, j, '(3\ 4))$

;;;If (exist-intersect-8-12-3-4 n l g) holds,
 ;;;then so does (exist-union g n '(3 4)).

THEOREM: int-8-12-3-4-then-un34
 $\text{exist-intersect-8-12-3-4}(n, l, g) \rightarrow \text{exist-union}(g, n, '(3\ 4))$

;*lemmas for a1

;;;i is the witness of
 ;;;(exist-intersect-8-12-3-4 n lp gp).

THEOREM: int-wtn
 $((j \in \text{nset}(n)) \wedge \text{intersect-8-12-3-4-at-n}(j, lp, gp))$
 $\rightarrow \text{exist-intersect-8-12-3-4}(n, lp, gp)$

;;;If there exists j such that j's entry in lp
 ;;;is between 8..12 and entry in gp is either 3 or 4
 ;;;then (intersect-8-12-3-4-at-n j lp gp) holds.

THEOREM: un8-12-and-un34-then-int
 $(\text{union-at-n}(lp, j, '(8\ 9\ 10\ 11\ 12)) \wedge \text{union-at-n}(gp, j, '(3\ 4)))$
 $\rightarrow \text{intersect-8-12-3-4-at-n}(j, lp, gp)$

;;;By the two lemmas above,
 ;;;(exist-intersect-8-12-3-4 n lp gp) holds provided
 ;;;that there exists j such that j's entry in lp is
 ;;;between 8..12 and entry in gp is either 3 or 4.

;* ep-18-12

;;;If the k's entry in l is 5, then the k's entry
 ;;;in g is 3 by lg.

THEOREM: lg-15-g3
 $((k \in \text{nset}(n)) \wedge \text{lg}(n, l, g) \wedge \text{at}(l, k, 5)) \rightarrow \text{at}(g, k, 3)$

;;;If the k's entry in gp is 3 then certainly
;;;it is either 3 or 4.

THEOREM: gp3-then-un34
 $\text{at}(gp, k, 3) \rightarrow \text{union-at-n}(gp, k, '(3\ 4))$

;;;nep-18-12

;;;If the k's entry in l is between 8..12 then
;;;it is either between 8..11 or equal to 12.

THEOREM: case-k
 $(\text{union-at-n}(l, k, '(8\ 9\ 10\ 11\ 12))$
 $\wedge (\neg \text{union-at-n}(l, k, '(8\ 9\ 10\ 11))))$
 $\rightarrow \text{at}(l, k, 12)$

;;;;k-not-18-12

;;;If (exist-intersect-8-12-3-4 n l g) holds
;;;then the witness has its entry in g either equal
;;;to 3 or 4.

THEOREM: intersect-8-12-3-4-then-3-4
 $\text{exist-intersect-8-12-3-4}(n, l, g)$
 $\rightarrow \text{union-at-n}(g, \text{exist-intersect-8-12-3-4}(n, l, g), '(3\ 4))$

;;;If (exist-intersect-8-12-3-4 n l g) holds,
;;; then the witness has its entry in g between 8 and 12.

THEOREM: intersect-8-12-3-4-then-8-12
 $\text{exist-intersect-8-12-3-4}(n, l, g)$
 $\rightarrow \text{union-at-n}(l, \text{exist-intersect-8-12-3-4}(n, l, g), '(8\ 9\ 10\ 11\ 12))$

;;;k-in-18-11

;;;If k's entry in lp is between 9 and 12,
;;;then it is certainly between 8 and 12.

THEOREM: un9-12-then-un8-12
 $\text{union-at-n}(lp, k, '(9\ 10\ 11\ 12))$
 $\rightarrow \text{union-at-n}(lp, k, '(8\ 9\ 10\ 11\ 12))$

;;;If the i's entry in l is between 9 and 12,
;;;then the k's entry in g is 4.

THEOREM: if4
 $((j \in \text{nset}(n)) \wedge \text{lg}(n, l, g) \wedge \text{union-at-n}(l, j, '(9\ 10\ 11\ 12)))$
 $\rightarrow \text{at}(g, j, 4)$

;;k-in-l12

;;;If (exist-union lp n '(8 9 10 11 12)) holds then
 ;;;its witness does not have its entry in lp equal to 1.

THEOREM: ex-lp8-12-not-in-lp0
 $\text{exist-union}(lp, n, '(8\ 9\ 10\ 11\ 12))$
 $\rightarrow (\neg \text{at}(lp, \text{exist-union}(lp, n, '(8\ 9\ 10\ 11\ 12)), 0))$

;;;If k's entry in lp is between 8 and 12,
 ;;; then it is either between 8 and 11 or 12.

THEOREM: k-in-lp9-12-or-lp8
 $(\text{union-at-n}(lp, k, '(8\ 9\ 10\ 11\ 12))$
 $\wedge (\neg \text{union-at-n}(lp, k, '(9\ 10\ 11\ 12))))$
 $\rightarrow \text{at}(lp, k, 8)$

;;;If the k's entry is either 5 or 7,
 ;;;then it is between 5 and 7.

THEOREM: un57-then-un5-12
 $\text{union-at-n}(l, k, '(5\ 7)) \rightarrow \text{union-at-n}(l, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12))$

;;;If the k's entry in l is between 8 and 11,
 ;;;then it is between 5 and 12.

THEOREM: un8-11-then-un5-12
 $\text{union-at-n}(l, k, '(8\ 9\ 10\ 11))$
 $\rightarrow \text{union-at-n}(l, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12))$

;;;If the k's entry in l is between 8 and 12,
 ;;;then it is between 5 and 12.

THEOREM: un8-12-then-un5-12
 $\text{union-at-n}(l, k, '(8\ 9\ 10\ 11\ 12))$
 $\rightarrow \text{union-at-n}(l, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12))$

;*lemmas for a2

;;;i-eq-k-j-neq-k

;;;If the k's entry in l is either 10 or 11,
 ;;;then the k's entry in l is between 10 and 12.

THEOREM: un10-11-then-un10-12
union-at-n($l, k, '(10\ 11)$) \rightarrow union-at-n($l, k, '(10\ 11\ 12)$)

;;;If the j's entry in g is either 0 or 1 then
;;;the j's entry in l is not between 5 and 12.

THEOREM: if1
(($j \in \text{nset}(n)$) \wedge lg(n, l, g) \wedge union-at-n($g, j, '(0\ 1)$))
 \rightarrow (\neg union-at-n($l, j, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)$))

;;;j-eq-k-i-neq-k

;;;If the k's entry in l is between 5 and 7,
;;;then it is certainly between 5 and 12.

THEOREM: un5-7-then-un5-11
union-at-n($l, k, '(5\ 6\ 7)$) \rightarrow union-at-n($l, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11)$)

;;;If the k's entry in lp is between 5 and 7 then
;;;it is certain between 5 and 11.

THEOREM: un57-then-un5-11
union-at-n($l, k, '(5\ 7)$) \rightarrow union-at-n($l, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11)$)

;;;If the k's entry in l is between 8 and 11,
;;;then it is certainly between 5 and 11.

THEOREM: un8-11-then-un5-11
union-at-n($l, k, '(8\ 9\ 10\ 11)$)
 \rightarrow union-at-n($l, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11)$)

;;;If the k's entry in lp is between 5 and 12 and
;;;the k's entry in lp is between 5 and 7, then
;;;the k's entry in lp in fact is between 9 and 12.

THEOREM: k-in-lp5-7-or-lp8-or-lp9-12
(union-at-n($lp, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)$)
 \wedge (\neg union-at-n($lp, k, '(5\ 6\ 7)$))
 \wedge (\neg at($lp, k, 8$))
 \rightarrow union-at-n($lp, k, '(9\ 10\ 11\ 12)$))

;;;If the k's entry in l is between 5 and 11,
;;; then it is certainly between 5 and 12.

THEOREM: un5-11-then-un5-12
union-at-n($l, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11)$)
 \rightarrow union-at-n($l, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)$)

;;;If the k's entry in l is between 10 and 12,
 ;;; then it is certainly between 8 and 12.

THEOREM: un10-12-then-un8-12
 $\text{union-at-n}(l, i, '(10\ 11\ 12)) \rightarrow \text{union-at-n}(l, i, '(8\ 9\ 10\ 11\ 12))$

;;;j-eq-k-i-neq-k

;;;If (exist-union l n '(8 9 10 11 12)) does not hold,
 ;;;then the i's entry in l is not between 10 and 12.

THEOREM: i-not-l10-12
 $((i \in \text{nset}(n)) \wedge (\neg \text{exist-union}(l, n, '(8\ 9\ 10\ 11\ 12))))$
 $\rightarrow (\neg \text{union-at-n}(l, i, '(10\ 11\ 12)))$

*lemmas for a3

;;;j-eq-k-i-neq-k

;;;If the k's entry in l is between 5 and 11,
 ;;;then the k's entry in l is between 9 and 11.

THEOREM: un5-11-eq-un58-or-un8-11
 $(\text{union-at-n}(l, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11))$
 $\wedge (\neg \text{union-at-n}(l, k, '(5\ 6\ 7\ 8))))$
 $\rightarrow \text{union-at-n}(l, k, '(9\ 10\ 11))$

;;;If the k's entry in g is 4,
 ;;;then the k's entry in l is between 5 and 8.

THEOREM: a3-if4
 $((k \in \text{nset}(n)) \wedge \text{lg}(n, l, g) \wedge \text{at}(g, k, 4))$
 $\rightarrow (\neg \text{union-at-n}(l, k, '(5\ 6\ 7\ 8)))$

;;;If the k's entry in l is between 5 and 11,
 ;;;and the k's entry in l is between 5 and 12,
 ;;;then the k's entry in l is 9 and 11.

THEOREM: k-in-l5-11-g4-then-l9-11
 $((k \in \text{nset}(n))$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{union-at-n}(l, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11))$
 $\wedge \text{at}(g, k, 4))$
 $\rightarrow \text{union-at-n}(l, k, '(9\ 10\ 11))$

```

; dropped second use hint -- rsb

;;;If the i's entry in l is 12,
;;;then the i's entry in l is between 8 and 12.

THEOREM: l12-then-un8-12
at (l, i, 12) → union-at-n(l, i, '(8 9 10 11 12))

;;;If (exist-union l n '(8 9 10 11 12)) does not hold,
;;;then the i's entry in l is 12.

THEOREM: i-not-in-l12
((i ∈ nset(n)) ∧ (¬ exist-union(l, n, '(8 9 10 11 12))))
→ (¬ at(l, i, 12))

;;;j-neq-k-i-eq-k

;;;If the k's entry in l is 11,
;;; then the k's entry in l is between 10 and 12.

THEOREM: l11-then-un10-12
at (l, k, 11) → union-at-n(l, k, '(10 11 12))

;;;If the j's entry in g is either 2 or 3,
;;;then the j's entry in l is between 5 and 8 by lg.

THEOREM: if3
((j ∈ nset(n)) ∧ lg(n, l, g) ∧ (¬ union-at-n(g, j, '(2 3))))
→ (¬ union-at-n(l, j, '(5 6 7 8)))

;;;If the j's entry in l is between 5 and 12 and
;;;the j's entry in l is between 5 and 8, then
;;;the j's entry in l is 9 and 12.

THEOREM: l5-12-eq-l5-8-or-l9-12
(union-at-n(l, j, '(5 6 7 8 9 10 11 12))
 ∧ (¬ union-at-n(l, j, '(5 6 7 8))))
→ union-at-n(l, j, '(9 10 11 12))

;;;i-j-eq-k

;;;If the k's entry in lp is 12,
;;;then it is certainly between 5 and 12.

THEOREM: l12-then-un9-12
at (lp, k, 12) → union-at-n(lp, k, '(9 10 11 12))

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;*lemmas for b1a

;;;If the u's entry in g is 4,
;;;then the u's entry in l is between 8 and 12 by lg.

THEOREM: b1a-if4
((u ∈ nset (n)) ∧ lg (n, l, g) ∧ at (g, u, 4))
→ union-at-n (l, u, '(8 9 10 11 12))

;*lemmas for b1b

;;;If the k's entry in lp is between 9 and 12,
;;;then the k's entry in gp is iether 3 or 4 by lg.

THEOREM: lp9-12-then-k-in-g34
((k ∈ nset (n)) ∧ union-at-n (lp, k, '(9 10 11 12)) ∧ lg (n, lp, gp))
→ union-at-n (gp, k, '(3 4))

;;;If the k's entry in lp is between 8 and 12, and
;;;it is not 8, then it is certainly between 9 and 12.

THEOREM: un8-12-then-l8-or-l9-12
(union-at-n (lp, k, '(8 9 10 11 12)) ∧ (¬ at (lp, k, 8)))
→ union-at-n (lp, k, '(9 10 11 12))

;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;; defn.ev ;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;Well-formed-state.

DEFINITION:
ws (n, l, g)
= ((n ∈ N)
   ∧ listp (l)
   ∧ listp (g)
   ∧ (length (l) = n)
   ∧ (length (g) = n)
   ∧ all-union (l, n, '(0 1 2 3 4 5 6 7 8 9 10 11 12))
   ∧ all-union (g, n, '(0 1 2 3 4)))

EVENT: Disable ws.

;;;Transitions.

DEFINITION:
rhoi0 (n, i, l, g, lp, gp)
= (at (l, i, 0) ∧ (gp = g) ∧ (lp = move (l, i, 1)))

```

DEFINITION:

$$\begin{aligned} & \text{rhoi1a}(n, i, l, g, lp, gp) \\ &= (\text{at}(l, i, 1) \wedge (gp = g) \wedge (lp = \text{move}(l, i, 2))) \end{aligned}$$

DEFINITION:

$$\text{rhoi1b}(n, i, l, g, lp, gp) = (\text{at}(l, i, 1) \wedge (g = gp) \wedge (lp = l))$$

DEFINITION:

$$\begin{aligned} & \text{rhoi2}(n, i, l, g, lp, gp) \\ &= (\text{at}(l, i, 2) \wedge (lp = \text{move}(l, i, 3)) \wedge (gp = \text{move}(g, i, 1))) \end{aligned}$$

DEFINITION:

$$\begin{aligned} & \text{rhoi3a}(n, i, l, g, lp, gp) \\ &= (\text{at}(l, i, 3) \\ & \quad \wedge (gp = g) \\ & \quad \wedge (lp = \text{move}(l, i, 4)) \\ & \quad \wedge (\neg \text{exist-union}(g, n, '(3\ 4)))) \end{aligned}$$

DEFINITION:

$$\begin{aligned} & \text{rhoi3b}(n, i, l, g, lp, gp) \\ &= (\text{at}(l, i, 3) \\ & \quad \wedge (gp = g) \\ & \quad \wedge (lp = l) \\ & \quad \wedge \text{exist-union}(g, n, '(3\ 4))) \end{aligned}$$

DEFINITION:

$$\begin{aligned} & \text{rhoi4}(n, i, l, g, lp, gp) \\ &= (\text{at}(l, i, 4) \wedge (gp = \text{move}(g, i, 3)) \wedge (lp = \text{move}(l, i, 5))) \end{aligned}$$

DEFINITION:

$$\begin{aligned} & \text{rhoi5a}(n, i, l, g, lp, gp) \\ &= (\text{at}(l, i, 5) \\ & \quad \wedge (gp = g) \\ & \quad \wedge \text{exist-union}(g, n, '(1)) \\ & \quad \wedge (lp = \text{move}(l, i, 6))) \end{aligned}$$

DEFINITION:

$$\begin{aligned} & \text{rhoi5b}(n, i, l, g, lp, gp) \\ &= (\text{at}(l, i, 5) \\ & \quad \wedge (gp = g) \\ & \quad \wedge (\neg \text{exist-union}(g, n, '(1))) \\ & \quad \wedge (lp = \text{move}(l, i, 8))) \end{aligned}$$

DEFINITION:

$$\begin{aligned} & \text{rhoi6}(n, i, l, g, lp, gp) \\ &= (\text{at}(l, i, 6) \wedge (gp = \text{move}(g, i, 2)) \wedge (lp = \text{move}(l, i, 7))) \end{aligned}$$

DEFINITION:

$$\begin{aligned} & \text{rhoi7a}(n, i, l, g, lp, gp) \\ = & (\text{at}(l, i, 7) \\ & \wedge \text{exist-union}(g, n, '(4)) \\ & \wedge (lp = \text{move}(l, i, 8)) \\ & \wedge (gp = g)) \end{aligned}$$

DEFINITION:

$$\begin{aligned} & \text{rhoi7b}(n, i, l, g, lp, gp) \\ = & (\text{at}(l, i, 7) \\ & \wedge (\neg \text{exist-union}(g, n, '(4))) \\ & \wedge (lp = l) \\ & \wedge (gp = g)) \end{aligned}$$

DEFINITION:

$$\begin{aligned} & \text{rhoi8}(n, i, l, g, lp, gp) \\ = & (\text{at}(l, i, 8) \wedge (gp = \text{move}(g, i, 4)) \wedge (lp = \text{move}(l, i, 9))) \end{aligned}$$

DEFINITION:

$$\begin{aligned} & \text{phi9}(i, n, g) \\ = & \text{if } (g \simeq \text{nil}) \vee (i \notin \mathbf{N}) \vee (n \notin \mathbf{N}) \text{ then f} \\ & \text{elseif } n = 0 \text{ then t} \\ & \text{else } ((n \not\prec i) \wedge \text{phi9}(i, n - 1, g)) \\ & \quad \vee (\text{union-at-n}(g, n, '(0 1)) \wedge \text{phi9}(i, n - 1, g)) \text{ endif} \end{aligned}$$

EVENT: Disable phi9.

DEFINITION:

$$\begin{aligned} & \text{rhoi9a}(n, i, l, g, lp, gp) \\ = & (\text{at}(l, i, 9) \wedge (gp = g) \wedge \text{phi9}(i, n, g) \wedge (lp = \text{move}(l, i, 10))) \end{aligned}$$

DEFINITION:

$$\begin{aligned} & \text{rhoi9b}(n, i, l, g, lp, gp) \\ = & (\text{at}(l, i, 9) \wedge (gp = g) \wedge (\neg \text{phi9}(i, n, g)) \wedge (lp = l)) \end{aligned}$$

DEFINITION:

$$\begin{aligned} & \text{rhoi10}(n, i, l, g, lp, gp) \\ = & (\text{at}(l, i, 10) \wedge (lp = \text{move}(l, i, 11)) \wedge (gp = g)) \end{aligned}$$

DEFINITION:

$$\begin{aligned} & \text{phi11}(i, n, g) \\ = & \text{if } (g \simeq \text{nil}) \vee (i \notin \mathbf{N}) \vee (n \notin \mathbf{N}) \text{ then f} \\ & \text{elseif } n = 0 \text{ then t} \\ & \text{else } ((i \not\prec n) \wedge \text{phi11}(i, n - 1, g)) \\ & \quad \vee ((\neg \text{union-at-n}(g, n, '(2 3))) \\ & \quad \wedge \text{phi11}(i, n - 1, g)) \text{ endif} \end{aligned}$$

EVENT: Disable phi11.

DEFINITION:

$$\begin{aligned} \text{rhoi11a}(n, i, l, g, lp, gp) \\ = & (\text{at}(l, i, 11) \\ & \wedge (gp = g) \\ & \wedge \text{phi11}(i, n, g) \\ & \wedge (lp = \text{move}(l, i, 12))) \end{aligned}$$

DEFINITION:

$$\begin{aligned} \text{rhoi11b}(n, i, l, g, lp, gp) \\ = & (\text{at}(l, i, 11) \wedge (gp = g) \wedge (\neg \text{phi11}(i, n, g)) \wedge (lp = l)) \end{aligned}$$

DEFINITION:

$$\begin{aligned} \text{rhoi12}(n, i, l, g, lp, gp) \\ = & (\text{at}(l, i, 12) \wedge (gp = \text{move}(g, i, 0)) \wedge (lp = \text{move}(l, i, 0))) \end{aligned}$$

;;;The transition operates on i'th.

DEFINITION:

$$\begin{aligned} \text{rhoi}(n, i, l, g, lp, gp) \\ = & (\text{rhoi0}(n, i, l, g, lp, gp) \\ & \vee \text{rhoi1a}(n, i, l, g, lp, gp) \\ & \vee \text{rhoi1b}(n, i, l, g, lp, gp) \\ & \vee \text{rhoi2}(n, i, l, g, lp, gp) \\ & \vee \text{rhoi3a}(n, i, l, g, lp, gp) \\ & \vee \text{rhoi3b}(n, i, l, g, lp, gp) \\ & \vee \text{rhoi4}(n, i, l, g, lp, gp) \\ & \vee \text{rhoi5a}(n, i, l, g, lp, gp) \\ & \vee \text{rhoi5b}(n, i, l, g, lp, gp) \\ & \vee \text{rhoi6}(n, i, l, g, lp, gp) \\ & \vee \text{rhoi7a}(n, i, l, g, lp, gp) \\ & \vee \text{rhoi7b}(n, i, l, g, lp, gp) \\ & \vee \text{rhoi8}(n, i, l, g, lp, gp) \\ & \vee \text{rhoi9a}(n, i, l, g, lp, gp) \\ & \vee \text{rhoi9b}(n, i, l, g, lp, gp) \\ & \vee \text{rhoi10}(n, i, l, g, lp, gp) \\ & \vee \text{rhoi11a}(n, i, l, g, lp, gp) \\ & \vee \text{rhoi11b}(n, i, l, g, lp, gp) \\ & \vee \text{rhoi12}(n, i, l, g, lp, gp)) \end{aligned}$$

EVENT: Disable rhoi.

;;; Propositions

;;;a0

DEFINITION:

a0(n, l, k)
= $((k \in \text{nset}(n)) \wedge \text{exist-union}(l, n, '(8\ 9\ 10\ 11\ 12)))$
→ $(\neg \text{at}(l, k, 4))$

EVENT: Disable a0.

;;;a1

DEFINITION:

a1(n, l, g)
= $(\text{exist-union}(l, n, '(8\ 9\ 10\ 11\ 12)))$
→ $\text{exist-intersect-8-12-3-4}(n, l, g)$

EVENT: Disable a1.

;;; a2

DEFINITION:

a2-at-n1-n2($n1, n2, l$)
= **if** $\text{union-at-n}(l, n1, '(10\ 11\ 12))$
then $\neg \text{union-at-n}(l, n2, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12))$
else t endif

EVENT: Disable a2-at-n1-n2.

DEFINITION:

a2-at-n2($n1, n2, l$)
= **if** $n2 \simeq 0$ **then t**
elseif $n2 \not\prec n1$ **then** a2-at-n2($n1, n2 - 1, l$)
else a2-at-n1-n2($n1, n2, l$) \wedge a2-at-n2($n1, n2 - 1, l$) **endif**

EVENT: Disable a2-at-n2.

DEFINITION:

a2($n1, n2, l$)
= **if** $n1 \simeq 0$ **then t**
else a2-at-n2($n1, n2, l$) \wedge a2($n1 - 1, n2, l$) **endif**

EVENT: Disable a2.

;;;a3

DEFINITION:

a3-at-n1-n2($n1, n2, l, g$)
= **if** at($l, n1, 12$) \wedge union-at-n($l, n2, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)$)
 then at($g, n2, 4$)
 else t endif

EVENT: Disable a3-at-n1-n2.

DEFINITION:

a3-at-n2($n1, n2, l, g$)
= **if** $n2 \simeq 0$ **then t**
 else a3-at-n1-n2($n1, n2, l, g$) \wedge a3-at-n2($n1, n2 - 1, l, g$) **endif**

EVENT: Disable a3-at-n2.

DEFINITION:

a3($n1, n2, l, g$)
= **if** $n1 \simeq 0$ **then t**
 else a3-at-n2($n1, n2, l, g$) \wedge a3($n1 - 1, n2, l, g$) **endif**

EVENT: Disable a3.

;; basic.ev ;;;
;;;ws implies that n is a number.

THEOREM: ws-num-n
 $ws(n, l, g) \rightarrow (n \in \mathbf{N})$

;;;ws implies that l is a list.

THEOREM: ws-list-l
 $ws(n, l, g) \rightarrow \text{listp}(l)$

;;;ws implies that g is a list.

THEOREM: ws-list-g
 $ws(n, l, g) \rightarrow \text{listp}(g)$

;;;ws implies that length of l is n.

THEOREM: ws-ln-l
 $ws(n, l, g) \rightarrow (\text{length}(l) = n)$

;;;ws implies that length of g is n.

THEOREM: ws-ln-g
 $ws(n, l, g) \rightarrow (\text{length}(g) = n)$

;;;ws and rho imply that lp is a list.

THEOREM: ws-ln-lp
 $(ws(n, l, g) \wedge (k \in \text{nset}(n)) \wedge \text{rhoi}(n, k, l, g, lp, gp)) \rightarrow \text{listp}(lp)$

;;;ws and rho imply that gp is a list.

THEOREM: ws-ln-gp
 $(ws(n, l, g) \wedge (k \in \text{nset}(n)) \wedge \text{rhoi}(n, k, l, g, lp, gp)) \rightarrow \text{listp}(gp)$

;;;ws implies that n is nonzero.

THEOREM: ws-n-not-0
 $ws(n, l, g) \rightarrow (n \neq 0)$

THEOREM: n-not-0
 $ws(n, l, g) \rightarrow (n \in \text{nset}(n))$

;*the rho! lemmas

;;;Auxiliary lemma.

THEOREM: lm-l-rholemma
 $(\text{listp}(l)$
 $\wedge (j \in \text{nset}(\text{length}(l)))$
 $\wedge (k \in \text{nset}(\text{length}(l)))$
 $\wedge \text{rhoi}(n, k, l, g, lp, gp)$
 $\wedge (k \neq j))$
 $\rightarrow (\text{nth}(l, j) = \text{nth}(lp, j))$

EVENT: Disable lm-l-rholemma.

;;;Rholemma for list l.

THEOREM: l-rholemma
 $(ws(n, l, g)$
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{rhoi}(n, k, l, g, lp, gp)$
 $\wedge (k \neq j))$
 $\rightarrow (\text{nth}(l, j) = \text{nth}(lp, j))$

;;;Auxiliary lemma.

THEOREM: lm-g-rholemma

(listp (g)
 \wedge (j \in nset (length (g)))
 \wedge (k \in nset (length (g)))
 \wedge rhoi (n, k, l, g, lp, gp)
 \wedge (k \neq j)
 \rightarrow (nth (g, j) = nth (gp, j))

EVENT: Disable lm-g-rholemma.

;;;Rholemma for list g.

THEOREM: g-rholemma

(ws (n, l, g)
 \wedge (j \in nset (n))
 \wedge (k \in nset (n))
 \wedge rhoi (n, k, l, g, lp, gp)
 \wedge (k \neq j)
 \rightarrow (nth (g, j) = nth (gp, j))

;;; lp-gp-same-l-g

;;;Another version of Rholemma for l.

;;;It applies to (union-at-n l j m) in stead of
;;;(nth l j).

THEOREM: lp-same-l

(ws (n, l, g)
 \wedge listp (m)
 \wedge (j \in nset (n))
 \wedge (k \in nset (n))
 \wedge rhoi (n, k, l, g, lp, gp)
 \wedge (j \neq k)
 \wedge union-at-n (l, j, m)
 \rightarrow union-at-n (lp, j, m)

;;;Contrast to the one above,

;;;the order of l and lp is reversed.

THEOREM: l-same-lp

(ws (n, l, g)
 \wedge listp (m)
 \wedge (j \in nset (n))

$\wedge (k \in \text{nset}(n))$
 $\wedge \text{rhoi}(n, k, l, g, lp, gp)$
 $\wedge (j \neq k)$
 $\wedge \text{union-at-n}(lp, j, m)$
 $\rightarrow \text{union-at-n}(l, j, m)$

THEOREM: lp-same-l-not

$(\text{ws}(n, l, g)$
 $\wedge \text{listp}(m)$
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{rhoi}(n, k, l, g, lp, gp)$
 $\wedge (j \neq k)$
 $\wedge (\neg \text{union-at-n}(l, j, m)))$
 $\rightarrow (\neg \text{union-at-n}(lp, j, m))$

;;;Another version of Rholemma for g.

THEOREM: gp-same-g

$(\text{ws}(n, l, g)$
 $\wedge \text{listp}(m)$
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{rhoi}(n, k, l, g, lp, gp)$
 $\wedge (j \neq k)$
 $\wedge \text{union-at-n}(g, j, m)$
 $\rightarrow \text{union-at-n}(gp, j, m)$

;;;Contrast to the one above,
 ;;;the order of g and gp is reversed.

THEOREM: g-same-gp

$(\text{ws}(n, l, g)$
 $\wedge \text{listp}(m)$
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{rhoi}(n, k, l, g, lp, gp)$
 $\wedge (j \neq k)$
 $\wedge \text{union-at-n}(gp, j, m)$
 $\rightarrow \text{union-at-n}(g, j, m)$

;;;It applies to (at l j m) in stead of
 ;;;(nth l j).

THEOREM: l-same-lp-at

$$\begin{aligned}
& (\text{ws } (n, l, g) \\
& \wedge (j \in \text{nset } (n)) \\
& \wedge (k \in \text{nset } (n)) \\
& \wedge (m \in \mathbf{N}) \\
& \wedge \text{rhoi } (n, k, l, g, lp, gp) \\
& \wedge (j \neq k) \\
& \wedge \text{at } (lp, j, m)) \\
& \rightarrow \text{at } (l, j, m)
\end{aligned}$$

THEOREM: gp-same-g-at

$$\begin{aligned}
& (\text{ws } (n, l, g) \\
& \wedge (j \in \text{nset } (n)) \\
& \wedge (k \in \text{nset } (n)) \\
& \wedge (m \in \mathbf{N}) \\
& \wedge \text{rhoi } (n, k, l, g, lp, gp) \\
& \wedge (j \neq k) \\
& \wedge \text{at } (g, j, m)) \\
& \rightarrow \text{at } (gp, j, m)
\end{aligned}$$

THEOREM: l-same-lp-at-not

$$\begin{aligned}
& (\text{ws } (n, l, g) \\
& \wedge (m \in \mathbf{N}) \\
& \wedge (j \in \text{nset } (n)) \\
& \wedge (k \in \text{nset } (n)) \\
& \wedge \text{rhoi } (n, k, l, g, lp, gp) \\
& \wedge (j \neq k) \\
& \wedge (\neg \text{at } (l, j, m))) \\
& \rightarrow (\neg \text{at } (lp, j, m))
\end{aligned}$$

;*basic properties of a2

;;;Auxiliary lemma.

THEOREM: lm-a2-at-n2-a2-at-n1-n2

$$\begin{aligned}
& ((n \in \mathbf{N}) \wedge (k \in \mathbf{N}) \wedge (j \in \text{nset } (n)) \wedge (j < k) \wedge \text{a2-at-n2 } (k, n, l)) \\
& \rightarrow \text{a2-at-n1-n2 } (k, j, l)
\end{aligned}$$

EVENT: Disable lm-a2-at-n2-a2-at-n1-n2.

THEOREM: a2-at-n2-a2-at-n1-n2

$$\begin{aligned}
& (\text{ws } (n, l, g) \\
& \wedge (k \in \text{nset } (n)) \\
& \wedge (j \in \text{nset } (n)) \\
& \wedge (j < k) \\
& \wedge \text{a2-at-n2 } (k, n, l)) \\
& \rightarrow \text{a2-at-n1-n2 } (k, j, l)
\end{aligned}$$

THEOREM: lm-a2-a2-at-n2
 $((n \in \mathbf{N}) \wedge (i \in \mathbf{N}) \wedge (k \in \text{nset}(n)) \wedge a2(n, i, l)) \rightarrow a2\text{-at-n2}(k, i, l)$

THEOREM: a2-a2-at-n2
 $(ws(n, l, g) \wedge (i \in \text{nset}(n)) \wedge (k \in \text{nset}(n)) \wedge a2(n, i, l))$
 $\rightarrow a2\text{-at-n2}(k, i, l)$

;*basic properties of a3

THEOREM: lm-a3-at-n2-a3-at-n1-n2
 $((n \in \mathbf{N}) \wedge (u \in \mathbf{N}) \wedge (j \in \text{nset}(n)) \wedge a3\text{-at-n2}(u, n, l, g))$
 $\rightarrow a3\text{-at-n1-n2}(u, j, l, g)$

EVENT: Disable lm-a3-at-n2-a3-at-n1-n2.

THEOREM: a3-at-n2-a3-at-n1-n2
 $(ws(n, l, g) \wedge (u \in \text{nset}(n)) \wedge (j \in \text{nset}(n)) \wedge a3\text{-at-n2}(u, n, l, g))$
 $\rightarrow a3\text{-at-n1-n2}(u, j, l, g)$

THEOREM: lm-a3-a3-at-n2
 $((n \in \mathbf{N}) \wedge (i \in \mathbf{N}) \wedge (u \in \text{nset}(n)) \wedge a3(n, i, l, g))$
 $\rightarrow a3\text{-at-n2}(u, i, l, g)$

EVENT: Disable lm-a3-a3-at-n2.

THEOREM: a3-a3-at-n2
 $(ws(n, l, g) \wedge (i \in \text{nset}(n)) \wedge (u \in \text{nset}(n)) \wedge a3(n, i, l, g))$
 $\rightarrow a3\text{-at-n2}(u, i, l, g)$

;;;;;Instances used in the proofs of
;;;;;a1 a2 and a3.

;;;(a2 n n l) and (a3 n n l g) are involved
;;;in double bounded quantifiers
;;;forall i \leq n forall j \leq n,
;;;with their quantifier-free formulas
;;;(a3-at-n1-n2 i j l g) and (a2-at-n1-n2 i j l)
;;;respectively. What follows are all instances of
;;;the following type: If (a3 n n l g) holds, then
;;;in particular, so its instance
;;;(a3-at-n1-n2 i j l g) does.

;;;The instances are i and j.

THEOREM: a3-i-j-a3-at-n1-n2
 $(ws(n, l, g) \wedge (i \in nset(n)) \wedge (j \in nset(n)) \wedge a3(n, n, l, g))$
 $\rightarrow a3-at-n1-n2(i, j, l, g)$

;;;The instances are k and
 ;;;(exist-union lp n '(8 9 10 11 12)).

THEOREM: a3-ex-a3-at-n1-n2
 $(ws(n, l, g)$
 $\wedge (k \in nset(n))$
 $\wedge a3(n, n, l, g)$
 $\wedge exist-union(lp, n, '(8 9 10 11 12)))$
 $\rightarrow a3-at-n1-n2(k, exist-union(lp, n, '(8 9 10 11 12)), l, g)$

THEOREM: a2-n-a2-at-n2
 $(ws(n, l, g) \wedge (k \in nset(n)) \wedge a2(n, n, l)) \rightarrow a2-at-n2(k, n, l)$

;;;The instances are i and j.

THEOREM: a2-i-j-a2-at-n1-n2
 $(ws(n, l, g)$
 $\wedge (i \in nset(n))$
 $\wedge (j \in nset(n))$
 $\wedge a2(n, n, l)$
 $\wedge (j < i))$
 $\rightarrow a2-at-n1-n2(i, j, l)$

;;;;;; ws.ev ;;;;;;

THEOREM: j-eq-k-move-member-g
 $(listp(g) \wedge listp(m) \wedge (i \in m) \wedge (k \in nset(length(g))))$
 $\rightarrow (nth(move(g, k, i), k) \in m)$

THEOREM: j-neq-k-move-member-g
 $(listp(g)$
 $\wedge listp(m)$
 $\wedge (k \in nset(length(g)))$
 $\wedge (j \neq k)$
 $\wedge (i \in m)$
 $\wedge (nth(g, j) \in m))$
 $\rightarrow (nth(move(g, k, i), j) \in m)$

THEOREM: move-member-g
 $(listp(g)$
 $\wedge listp(m)$
 $\wedge (i \in m)$

$\wedge (k \in \text{nset}(\text{length}(g)))$
 $\wedge (\text{nth}(g, j) \in m)$
 $\rightarrow (\text{nth}(\text{move}(g, k, i), j) \in m)$

THEOREM: move-member-l

$(\text{listp}(l)$
 $\wedge \text{listp}(m)$
 $\wedge (j \in \mathbf{N})$
 $\wedge (i \in m)$
 $\wedge (k \in \text{nset}(\text{length}(l)))$
 $\wedge (\text{nth}(l, j) \in m)$
 $\rightarrow (\text{nth}(\text{move}(l, k, i), j) \in m)$

THEOREM: ws-union-g

$\text{ws}(n, l, g) \rightarrow \text{all-union}(g, n, '(0\ 1\ 2\ 3\ 4))$

THEOREM: ws-union-l

$\text{ws}(n, l, g) \rightarrow \text{all-union}(l, n, '(0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12))$

THEOREM: rho0-preserves-union-g

$(\text{ws}(n, l, g) \wedge (k \in \text{nset}(n)) \wedge \text{rhoi0}(n, k, l, g, lp, gp))$
 $\rightarrow \text{all-union}(gp, n, '(0\ 1\ 2\ 3\ 4))$

THEOREM: rho1a-preserves-union-g

$(\text{ws}(n, l, g) \wedge (k \in \text{nset}(n)) \wedge \text{rhoi1a}(n, k, l, g, lp, gp))$
 $\rightarrow \text{all-union}(gp, n, '(0\ 1\ 2\ 3\ 4))$

THEOREM: rho1b-preserves-union-g

$(\text{ws}(n, l, g) \wedge (k \in \text{nset}(n)) \wedge \text{rhoi1b}(n, k, l, g, lp, gp))$
 $\rightarrow \text{all-union}(gp, n, '(0\ 1\ 2\ 3\ 4))$

THEOREM: lm-rho2-preserves-union-g

$(\text{listp}(g)$
 $\wedge (\text{length}(g) = n)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{all-union}(g, j, '(0\ 1\ 2\ 3\ 4))$
 $\wedge \text{rhoi2}(n, k, l, g, lp, gp))$
 $\rightarrow \text{all-union}(gp, j, '(0\ 1\ 2\ 3\ 4))$

THEOREM: rho2-preserves-union-g

$(\text{ws}(n, l, g) \wedge (k \in \text{nset}(n)) \wedge \text{rhoi2}(n, k, l, g, lp, gp))$
 $\rightarrow \text{all-union}(gp, n, '(0\ 1\ 2\ 3\ 4))$

THEOREM: rho3a-preserves-union-g

$(\text{ws}(n, l, g) \wedge (k \in \text{nset}(n)) \wedge \text{rhoi3a}(n, k, l, g, lp, gp))$
 $\rightarrow \text{all-union}(gp, n, '(0\ 1\ 2\ 3\ 4))$

THEOREM: rho3b-preserves-union-g
 $(ws(n, l, g) \wedge (k \in nset(n)) \wedge rhoi3b(n, k, l, g, lp, gp))$
 $\rightarrow all_union(gp, n, '(0\ 1\ 2\ 3\ 4))$

THEOREM: lm-rho4-preserves-union-g
 $(listp(g)$
 $\wedge (length(g) = n)$
 $\wedge (k \in nset(n))$
 $\wedge all_union(g, j, '(0\ 1\ 2\ 3\ 4))$
 $\wedge rhoi4(n, k, l, g, lp, gp))$
 $\rightarrow all_union(gp, j, '(0\ 1\ 2\ 3\ 4))$

THEOREM: rho4-preserves-union-g
 $(ws(n, l, g) \wedge (k \in nset(n)) \wedge rhoi4(n, k, l, g, lp, gp))$
 $\rightarrow all_union(gp, n, '(0\ 1\ 2\ 3\ 4))$

THEOREM: rho5a-preserves-union-g
 $(ws(n, l, g) \wedge (k \in nset(n)) \wedge rhoi5a(n, k, l, g, lp, gp))$
 $\rightarrow all_union(gp, n, '(0\ 1\ 2\ 3\ 4))$

THEOREM: rho5b-preserves-union-g
 $(ws(n, l, g) \wedge (k \in nset(n)) \wedge rhoi5b(n, k, l, g, lp, gp))$
 $\rightarrow all_union(gp, n, '(0\ 1\ 2\ 3\ 4))$

THEOREM: lm-rho6-preserves-union-g
 $(listp(g)$
 $\wedge (length(g) = n)$
 $\wedge (k \in nset(n))$
 $\wedge all_union(g, j, '(0\ 1\ 2\ 3\ 4))$
 $\wedge rhoi6(n, k, l, g, lp, gp))$
 $\rightarrow all_union(gp, j, '(0\ 1\ 2\ 3\ 4))$

THEOREM: rho6-preserves-union-g
 $(ws(n, l, g) \wedge (k \in nset(n)) \wedge rhoi6(n, k, l, g, lp, gp))$
 $\rightarrow all_union(gp, n, '(0\ 1\ 2\ 3\ 4))$

THEOREM: rho7a-preserves-union-g
 $(ws(n, l, g) \wedge (k \in nset(n)) \wedge rhoi7a(n, k, l, g, lp, gp))$
 $\rightarrow all_union(gp, n, '(0\ 1\ 2\ 3\ 4))$

THEOREM: rho7b-preserves-union-g
 $(ws(n, l, g) \wedge (k \in nset(n)) \wedge rhoi7b(n, k, l, g, lp, gp))$
 $\rightarrow all_union(gp, n, '(0\ 1\ 2\ 3\ 4))$

THEOREM: lm-rho8-preserves-union-g

(listp(g)
 \wedge (length(g) = n)
 \wedge ($k \in$ nset(n))
 \wedge all-union($g, j, '(0\ 1\ 2\ 3\ 4)$)
 \wedge rhoi8(n, k, l, g, lp, gp)
 \rightarrow all-union($gp, j, '(0\ 1\ 2\ 3\ 4)$)

THEOREM: rho8-preserves-union-g

(ws(n, l, g) \wedge ($k \in$ nset(n)) \wedge rhoi8(n, k, l, g, lp, gp)
 \rightarrow all-union($gp, n, '(0\ 1\ 2\ 3\ 4)$)

THEOREM: rho9a-preserves-union-g

(ws(n, l, g) \wedge ($k \in$ nset(n)) \wedge rhoi9a(n, k, l, g, lp, gp)
 \rightarrow all-union($gp, n, '(0\ 1\ 2\ 3\ 4)$)

THEOREM: rho9b-preserves-union-g

(ws(n, l, g) \wedge ($k \in$ nset(n)) \wedge rhoi9b(n, k, l, g, lp, gp)
 \rightarrow all-union($gp, n, '(0\ 1\ 2\ 3\ 4)$)

THEOREM: lm-rho10-preserves-union-g

(listp(g)
 \wedge (length(g) = n)
 \wedge ($k \in$ nset(n))
 \wedge all-union($g, j, '(0\ 1\ 2\ 3\ 4)$)
 \wedge rhoi10(n, k, l, g, lp, gp)
 \rightarrow all-union($gp, j, '(0\ 1\ 2\ 3\ 4)$)

THEOREM: rho10-preserves-union-g

(ws(n, l, g) \wedge ($k \in$ nset(n)) \wedge rhoi10(n, k, l, g, lp, gp)
 \rightarrow all-union($gp, n, '(0\ 1\ 2\ 3\ 4)$)

THEOREM: rho11a-preserves-union-g

(ws(n, l, g) \wedge ($k \in$ nset(n)) \wedge rhoi11a(n, k, l, g, lp, gp)
 \rightarrow all-union($gp, n, '(0\ 1\ 2\ 3\ 4)$)

THEOREM: rho11b-preserves-union-g

(ws(n, l, g) \wedge ($k \in$ nset(n)) \wedge rhoi11b(n, k, l, g, lp, gp)
 \rightarrow all-union($gp, n, '(0\ 1\ 2\ 3\ 4)$)

THEOREM: lm-rho12-preserves-union-g

(listp(g)
 \wedge (length(g) = n)
 \wedge ($k \in$ nset(n))
 \wedge all-union($g, j, '(0\ 1\ 2\ 3\ 4)$)
 \wedge rhoi12(n, k, l, g, lp, gp)
 \rightarrow all-union($gp, j, '(0\ 1\ 2\ 3\ 4)$)

THEOREM: rho12-preserves-union-g
 $(ws(n, l, g) \wedge (k \in nset(n)) \wedge rhoi12(n, k, l, g, lp, gp))$
 \rightarrow all-union($gp, n, '(0\ 1\ 2\ 3\ 4)$)

THEOREM: rho-preserves-union-g
 $(ws(n, l, g) \wedge (k \in nset(n)) \wedge rhoi(n, k, l, g, lp, gp))$
 \rightarrow all-union($gp, n, '(0\ 1\ 2\ 3\ 4)$)

THEOREM: lm-rho0-preserves-union-l
(listp(l)
 \wedge (length(l) = n)
 \wedge ($k \in nset(n)$)
 \wedge all-union($l, j, '(0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)$)
 \wedge rhoi0(n, k, l, g, lp, gp)
 \rightarrow all-union($lp, j, '(0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)$)

THEOREM: rho0-preserves-union-l
 $(ws(n, l, g) \wedge (k \in nset(n)) \wedge rhoi0(n, k, l, g, lp, gp))$
 \rightarrow all-union($lp, n, '(0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)$)

THEOREM: lm-rho1a-preserves-union-l
(listp(l)
 \wedge (length(l) = n)
 \wedge ($k \in nset(n)$)
 \wedge all-union($l, j, '(0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)$)
 \wedge rhoi1a(n, k, l, g, lp, gp)
 \rightarrow all-union($lp, j, '(0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)$)

THEOREM: rho1a-preserves-union-l
 $(ws(n, l, g) \wedge (k \in nset(n)) \wedge rhoi1a(n, k, l, g, lp, gp))$
 \rightarrow all-union($lp, n, '(0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)$)

THEOREM: lm-rho1b-preserves-union-l
(listp(l)
 \wedge (length(l) = n)
 \wedge ($k \in nset(n)$)
 \wedge all-union($l, j, '(0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)$)
 \wedge rhoi1b(n, k, l, g, lp, gp)
 \rightarrow all-union($lp, j, '(0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)$)

THEOREM: rho1b-preserves-union-l
 $(ws(n, l, g) \wedge (k \in nset(n)) \wedge rhoi1b(n, k, l, g, lp, gp))$
 \rightarrow all-union($lp, n, '(0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)$)

THEOREM: lm-rho2-preserves-union-l

$(\text{listp}(l)$
 $\wedge (\text{length}(l) = n)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{all-union}(l, j, '(0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12))$
 $\wedge \text{rhoi2}(n, k, l, g, lp, gp))$
 $\rightarrow \text{all-union}(lp, j, '(0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12))$

THEOREM: rho2-preserves-union-l

$(\text{ws}(n, l, g) \wedge (k \in \text{nset}(n)) \wedge \text{rhoi2}(n, k, l, g, lp, gp))$
 $\rightarrow \text{all-union}(lp, n, '(0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12))$

THEOREM: lm-rho3a-preserves-union-l

$(\text{listp}(l)$
 $\wedge (\text{length}(l) = n)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{all-union}(l, j, '(0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12))$
 $\wedge \text{rhoi3a}(n, k, l, g, lp, gp))$
 $\rightarrow \text{all-union}(lp, j, '(0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12))$

THEOREM: rho3a-preserves-union-l

$(\text{ws}(n, l, g) \wedge (k \in \text{nset}(n)) \wedge \text{rhoi3a}(n, k, l, g, lp, gp))$
 $\rightarrow \text{all-union}(lp, n, '(0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12))$

THEOREM: lm-rho3b-preserves-union-l

$(\text{listp}(l)$
 $\wedge (\text{length}(l) = n)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{all-union}(l, j, '(0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12))$
 $\wedge \text{rhoi3b}(n, k, l, g, lp, gp))$
 $\rightarrow \text{all-union}(lp, j, '(0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12))$

THEOREM: rho3b-preserves-union-l

$(\text{ws}(n, l, g) \wedge (k \in \text{nset}(n)) \wedge \text{rhoi3b}(n, k, l, g, lp, gp))$
 $\rightarrow \text{all-union}(lp, n, '(0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12))$

THEOREM: lm-rho4-preserves-union-l

$(\text{listp}(l)$
 $\wedge (\text{length}(l) = n)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{all-union}(l, j, '(0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12))$
 $\wedge \text{rhoi4}(n, k, l, g, lp, gp))$
 $\rightarrow \text{all-union}(lp, j, '(0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12))$

THEOREM: rho4-preserves-union-l

$(\text{ws}(n, l, g) \wedge (k \in \text{nset}(n)) \wedge \text{rhoi4}(n, k, l, g, lp, gp))$
 $\rightarrow \text{all-union}(lp, n, '(0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12))$

THEOREM: lm-rho5a-preserves-union-l

(listp (l)
 \wedge (length (l) = n)
 \wedge ($k \in$ nset (n))
 \wedge all-union ($l, j, '(0 1 2 3 4 5 6 7 8 9 10 11 12)$)
 \wedge rhoi5a (n, k, l, g, lp, gp)
 \rightarrow all-union ($lp, j, '(0 1 2 3 4 5 6 7 8 9 10 11 12)$))

THEOREM: rho5a-preserves-union-l

(ws (n, l, g) \wedge ($k \in$ nset (n)) \wedge rhoi5a (n, k, l, g, lp, gp)
 \rightarrow all-union ($lp, n, '(0 1 2 3 4 5 6 7 8 9 10 11 12)$))

THEOREM: lm-rho5b-preserves-union-l

(listp (l)
 \wedge (length (l) = n)
 \wedge ($k \in$ nset (n))
 \wedge all-union ($l, j, '(0 1 2 3 4 5 6 7 8 9 10 11 12)$)
 \wedge rhoi5b (n, k, l, g, lp, gp)
 \rightarrow all-union ($lp, j, '(0 1 2 3 4 5 6 7 8 9 10 11 12)$))

THEOREM: rho5b-preserves-union-l

(ws (n, l, g) \wedge ($k \in$ nset (n)) \wedge rhoi5b (n, k, l, g, lp, gp)
 \rightarrow all-union ($lp, n, '(0 1 2 3 4 5 6 7 8 9 10 11 12)$))

THEOREM: lm-rho6-preserves-union-l

(listp (l)
 \wedge (length (l) = n)
 \wedge ($k \in$ nset (n))
 \wedge all-union ($l, j, '(0 1 2 3 4 5 6 7 8 9 10 11 12)$)
 \wedge rhoi6 (n, k, l, g, lp, gp)
 \rightarrow all-union ($lp, j, '(0 1 2 3 4 5 6 7 8 9 10 11 12)$))

THEOREM: rho6-preserves-union-l

(ws (n, l, g) \wedge ($k \in$ nset (n)) \wedge rhoi6 (n, k, l, g, lp, gp)
 \rightarrow all-union ($lp, n, '(0 1 2 3 4 5 6 7 8 9 10 11 12)$))

THEOREM: lm-rho7a-preserves-union-l

(listp (l)
 \wedge (length (l) = n)
 \wedge ($k \in$ nset (n))
 \wedge all-union ($l, j, '(0 1 2 3 4 5 6 7 8 9 10 11 12)$)
 \wedge rhoi7a (n, k, l, g, lp, gp)
 \rightarrow all-union ($lp, j, '(0 1 2 3 4 5 6 7 8 9 10 11 12)$))

THEOREM: rho7a-preserves-union-l

(ws (n, l, g) \wedge ($k \in$ nset (n)) \wedge rhoi7a (n, k, l, g, lp, gp)
 \rightarrow all-union ($lp, n, '(0 1 2 3 4 5 6 7 8 9 10 11 12)$))

THEOREM: lm-rho7b-preserves-union-l
(listp (*l*)
 \wedge (length (*l*) = *n*)
 \wedge (*k* \in nset (*n*))
 \wedge all-union (*l*, *j*, '(0 1 2 3 4 5 6 7 8 9 10 11 12))
 \wedge rhoi7b (*n*, *k*, *l*, *g*, *lp*, *gp*)
 \rightarrow all-union (*lp*, *j*, '(0 1 2 3 4 5 6 7 8 9 10 11 12))

THEOREM: rho7b-preserves-union-l
(ws (*n*, *l*, *g*) \wedge (*k* \in nset (*n*)) \wedge rhoi7b (*n*, *k*, *l*, *g*, *lp*, *gp*)
 \rightarrow all-union (*lp*, *n*, '(0 1 2 3 4 5 6 7 8 9 10 11 12))

THEOREM: lm-rho8-preserves-union-l
(listp (*l*)
 \wedge (length (*l*) = *n*)
 \wedge (*k* \in nset (*n*))
 \wedge all-union (*l*, *j*, '(0 1 2 3 4 5 6 7 8 9 10 11 12))
 \wedge rhoi8 (*n*, *k*, *l*, *g*, *lp*, *gp*)
 \rightarrow all-union (*lp*, *j*, '(0 1 2 3 4 5 6 7 8 9 10 11 12))

THEOREM: rho8-preserves-union-l
(ws (*n*, *l*, *g*) \wedge (*k* \in nset (*n*)) \wedge rhoi8 (*n*, *k*, *l*, *g*, *lp*, *gp*)
 \rightarrow all-union (*lp*, *n*, '(0 1 2 3 4 5 6 7 8 9 10 11 12))

THEOREM: lm-rho9a-preserves-union-l
(listp (*l*)
 \wedge (length (*l*) = *n*)
 \wedge (*k* \in nset (*n*))
 \wedge all-union (*l*, *j*, '(0 1 2 3 4 5 6 7 8 9 10 11 12))
 \wedge rhoi9a (*n*, *k*, *l*, *g*, *lp*, *gp*)
 \rightarrow all-union (*lp*, *j*, '(0 1 2 3 4 5 6 7 8 9 10 11 12))

THEOREM: rho9a-preserves-union-l
(ws (*n*, *l*, *g*) \wedge (*k* \in nset (*n*)) \wedge rhoi9a (*n*, *k*, *l*, *g*, *lp*, *gp*)
 \rightarrow all-union (*lp*, *n*, '(0 1 2 3 4 5 6 7 8 9 10 11 12))

THEOREM: lm-rho9b-preserves-union-l
(listp (*l*)
 \wedge (length (*l*) = *n*)
 \wedge (*k* \in nset (*n*))
 \wedge all-union (*l*, *j*, '(0 1 2 3 4 5 6 7 8 9 10 11 12))
 \wedge rhoi9b (*n*, *k*, *l*, *g*, *lp*, *gp*)
 \rightarrow all-union (*lp*, *j*, '(0 1 2 3 4 5 6 7 8 9 10 11 12))

THEOREM: rho9b-preserves-union-l
(ws (*n*, *l*, *g*) \wedge (*k* \in nset (*n*)) \wedge rhoi9b (*n*, *k*, *l*, *g*, *lp*, *gp*)
 \rightarrow all-union (*lp*, *n*, '(0 1 2 3 4 5 6 7 8 9 10 11 12))

THEOREM: lm-rho10-preserves-union-l
(listp (*l*)
 \wedge (length (*l*) = *n*)
 \wedge (*k* \in nset (*n*))
 \wedge all-union (*l*, *j*, '(0 1 2 3 4 5 6 7 8 9 10 11 12))
 \wedge rhoi10 (*n*, *k*, *l*, *g*, *lp*, *gp*)
 \rightarrow all-union (*lp*, *j*, '(0 1 2 3 4 5 6 7 8 9 10 11 12))

THEOREM: rho10-preserves-union-l
(ws (*n*, *l*, *g*) \wedge (*k* \in nset (*n*)) \wedge rhoi10 (*n*, *k*, *l*, *g*, *lp*, *gp*)
 \rightarrow all-union (*lp*, *n*, '(0 1 2 3 4 5 6 7 8 9 10 11 12))

THEOREM: lm-rho11a-preserves-union-l
(listp (*l*)
 \wedge (length (*l*) = *n*)
 \wedge (*k* \in nset (*n*))
 \wedge all-union (*l*, *j*, '(0 1 2 3 4 5 6 7 8 9 10 11 12))
 \wedge rhoi11a (*n*, *k*, *l*, *g*, *lp*, *gp*)
 \rightarrow all-union (*lp*, *j*, '(0 1 2 3 4 5 6 7 8 9 10 11 12))

THEOREM: rho11a-preserves-union-l
(ws (*n*, *l*, *g*) \wedge (*k* \in nset (*n*)) \wedge rhoi11a (*n*, *k*, *l*, *g*, *lp*, *gp*)
 \rightarrow all-union (*lp*, *n*, '(0 1 2 3 4 5 6 7 8 9 10 11 12))

THEOREM: lm-rho11b-preserves-union-l
(listp (*l*)
 \wedge (length (*l*) = *n*)
 \wedge (*k* \in nset (*n*))
 \wedge all-union (*l*, *j*, '(0 1 2 3 4 5 6 7 8 9 10 11 12))
 \wedge rhoi11b (*n*, *k*, *l*, *g*, *lp*, *gp*)
 \rightarrow all-union (*lp*, *j*, '(0 1 2 3 4 5 6 7 8 9 10 11 12))

THEOREM: rho11b-preserves-union-l
(ws (*n*, *l*, *g*) \wedge (*k* \in nset (*n*)) \wedge rhoi11b (*n*, *k*, *l*, *g*, *lp*, *gp*)
 \rightarrow all-union (*lp*, *n*, '(0 1 2 3 4 5 6 7 8 9 10 11 12))

THEOREM: lm-rho12-preserves-union-l
(listp (*l*)
 \wedge (length (*l*) = *n*)
 \wedge (*k* \in nset (*n*))
 \wedge all-union (*l*, *j*, '(0 1 2 3 4 5 6 7 8 9 10 11 12))
 \wedge rhoi12 (*n*, *k*, *l*, *g*, *lp*, *gp*)
 \rightarrow all-union (*lp*, *j*, '(0 1 2 3 4 5 6 7 8 9 10 11 12))

THEOREM: rho12-preserves-union-l
(ws (*n*, *l*, *g*) \wedge (*k* \in nset (*n*)) \wedge rhoi12 (*n*, *k*, *l*, *g*, *lp*, *gp*)
 \rightarrow all-union (*lp*, *n*, '(0 1 2 3 4 5 6 7 8 9 10 11 12))

THEOREM: rho-preserves-union-l
 $(ws(n, l, g) \wedge (k \in nset(n)) \wedge rhoi(n, k, l, g, lp, gp))$
 $\rightarrow all\text{-}union(lp, n, '(0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12))$

THEOREM: lm-rho-preserves-ln-l
 $(listp(l) \wedge (length(l) = n) \wedge (k \in nset(n)) \wedge rhoi(n, k, l, g, lp, gp))$
 $\rightarrow (length(lp) = n)$

THEOREM: rho-preserves-ln-l
 $(ws(n, l, g) \wedge (k \in nset(n)) \wedge rhoi(n, k, l, g, lp, gp))$
 $\rightarrow (length(lp) = n)$

THEOREM: lm-rho-preserves-ln-g
 $(listp(g) \wedge (length(g) = n) \wedge (k \in nset(n)) \wedge rhoi(n, k, l, g, lp, gp))$
 $\rightarrow (length(gp) = n)$

THEOREM: rho-preserves-ln-g
 $(ws(n, l, g) \wedge (k \in nset(n)) \wedge rhoi(n, k, l, g, lp, gp))$
 $\rightarrow (length(gp) = n)$

THEOREM: lm-rho-preserves-ws
 $((n \in \mathbf{N})$
 $\wedge listp(lp)$
 $\wedge listp(gp)$
 $\wedge (length(lp) = n)$
 $\wedge (length(gp) = n)$
 $\wedge all\text{-}union(lp, n, '(0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12))$
 $\wedge all\text{-}union(gp, n, '(0\ 1\ 2\ 3\ 4))$
 $\rightarrow ws(n, lp, gp)$

THEOREM: rho-preserves-ws
 $(ws(n, l, g) \wedge (k \in nset(n)) \wedge rhoi(n, k, l, g, lp, gp)) \rightarrow ws(n, lp, gp)$

;;;;;;;;;;;;; lg.ev ;;;;;;;;;;;;;;
 ;;rhoi0

THEOREM: n-neq-k-rhoi0
 $(listp(l)$
 $\wedge listp(g)$
 $\wedge (n \in \mathbf{N})$
 $\wedge (k \in nset(length(l)))$
 $\wedge (k \neq n)$
 $\wedge at(l, k, 0)$
 $\wedge lg\text{-}at\text{-}n(n, l, g)$
 $\rightarrow lg\text{-}at\text{-}n(n, move(l, k, 1), g)$

EVENT: Disable n-neq-k-rhoi0.

THEOREM: n-eq-k-rhoi0
(listp (l)
 \wedge listp (g)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge at ($l, k, 0$)
 \wedge lg-at-n (k, l, g)
 \rightarrow lg-at-n ($k, \text{move}(l, k, 1), g$)

EVENT: Disable n-eq-k-rhoi0.

THEOREM: lg-at-rhoi0
(listp (l)
 \wedge listp (g)
 \wedge ($n \in \mathbf{N}$)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge at ($l, k, 0$)
 \wedge lg-at-n (n, l, g)
 \rightarrow lg-at-n ($n, \text{move}(l, k, 1), g$)

EVENT: Disable lg-at-rhoi0.

THEOREM: lg-rhoi0
(listp (l)
 \wedge listp (g)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge ($n \in \mathbf{N}$)
 \wedge at ($l, k, 0$)
 \wedge lg (n, l, g)
 \rightarrow lg ($n, \text{move}(l, k, 1), g$)

EVENT: Disable lg-rhoi0.

THEOREM: rhoi0-preserves-lg
(ws (n, l, g) \wedge ($k \in \text{nset}(n)$) \wedge rhoi0 (n, k, l, g, lp, gp) \wedge lg (n, l, g)
 \rightarrow lg (n, lp, gp)

;;;rhoi1a

THEOREM: n-neq-k-rhoi1a
(listp (l)
 \wedge listp (g)

$$\begin{aligned}
& \wedge (n \in \mathbf{N}) \\
& \wedge (k \in \text{nset}(\text{length}(l))) \\
& \wedge (k \neq n) \\
& \wedge \text{at}(l, k, 1) \\
& \wedge \text{lg-at-n}(n, l, g) \\
& \rightarrow \text{lg-at-n}(n, \text{move}(l, k, 2), g)
\end{aligned}$$

EVENT: Disable n-neq-k-rhoila.

THEOREM: n-eq-k-rhoila

$$\begin{aligned}
& (\text{listp}(l) \\
& \wedge \text{listp}(g) \\
& \wedge (k \in \text{nset}(\text{length}(l))) \\
& \wedge \text{at}(l, k, 1) \\
& \wedge \text{lg-at-n}(k, l, g) \\
& \rightarrow \text{lg-at-n}(n, \text{move}(l, k, 2), g)
\end{aligned}$$

EVENT: Disable n-eq-k-rhoila.

THEOREM: lg-at-rhoila

$$\begin{aligned}
& (\text{listp}(l) \\
& \wedge \text{listp}(g) \\
& \wedge (n \in \mathbf{N}) \\
& \wedge (k \in \text{nset}(\text{length}(l))) \\
& \wedge \text{at}(l, k, 1) \\
& \wedge \text{lg-at-n}(n, l, g) \\
& \rightarrow \text{lg-at-n}(n, \text{move}(l, k, 2), g)
\end{aligned}$$

EVENT: Disable lg-at-rhoila.

THEOREM: lg-rhoila

$$\begin{aligned}
& (\text{listp}(l) \\
& \wedge \text{listp}(g) \\
& \wedge (k \in \text{nset}(\text{length}(l))) \\
& \wedge (n \in \mathbf{N}) \\
& \wedge \text{at}(l, k, 1) \\
& \wedge \text{lg}(n, l, g) \\
& \rightarrow \text{lg}(n, \text{move}(l, k, 2), g)
\end{aligned}$$

EVENT: Disable lg-rhoila.

THEOREM: rhoila-preserves-lg

$$\begin{aligned}
& (\text{ws}(n, l, g) \wedge (k \in \text{nset}(n)) \wedge \text{rhoila}(n, k, l, g, lp, gp) \wedge \text{lg}(n, l, g)) \\
& \rightarrow \text{lg}(n, lp, gp)
\end{aligned}$$

;;;rhoi1b

THEOREM: rhoi1b-preserves-lg

$(ws(n, l, g) \wedge (k \in nset(n)) \wedge rhoi1b(n, k, l, g, lp, gp) \wedge lg(n, l, g))$
 $\rightarrow lg(n, lp, gp)$

;;;rhoi2

THEOREM: n-neq-k-rhoi2

$(listp(l)$
 $\wedge listp(g)$
 $\wedge (n \in \mathbf{N})$
 $\wedge (k \in nset(length(l)))$
 $\wedge (k \neq n)$
 $\wedge at(l, k, 2)$
 $\wedge lg-at-n(n, l, g))$
 $\rightarrow lg-at-n(n, move(l, k, 3), move(g, k, 1))$

EVENT: Disable n-neq-k-rhoi2.

THEOREM: n-eq-k-rhoi2

$(listp(l)$
 $\wedge listp(g)$
 $\wedge (k \in nset(length(l)))$
 $\wedge at(l, k, 2)$
 $\wedge lg-at-n(k, l, g))$
 $\rightarrow lg-at-n(n, move(l, k, 3), move(g, k, 1))$

EVENT: Disable n-eq-k-rhoi2.

THEOREM: lg-at-rhoi2

$(listp(l)$
 $\wedge listp(g)$
 $\wedge (n \in \mathbf{N})$
 $\wedge (k \in nset(length(l)))$
 $\wedge at(l, k, 2)$
 $\wedge lg-at-n(n, l, g))$
 $\rightarrow lg-at-n(n, move(l, k, 3), move(g, k, 1))$

EVENT: Disable lg-at-rhoi2.

THEOREM: lg-rhoi2

$(listp(l)$
 $\wedge listp(g)$

$$\begin{aligned}
& \wedge (k \in \text{nset}(\text{length}(l))) \\
& \wedge (n \in \mathbf{N}) \\
& \wedge \text{at}(l, k, 2) \\
& \wedge \text{lg}(n, l, g) \\
& \rightarrow \text{lg}(n, \text{move}(l, k, 3), \text{move}(g, k, 1))
\end{aligned}$$

EVENT: Disable lg-rhoi2.

THEOREM: rhoi2-preserves-lg
 $(\text{ws}(n, l, g) \wedge (k \in \text{nset}(n)) \wedge \text{rhoi2}(n, k, l, g, lp, gp) \wedge \text{lg}(n, l, g))$
 $\rightarrow \text{lg}(n, lp, gp)$

;;;rhoi3a

THEOREM: n-neq-k-rhoi3a
 $(\text{listp}(l)$
 $\wedge \text{listp}(g)$
 $\wedge (n \in \mathbf{N})$
 $\wedge (k \in \text{nset}(\text{length}(l)))$
 $\wedge (k \neq n)$
 $\wedge \text{at}(l, k, 3)$
 $\wedge \text{lg-at-n}(n, l, g))$
 $\rightarrow \text{lg-at-n}(n, \text{move}(l, k, 4), g)$

EVENT: Disable n-neq-k-rhoi3a.

THEOREM: n-eq-k-rhoi3a
 $(\text{listp}(l)$
 $\wedge \text{listp}(g)$
 $\wedge (k \in \text{nset}(\text{length}(l)))$
 $\wedge \text{at}(l, k, 3)$
 $\wedge \text{lg-at-n}(k, l, g))$
 $\rightarrow \text{lg-at-n}(k, \text{move}(l, k, 4), g)$

EVENT: Disable n-eq-k-rhoi3a.

THEOREM: lg-at-rhoi3a
 $(\text{listp}(l)$
 $\wedge \text{listp}(g)$
 $\wedge (n \in \mathbf{N})$
 $\wedge (k \in \text{nset}(\text{length}(l)))$
 $\wedge \text{at}(l, k, 3)$
 $\wedge \text{lg-at-n}(n, l, g))$
 $\rightarrow \text{lg-at-n}(n, \text{move}(l, k, 4), g)$

EVENT: Disable lg-at-rhoi3a.

THEOREM: lg-rhoi3a

(listp(l)
 \wedge listp(g)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge ($n \in \mathbf{N}$)
 \wedge at($l, k, 3$)
 \wedge lg(n, l, g)
 \rightarrow lg($n, \text{move}(l, k, 4), g$))

EVENT: Disable lg-rhoi3a.

THEOREM: rhoi3a-preserves-lg

(ws(n, l, g) \wedge ($k \in \text{nset}(n)$) \wedge rhoi3a(n, k, l, g, lp, gp) \wedge lg(n, l, g))
 \rightarrow lg(n, lp, gp)

;;;rhoi3b

THEOREM: rhoi3b-preserves-lg

(ws(n, l, g) \wedge ($k \in \text{nset}(n)$) \wedge rhoi3b(n, k, l, g, lp, gp) \wedge lg(n, l, g))
 \rightarrow lg(n, lp, gp)

;;;rhoi4

THEOREM: n-neq-k-rhoi4

(listp(l)
 \wedge listp(g)
 \wedge ($n \in \mathbf{N}$)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge ($k \neq n$)
 \wedge at($l, k, 4$)
 \wedge lg-at-n(n, l, g)
 \rightarrow lg-at-n($n, \text{move}(l, k, 5), \text{move}(g, k, 3)$))

EVENT: Disable n-neq-k-rhoi4.

THEOREM: n-eq-k-rhoi4

(listp(l)
 \wedge listp(g)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge at($l, k, 4$)
 \wedge lg-at-n(k, l, g)
 \rightarrow lg-at-n($n, \text{move}(l, k, 5), \text{move}(g, k, 3)$))

EVENT: Disable n-eq-k-rhoi4.

THEOREM: lg-at-rhoi4

(listp (l)
 \wedge listp (g)
 \wedge ($n \in \mathbf{N}$)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge at ($l, k, 4$)
 \wedge lg-at-n (n, l, g)
 \rightarrow lg-at-n ($n, \text{move}(l, k, 5), \text{move}(g, k, 3)$)

EVENT: Disable lg-at-rhoi4.

THEOREM: lg-rhoi4

(listp (l)
 \wedge listp (g)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge ($n \in \mathbf{N}$)
 \wedge at ($l, k, 4$)
 \wedge lg (n, l, g)
 \rightarrow lg ($n, \text{move}(l, k, 5), \text{move}(g, k, 3)$)

EVENT: Disable lg-rhoi4.

THEOREM: rhoi4-preserves-lg

(ws (n, l, g) \wedge ($k \in \text{nset}(n)$) \wedge rhoi4 (n, k, l, g, lp, gp) \wedge lg (n, l, g)
 \rightarrow lg (n, lp, gp)

;;;rhoi5a

THEOREM: n-neq-k-rhoi5a

(listp (l)
 \wedge listp (g)
 \wedge ($n \in \mathbf{N}$)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge ($k \neq n$)
 \wedge at ($l, k, 5$)
 \wedge lg-at-n (n, l, g)
 \rightarrow lg-at-n ($n, \text{move}(l, k, 6), g$)

EVENT: Disable n-neq-k-rhoi5a.

THEOREM: n-eq-k-rhoi5a

$$\begin{aligned}
& (\text{listp } (l)) \\
& \wedge \text{listp } (g) \\
& \wedge (k \in \text{nset } (\text{length } (l))) \\
& \wedge \text{at } (l, k, 5) \\
& \wedge \text{lg-at-n } (k, l, g) \\
& \rightarrow \text{lg-at-n } (k, \text{move } (l, k, 6), g)
\end{aligned}$$

EVENT: Disable n-eq-k-rhoi5a.

THEOREM: lg-at-rhoi5a

$$\begin{aligned}
& (\text{listp } (l)) \\
& \wedge \text{listp } (g) \\
& \wedge (n \in \mathbf{N}) \\
& \wedge (k \in \text{nset } (\text{length } (l))) \\
& \wedge \text{at } (l, k, 5) \\
& \wedge \text{lg-at-n } (n, l, g) \\
& \rightarrow \text{lg-at-n } (n, \text{move } (l, k, 6), g)
\end{aligned}$$

EVENT: Disable lg-at-rhoi5a.

THEOREM: lg-rhoi5a

$$\begin{aligned}
& (\text{listp } (l)) \\
& \wedge \text{listp } (g) \\
& \wedge (k \in \text{nset } (\text{length } (l))) \\
& \wedge (n \in \mathbf{N}) \\
& \wedge \text{at } (l, k, 5) \\
& \wedge \text{lg } (n, l, g) \\
& \rightarrow \text{lg } (n, \text{move } (l, k, 6), g)
\end{aligned}$$

EVENT: Disable lg-rhoi5a.

THEOREM: rhoi5a-preserves-lg

$$\begin{aligned}
& (\text{ws } (n, l, g) \wedge (k \in \text{nset } (n)) \wedge \text{rhoi5a } (n, k, l, g, lp, gp) \wedge \text{lg } (n, l, g)) \\
& \rightarrow \text{lg } (n, lp, gp)
\end{aligned}$$

;;;rhoi5b

THEOREM: n-neq-k-rhoi5b

$$\begin{aligned}
& (\text{listp } (l)) \\
& \wedge \text{listp } (g) \\
& \wedge (n \in \mathbf{N}) \\
& \wedge (k \in \text{nset } (\text{length } (l))) \\
& \wedge (k \neq n)
\end{aligned}$$

\wedge at($l, k, 5$)
 \wedge lg-at-n(n, l, g)
 \rightarrow lg-at-n($n, \text{move}(l, k, 8), g$)

EVENT: Disable n-neq-k-rhoi5b.

THEOREM: n-eq-k-rhoi5b
 (listp(l)
 \wedge listp(g)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge at($l, k, 5$)
 \wedge lg-at-n(k, l, g)
 \rightarrow lg-at-n($k, \text{move}(l, k, 8), g$)

EVENT: Disable n-eq-k-rhoi5b.

THEOREM: lg-at-rhoi5b
 (listp(l)
 \wedge listp(g)
 \wedge ($n \in \mathbf{N}$)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge at($l, k, 5$)
 \wedge lg-at-n(n, l, g)
 \rightarrow lg-at-n($n, \text{move}(l, k, 8), g$)

EVENT: Disable lg-at-rhoi5b.

THEOREM: lg-rhoi5b
 (listp(l)
 \wedge listp(g)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge ($n \in \mathbf{N}$)
 \wedge at($l, k, 5$)
 \wedge lg(n, l, g)
 \rightarrow lg($n, \text{move}(l, k, 8), g$)

EVENT: Disable lg-rhoi5b.

THEOREM: rhoi5b-preserves-lg
 (ws(n, l, g) \wedge ($k \in \text{nset}(n)$) \wedge rhoi5b(n, k, l, g, lp, gp) \wedge lg(n, l, g))
 \rightarrow lg(n, lp, gp)

;;;rhoi6

THEOREM: n-neq-k-rhoi6

(listp (l)
 \wedge listp (g)
 \wedge ($n \in \mathbf{N}$)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge ($k \neq n$)
 \wedge at ($l, k, 6$)
 \wedge lg-at-n (n, l, g)
 \rightarrow lg-at-n ($n, \text{move}(l, k, 7), \text{move}(g, k, 2)$))

EVENT: Disable n-neq-k-rhoi6.

THEOREM: n-eq-k-rhoi6

(listp (l)
 \wedge listp (g)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge at ($l, k, 6$)
 \wedge lg-at-n (k, l, g)
 \rightarrow lg-at-n ($n, \text{move}(l, k, 7), \text{move}(g, k, 2)$))

EVENT: Disable n-eq-k-rhoi6.

THEOREM: lg-at-rhoi6

(listp (l)
 \wedge listp (g)
 \wedge ($n \in \mathbf{N}$)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge at ($l, k, 6$)
 \wedge lg-at-n (n, l, g)
 \rightarrow lg-at-n ($n, \text{move}(l, k, 7), \text{move}(g, k, 2)$))

EVENT: Disable lg-at-rhoi6.

THEOREM: lg-rhoi6

(listp (l)
 \wedge listp (g)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge ($n \in \mathbf{N}$)
 \wedge at ($l, k, 6$)
 \wedge lg (n, l, g)
 \rightarrow lg ($n, \text{move}(l, k, 7), \text{move}(g, k, 2)$))

EVENT: Disable lg-rhoi6.

THEOREM: rhoi6-preserves-lg
 $(ws(n, l, g) \wedge (k \in \text{nset}(n))) \wedge \text{rhoi6}(n, k, l, g, lp, gp) \wedge \text{lg}(n, l, g)$
 $\rightarrow \text{lg}(n, lp, gp)$

;;;rhoi7a

THEOREM: n-neq-k-rhoi7a
 $(\text{listp}(l)$
 $\wedge \text{listp}(g)$
 $\wedge (n \in \mathbf{N})$
 $\wedge (k \in \text{nset}(\text{length}(l)))$
 $\wedge (k \neq n)$
 $\wedge \text{at}(l, k, 7)$
 $\wedge \text{lg-at-n}(n, l, g))$
 $\rightarrow \text{lg-at-n}(n, \text{move}(l, k, 8), g)$

EVENT: Disable n-neq-k-rhoi7a.

THEOREM: n-eq-k-rhoi7a
 $(\text{listp}(l)$
 $\wedge \text{listp}(g)$
 $\wedge (k \in \text{nset}(\text{length}(l)))$
 $\wedge \text{at}(l, k, 7)$
 $\wedge \text{lg-at-n}(k, l, g))$
 $\rightarrow \text{lg-at-n}(k, \text{move}(l, k, 8), g)$

EVENT: Disable n-eq-k-rhoi7a.

THEOREM: lg-at-rhoi7a
 $(\text{listp}(l)$
 $\wedge \text{listp}(g)$
 $\wedge (n \in \mathbf{N})$
 $\wedge (k \in \text{nset}(\text{length}(l)))$
 $\wedge \text{at}(l, k, 7)$
 $\wedge \text{lg-at-n}(n, l, g))$
 $\rightarrow \text{lg-at-n}(n, \text{move}(l, k, 8), g)$

EVENT: Disable lg-at-rhoi7a.

THEOREM: lg-rhoi7a
 $(\text{listp}(l)$
 $\wedge \text{listp}(g)$
 $\wedge (k \in \text{nset}(\text{length}(l)))$

$$\begin{aligned}
& \wedge (n \in \mathbf{N}) \\
& \wedge \text{at}(l, k, 7) \\
& \wedge \text{lg}(n, l, g) \\
& \rightarrow \text{lg}(n, \text{move}(l, k, 8), g)
\end{aligned}$$

EVENT: Disable lg-rhoi7a.

THEOREM: rhoi7a-preserves-lg
 $(\text{ws}(n, l, g) \wedge (k \in \text{nset}(n)) \wedge \text{rhoi7a}(n, k, l, g, lp, gp) \wedge \text{lg}(n, l, g))$
 $\rightarrow \text{lg}(n, lp, gp)$

;;;rhoi7b

THEOREM: rhoi7b-preserves-lg
 $(\text{ws}(n, l, g) \wedge (k \in \text{nset}(n)) \wedge \text{rhoi7b}(n, k, l, g, lp, gp) \wedge \text{lg}(n, l, g))$
 $\rightarrow \text{lg}(n, lp, gp)$

;;;rhoi8

THEOREM: n-neq-k-rhoi8
 $(\text{listp}(l)$
 $\wedge \text{listp}(g)$
 $\wedge (n \in \mathbf{N})$
 $\wedge (k \in \text{nset}(\text{length}(l)))$
 $\wedge (k \neq n)$
 $\wedge \text{at}(l, k, 8)$
 $\wedge \text{lg-at-n}(n, l, g))$
 $\rightarrow \text{lg-at-n}(n, \text{move}(l, k, 9), \text{move}(g, k, 4))$

EVENT: Disable n-neq-k-rhoi8.

THEOREM: n-eq-k-rhoi8
 $(\text{listp}(l)$
 $\wedge \text{listp}(g)$
 $\wedge (k \in \text{nset}(\text{length}(l)))$
 $\wedge \text{at}(l, k, 8)$
 $\wedge \text{lg-at-n}(k, l, g))$
 $\rightarrow \text{lg-at-n}(n, \text{move}(l, k, 9), \text{move}(g, k, 4))$

EVENT: Disable n-eq-k-rhoi8.

THEOREM: lg-at-rhoi8
 $(\text{listp}(l)$
 $\wedge \text{listp}(g)$

$$\begin{aligned}
& \wedge (n \in \mathbf{N}) \\
& \wedge (k \in \text{nset}(\text{length}(l))) \\
& \wedge \text{at}(l, k, 8) \\
& \wedge \text{lg-at-n}(n, l, g) \\
& \rightarrow \text{lg-at-n}(n, \text{move}(l, k, 9), \text{move}(g, k, 4))
\end{aligned}$$

EVENT: Disable lg-at-rhoi8.

THEOREM: lg-rhoi8

$$\begin{aligned}
& (\text{listp}(l) \\
& \wedge \text{listp}(g) \\
& \wedge (k \in \text{nset}(\text{length}(l))) \\
& \wedge (n \in \mathbf{N}) \\
& \wedge \text{at}(l, k, 8) \\
& \wedge \text{lg}(n, l, g) \\
& \rightarrow \text{lg}(n, \text{move}(l, k, 9), \text{move}(g, k, 4))
\end{aligned}$$

EVENT: Disable lg-rhoi8.

THEOREM: rhoi8-preserves-lg

$$\begin{aligned}
& (\text{ws}(n, l, g) \wedge (k \in \text{nset}(n)) \wedge \text{rhoi8}(n, k, l, g, lp, gp) \wedge \text{lg}(n, l, g)) \\
& \rightarrow \text{lg}(n, lp, gp)
\end{aligned}$$

;;;rhoi9a

THEOREM: n-neq-k-rhoi9a

$$\begin{aligned}
& (\text{listp}(l) \\
& \wedge \text{listp}(g) \\
& \wedge (n \in \mathbf{N}) \\
& \wedge (k \in \text{nset}(\text{length}(l))) \\
& \wedge (k \neq n) \\
& \wedge \text{at}(l, k, 9) \\
& \wedge \text{lg-at-n}(n, l, g) \\
& \rightarrow \text{lg-at-n}(n, \text{move}(l, k, 10), g)
\end{aligned}$$

EVENT: Disable n-neq-k-rhoi9a.

THEOREM: n-eq-k-rhoi9a

$$\begin{aligned}
& (\text{listp}(l) \\
& \wedge \text{listp}(g) \\
& \wedge (k \in \text{nset}(\text{length}(l))) \\
& \wedge \text{at}(l, k, 9) \\
& \wedge \text{lg-at-n}(k, l, g) \\
& \rightarrow \text{lg-at-n}(k, \text{move}(l, k, 10), g)
\end{aligned}$$

EVENT: Disable n-eq-k-rhoi9a.

THEOREM: lg-at-rhoi9a

(listp(l)
 \wedge listp(g)
 \wedge ($n \in \mathbf{N}$)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge at($l, k, 9$)
 \wedge lg-at-n(n, l, g)
 \rightarrow lg-at-n($n, \text{move}(l, k, 10), g$)

EVENT: Disable lg-at-rhoi9a.

THEOREM: lg-rhoi9a

(listp(l)
 \wedge listp(g)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge ($n \in \mathbf{N}$)
 \wedge at($l, k, 9$)
 \wedge lg(n, l, g)
 \rightarrow lg($n, \text{move}(l, k, 10), g$)

EVENT: Disable lg-rhoi9a.

THEOREM: rhoi9a-preserves-lg

(ws(n, l, g) \wedge ($k \in \text{nset}(n)$) \wedge rhoi9a(n, k, l, g, lp, gp) \wedge lg(n, l, g)
 \rightarrow lg(n, lp, gp)

;;;rhoi9b

THEOREM: rhoi9b-preserves-lg

(ws(n, l, g) \wedge ($k \in \text{nset}(n)$) \wedge rhoi9b(n, k, l, g, lp, gp) \wedge lg(n, l, g)
 \rightarrow lg(n, lp, gp)

;;;rhoi10

THEOREM: n-neq-k-rhoi10

(listp(l)
 \wedge listp(g)
 \wedge ($n \in \mathbf{N}$)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge ($k \neq n$)
 \wedge at($l, k, 10$)
 \wedge lg-at-n(n, l, g)
 \rightarrow lg-at-n($n, \text{move}(l, k, 11), g$)

EVENT: Disable n-neq-k-rhoi10.

THEOREM: n-eq-k-rhoi10
(listp (l)
 \wedge listp (g)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge at ($l, k, 10$)
 \wedge lg-at-n (k, l, g)
 \rightarrow lg-at-n ($k, \text{move}(l, k, 11), g$)

EVENT: Disable n-eq-k-rhoi10.

THEOREM: lg-at-rhoi10
(listp (l)
 \wedge listp (g)
 \wedge ($n \in \mathbf{N}$)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge at ($l, k, 10$)
 \wedge lg-at-n (n, l, g)
 \rightarrow lg-at-n ($n, \text{move}(l, k, 11), g$)

EVENT: Disable lg-at-rhoi10.

THEOREM: lg-rhoi10
(listp (l)
 \wedge listp (g)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge ($n \in \mathbf{N}$)
 \wedge at ($l, k, 10$)
 \wedge lg (n, l, g)
 \rightarrow lg ($n, \text{move}(l, k, 11), g$)

EVENT: Disable lg-rhoi10.

THEOREM: rhoi10-preserves-lg
(ws (n, l, g) \wedge ($k \in \text{nset}(n)$) \wedge rhoi10 (n, k, l, g, lp, gp) \wedge lg (n, l, g)
 \rightarrow lg (n, lp, gp)

;;;rhoi11a

THEOREM: n-neq-k-rhoi11a
(listp (l)
 \wedge listp (g)

$$\begin{aligned}
& \wedge (n \in \mathbf{N}) \\
& \wedge (k \in \text{nset}(\text{length}(l))) \\
& \wedge (k \neq n) \\
& \wedge \text{at}(l, k, 11) \\
& \wedge \text{lg-at-n}(n, l, g) \\
& \rightarrow \text{lg-at-n}(n, \text{move}(l, k, 12), g)
\end{aligned}$$

EVENT: Disable n-neq-k-rhoi1a.

THEOREM: n-eq-k-rhoi1a

$$\begin{aligned}
& (\text{listp}(l) \\
& \wedge \text{listp}(g) \\
& \wedge (k \in \text{nset}(\text{length}(l))) \\
& \wedge \text{at}(l, k, 11) \\
& \wedge \text{lg-at-n}(k, l, g) \\
& \rightarrow \text{lg-at-n}(k, \text{move}(l, k, 12), g)
\end{aligned}$$

EVENT: Disable n-eq-k-rhoi1a.

THEOREM: lg-at-rhoi1a

$$\begin{aligned}
& (\text{listp}(l) \\
& \wedge \text{listp}(g) \\
& \wedge (n \in \mathbf{N}) \\
& \wedge (k \in \text{nset}(\text{length}(l))) \\
& \wedge \text{at}(l, k, 11) \\
& \wedge \text{lg-at-n}(n, l, g) \\
& \rightarrow \text{lg-at-n}(n, \text{move}(l, k, 12), g)
\end{aligned}$$

EVENT: Disable lg-at-rhoi1a.

THEOREM: lg-rhoi1a

$$\begin{aligned}
& (\text{listp}(l) \\
& \wedge \text{listp}(g) \\
& \wedge (k \in \text{nset}(\text{length}(l))) \\
& \wedge (n \in \mathbf{N}) \\
& \wedge \text{at}(l, k, 11) \\
& \wedge \text{lg}(n, l, g) \\
& \rightarrow \text{lg}(n, \text{move}(l, k, 12), g)
\end{aligned}$$

EVENT: Disable lg-rhoi1a.

THEOREM: rhoi1a-preserves-lg

$$\begin{aligned}
& (\text{ws}(n, l, g) \wedge (k \in \text{nset}(n)) \wedge \text{rhoi1a}(n, k, l, g, lp, gp) \wedge \text{lg}(n, l, g)) \\
& \rightarrow \text{lg}(n, lp, gp)
\end{aligned}$$

;;;rhoi11b

THEOREM: rhoi11b-preserves-lg

$(ws(n, l, g) \wedge (k \in \text{nset}(n)) \wedge \text{rhoi11b}(n, k, l, g, lp, gp) \wedge \text{lg}(n, l, g))$
 $\rightarrow \text{lg}(n, lp, gp)$

;;;rhoi12

THEOREM: n-neq-k-rhoi12

$(\text{listp}(l)$
 $\wedge \text{listp}(g)$
 $\wedge (n \in \mathbf{N})$
 $\wedge (k \in \text{nset}(\text{length}(l)))$
 $\wedge (k \neq n)$
 $\wedge \text{at}(l, k, 12)$
 $\wedge \text{lg-at-n}(n, l, g))$
 $\rightarrow \text{lg-at-n}(n, \text{move}(l, k, 0), \text{move}(g, k, 0))$

EVENT: Disable n-neq-k-rhoi12.

THEOREM: n-eq-k-rhoi12

$(\text{listp}(l)$
 $\wedge \text{listp}(g)$
 $\wedge (k \in \text{nset}(\text{length}(l)))$
 $\wedge \text{at}(l, k, 12)$
 $\wedge \text{lg-at-n}(k, l, g))$
 $\rightarrow \text{lg-at-n}(n, \text{move}(l, k, 0), \text{move}(g, k, 0))$

EVENT: Disable n-eq-k-rhoi12.

THEOREM: lg-at-rhoi12

$(\text{listp}(l)$
 $\wedge \text{listp}(g)$
 $\wedge (n \in \mathbf{N})$
 $\wedge (k \in \text{nset}(\text{length}(l)))$
 $\wedge \text{at}(l, k, 12)$
 $\wedge \text{lg-at-n}(n, l, g))$
 $\rightarrow \text{lg-at-n}(n, \text{move}(l, k, 0), \text{move}(g, k, 0))$

EVENT: Disable lg-at-rhoi12.

THEOREM: lg-rhoi12

$(\text{listp}(l)$
 $\wedge \text{listp}(g)$

$$\begin{aligned}
& \wedge (k \in \text{nset}(\text{length}(l))) \\
& \wedge (n \in \mathbf{N}) \\
& \wedge \text{at}(l, k, 12) \\
& \wedge \text{lg}(n, l, g) \\
& \rightarrow \text{lg}(n, \text{move}(l, k, 0), \text{move}(g, k, 0))
\end{aligned}$$

EVENT: Disable lg-rhoi12.

THEOREM: rhoi12-preserves-lg
 $(\text{ws}(n, l, g) \wedge (k \in \text{nset}(n)) \wedge \text{rhoi12}(n, k, l, g, lp, gp) \wedge \text{lg}(n, l, g))$
 $\rightarrow \text{lg}(n, lp, gp)$

THEOREM: rho-preserves-lg
 $(\text{ws}(n, l, g) \wedge (k \in \text{nset}(n)) \wedge \text{rhoi}(n, k, l, g, lp, gp) \wedge \text{lg}(n, l, g))$
 $\rightarrow \text{lg}(n, lp, gp)$

EVENT: Disable rhoi0-preserves-lg.

EVENT: Disable rhoi1a-preserves-lg.

EVENT: Disable rhoi1b-preserves-lg.

EVENT: Disable rhoi2-preserves-lg.

EVENT: Disable rhoi3a-preserves-lg.

EVENT: Disable rhoi3b-preserves-lg.

EVENT: Disable rhoi4-preserves-lg.

EVENT: Disable rhoi5a-preserves-lg.

EVENT: Disable rhoi5b-preserves-lg.

EVENT: Disable rhoi6-preserves-lg.

EVENT: Disable rhoi7a-preserves-lg.

EVENT: Disable rhoi7b-preserves-lg.

EVENT: Disable rhoi8-preserves-lg.

EVENT: Disable rhoi9a-preserves-lg.

EVENT: Disable rhoi9b-preserves-lg.

EVENT: Disable rhoi10-preserves-lg.

EVENT: Disable rhoi11a-preserves-lg.

EVENT: Disable rhoi11b-preserves-lg.

EVENT: Disable rhoi12-preserves-lg.

```
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;; a0.ev ;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;(exist-union lp n '(8 9 10 11 12)) and
;;;(not (exist-union l n '(8 9 10 11 12)))
;;;implies that k is the witness of
;;;(exist-union lp n '(8 9 10 11 12)). This
;;;proposition would have been more natural
;;;if we had been able to prove:
;;;(prove-lemma exist-l8-12 (rewrite)
;;;  (implies (and (ws n l g)
;;;                (member k (nset n))
;;;                (rhoi n k l g lp gp)
;;;                (exist-union lp n '(8 9 10 11 12))
;;;                (not (exist-union l n '(8 9 10 11 12))))
;;;            (equal k (exist-union lp n '(8 9 10 11 12)))).
;;;However Bmp refused to rewrite equal clause.
```

THEOREM: exist-l8-12

$$\begin{aligned} & (\text{ws } (n, l, g) \\ & \wedge (k \in \text{nset } (n)) \\ & \wedge \text{rhoi } (n, k, l, g, lp, gp) \\ & \wedge \text{exist-union } (lp, n, '(8 9 10 11 12)) \\ & \wedge (k \neq \text{exist-union } (lp, n, '(8 9 10 11 12)))) \\ & \rightarrow \text{exist-union } (l, n, '(8 9 10 11 12)) \end{aligned}$$

```
;;;If (exist-union lp n '(8 9 10 11 12)) and
;;;(not (exist-union l n '(8 9 10 11 12))) hold,
```

;;;then the k's entry of lp is between 8..12.

THEOREM: k-in-lp8-12

(ws (n, l, g)
 ∧ (k ∈ nset (n))
 ∧ rhoi (n, k, l, g, lp, gp)
 ∧ exist-union (lp, n, '(8 9 10 11 12))
 ∧ (¬ exist-union (l, n, '(8 9 10 11 12))))
 → union-at-n (lp, k, '(8 9 10 11 12))

;;;If k's entry in lp is between 8..12 and
;;;k's entry of l is not between 8..12,
;;;then k's entry of l is either 5 or 7.

THEOREM: k-not-in-l8-12-then-l57

(ws (n, l, g)
 ∧ (k ∈ nset (n))
 ∧ rhoi (n, k, l, g, lp, gp)
 ∧ union-at-n (lp, k, '(8 9 10 11 12))
 ∧ (¬ union-at-n (l, k, '(8 9 10 11 12))))
 ∧ (¬ at (l, k, 7)))
 → at (l, k, 5)

;;;If k's entry in lp is between 8..12 and
;;;(not (exist-union l n '(8 9 10 11 12))) holds,
;;;then k's entry of l is either 5 or 7.

THEOREM: k-in-l57

(ws (n, l, g)
 ∧ (k ∈ nset (n))
 ∧ rhoi (n, k, l, g, lp, gp)
 ∧ union-at-n (lp, k, '(8 9 10 11 12))
 ∧ (¬ exist-union (l, n, '(8 9 10 11 12))))
 ∧ (¬ at (l, k, 7)))
 → at (l, k, 5)

;;;If (exist-union lp n '(8 9 10 11 12)) and
;;;(not (exist-union l n '(8 9 10 11 12))) hold,
;;;then the k's entry of l is between either 5 or 7.

THEOREM: ex-k-in-l57

(ws (n, l, g)
 ∧ (k ∈ nset (n))
 ∧ rhoi (n, k, l, g, lp, gp)
 ∧ exist-union (lp, n, '(8 9 10 11 12))

$\wedge (\neg \text{exist-union}(l, n, '(8\ 9\ 10\ 11\ 12)))$
 $\wedge (\neg \text{at}(l, k, 7))$
 $\rightarrow \text{at}(l, k, 5)$

;;;Auxiliary lemma for ex-cond-rhoi5.

THEOREM: cond-rhoi5

$(\text{ws}(n, l, g)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{rhoi}(n, k, l, g, lp, gp)$
 $\wedge \text{union-at-n}(lp, k, '(8\ 9\ 10\ 11\ 12))$
 $\wedge \text{at}(l, k, 5))$
 $\rightarrow (\neg \text{exist-union}(g, n, '(1)))$

;;;If $(\text{exist-union } lp\ n\ '(8\ 9\ 10\ 11\ 12))$ and
 ;;; $(\neg (\text{exist-union } l\ n\ '(8\ 9\ 10\ 11\ 12)))$ and
 ;;;the k 's entry in l is 5 then
 ;;; $(\neg (\text{exist-union } g\ n\ '(1)))$ holds.

THEOREM: ex-cond-rhoi5

$(\text{ws}(n, l, g)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{rhoi}(n, k, l, g, lp, gp)$
 $\wedge \text{exist-union}(lp, n, '(8\ 9\ 10\ 11\ 12))$
 $\wedge (\neg \text{exist-union}(l, n, '(8\ 9\ 10\ 11\ 12)))$
 $\wedge \text{at}(l, k, 5))$
 $\rightarrow (\neg \text{exist-union}(g, n, '(1)))$

;;;Auxiliary lemma for ex-cond-rhoi7.

THEOREM: cond-rhoi7

$(\text{ws}(n, l, g)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{rhoi}(n, k, l, g, lp, gp)$
 $\wedge \text{union-at-n}(lp, k, '(8\ 9\ 10\ 11\ 12))$
 $\wedge \text{at}(l, k, 7))$
 $\rightarrow \text{exist-union}(g, n, '(4))$

;;;If $(\text{exist-union } lp\ n\ '(8\ 9\ 10\ 11\ 12))$ and
 ;;; $(\neg (\text{exist-union } l\ n\ '(8\ 9\ 10\ 11\ 12)))$ and
 ;;;the k 's entry in l is 7, then
 ;;; $(\text{exist-union } g\ n\ '(4))$ holds.

THEOREM: ex-cond-rhoi7

$(\text{ws}(n, l, g)$

$\wedge (k \in \text{nset}(n))$
 $\wedge \text{rhoi}(n, k, l, g, lp, gp)$
 $\wedge \text{exist-union}(lp, n, '(8\ 9\ 10\ 11\ 12))$
 $\wedge (\neg \text{exist-union}(l, n, '(8\ 9\ 10\ 11\ 12)))$
 $\wedge \text{at}(l, k, 7)$
 $\rightarrow \text{exist-union}(g, n, '(4))$

;;;If (exist-union lp n '(8 9 10 11 12))
 ;;;and (not (exist-union l n '(8 9 10 11 12))),
 ;;;then (not (exist-union g n '(1))) holds.

THEOREM: l5-only-lp8

$(\text{ws}(n, l, g)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{rhoi}(n, k, l, g, lp, gp)$
 $\wedge (\neg \text{exist-union}(l, n, '(8\ 9\ 10\ 11\ 12)))$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{exist-union}(lp, n, '(8\ 9\ 10\ 11\ 12)))$
 $\rightarrow (\neg \text{exist-union}(g, n, '(1)))$

;;;If j is not equal to k and j's entry of l
 ;;;is neither 3 or 4, then j's entry of lp
 ;;;is not 4.

THEOREM: j-neq-k-j-not-in-lp4

$(\text{ws}(n, l, g)$
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{rhoi}(n, k, l, g, lp, gp)$
 $\wedge (j \neq k)$
 $\wedge (\neg \text{union-at-n}(l, j, '(3\ 4)))$
 $\rightarrow (\neg \text{at}(lp, j, 4))$

;;;If k's entry of l is neither 3 or 4, then
 ;;;k's entry of lp is not 4.

THEOREM: j-eq-k-j-not-in-lp4

$(\text{ws}(n, l, g)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{rhoi}(n, k, l, g, lp, gp)$
 $\wedge (\neg \text{union-at-n}(l, k, '(3\ 4)))$
 $\rightarrow (\neg \text{at}(lp, k, 4))$

;;;If j's entry of l is neither 3 or 4, then
 ;;;j's entry of lp is not 4.

THEOREM: lp4-empty

(ws (n, l, g)
 \wedge ($j \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge rhoi(n, k, l, g, lp, gp)
 \wedge ($\neg \text{union-at-n}(l, j, '(3\ 4))$))
 \rightarrow ($\neg \text{at}(lp, j, 4)$)

;;;If (not (exist-union l n '(8 9 10 11 12)))
;;; and (exist-union lp n '(8 9 10 11 12)) hold,
;;;then there is no entry 4 in l.

THEOREM: l8-l12-empty

(ws (n, l, g)
 \wedge ($j \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge rhoi(n, k, l, g, lp, gp)
 \wedge ($\neg \text{exist-union}(l, n, '(8\ 9\ 10\ 11\ 12))$))
 \wedge lg(n, l, g)
 \rightarrow a0(n, lp, j)

;;;If (exist-union g n '(3 4)) holds and
;;;the k's entry in l is not 4, then
;;;the k's entry in lp is not 4 either.
;;;(Doorway is locked.)

THEOREM: dwy-lckd

(ws (n, l, g)
 \wedge ($k \in \text{nset}(n)$)
 \wedge rhoi(n, k, l, g, lp, gp)
 \wedge exist-union($g, n, '(3\ 4)$)
 \wedge ($\neg \text{at}(l, k, 4)$))
 \rightarrow ($\neg \text{at}(lp, k, 4)$)

;;;If (exist-union l n '(8 9 10 11 12))
;;;holds and j is equal to k, then
;;;j's entry in lp is not 4.

THEOREM: j-eq-k-l8-l12-nonemp

(ws (n, l, g)
 \wedge ($k \in \text{nset}(n)$)
 \wedge rhoi(n, k, l, g, lp, gp)
 \wedge exist-union($l, n, '(8\ 9\ 10\ 11\ 12)$)
 \wedge a0(n, l, k)
 \wedge a1(n, l, g)
 \rightarrow a0(n, lp, k)

```

;;;If (exist-union l n '(8 9 10 11 12))
;;; holds and j is not equal to k, then
;;;the j's entry in lp is not 4.

```

THEOREM: j-neq-k-l8-l12-nonemp

```

(ws (n, l, g)
  ^ (j ∈ nset (n))
  ^ (k ∈ nset (n))
  ^ rhoi (n, k, l, g, lp, gp)
  ^ (j ≠ k)
  ^ a0 (n, l, j)
  ^ exist-union (l, n, '(8 9 10 11 12)))
→ a0 (n, lp, j)

```

```

;;;If (exist-union l n '(8 9 10 11 12))
;;;holds then there is no entry 4 in lp.
;;;The order of the use hints is critical.
;;;Change the order and we fail.

```

THEOREM: l8-l12-nonemp

```

(ws (n, l, g)
  ^ (j ∈ nset (n))
  ^ (k ∈ nset (n))
  ^ rhoi (n, k, l, g, lp, gp)
  ^ exist-union (l, n, '(8 9 10 11 12))
  ^ a0 (n, l, j)
  ^ a1 (n, l, g))
→ a0 (n, lp, j)

```

```

;;;If (exist-union lp n '(8 9 10 11 12))
;;;holds, then there is no entry 4 in lp.

```

THEOREM: rho-preserves-a0

```

(ws (n, l, g)
  ^ (j ∈ nset (n))
  ^ (k ∈ nset (n))
  ^ rhoi (n, k, l, g, lp, gp)
  ^ lg (n, l, g)
  ^ a0 (n, l, j)
  ^ a1 (n, l, g))
→ a0 (n, lp, j)

```

```

;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;; a1.ev ;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;* ep-18-12

```

```

;;;Auxiliary lemma for at-gp-rhoi5

```

THEOREM: gp-rhoi5

(ws (n, l, g)
 \wedge ($k \in \text{nset}(n)$)
 \wedge rhoi (n, k, l, g, lp, gp)
 \wedge union-at-n ($lp, k, '(8\ 9\ 10\ 11\ 12)$)
 \wedge at ($l, k, 5$)
 \wedge at ($g, k, 3$)
 \rightarrow at ($gp, k, 3$)

;;;If (not (exist-union l n '(8 9 10 11 12))),
;;;(exist-union lp n '(8 9 10 11 12)) and the k's
;;;entry in l is 5 then the k's entry in gp is 3.

THEOREM: ex-gp-rhoi5

(ws (n, l, g)
 \wedge ($k \in \text{nset}(n)$)
 \wedge rhoi (n, k, l, g, lp, gp)
 \wedge lg (n, l, g)
 \wedge (\neg exist-union ($l, n, '(8\ 9\ 10\ 11\ 12)$))
 \wedge exist-union ($lp, n, '(8\ 9\ 10\ 11\ 12)$)
 \wedge at ($l, k, 5$)
 \rightarrow at ($gp, k, 3$)

;;;If (not (exist-union l n '(8 9 10 11 12)))
;;;and (exist-union lp n '(8 9 10 11 12)) holds,
;;;then the k's entry is either 3 or 4.

THEOREM: k-in-gp34

(ws (n, l, g)
 \wedge ($k \in \text{nset}(n)$)
 \wedge rhoi (n, k, l, g, lp, gp)
 \wedge lg (n, l, g)
 \wedge (\neg exist-union ($l, n, '(8\ 9\ 10\ 11\ 12)$))
 \wedge exist-union ($lp, n, '(8\ 9\ 10\ 11\ 12)$))
 \rightarrow union-at-n ($gp, k, '(3\ 4)$)

;;;If (exist-union lp n '(8 9 10 11 12)) and
;;;(not (exist-union l n '(8 9 10 11 12))) holds,
;;;then so does (exist-intersect-8-12-3-4 n lp gp).

THEOREM: lm-a1-ep-l8-12

(ws (n, l, g)
 \wedge ($k \in \text{nset}(n)$)
 \wedge rhoi (n, k, l, g, lp, gp)
 \wedge exist-union ($lp, n, '(8\ 9\ 10\ 11\ 12)$)

$\wedge (\neg \text{exist-union}(l, n, '(8\ 9\ 10\ 11\ 12)))$
 $\wedge \text{lg}(n, l, g)$
 $\rightarrow \text{exist-intersect-8-12-3-4}(n, lp, gp)$
 ;;;If (not (exist-union l n '(8 9 10 11 12))) holds,
 ;;;then so does a1.

THEOREM: a1-ep-l8-12

$(\text{ws}(n, l, g)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{rhoi}(n, k, l, g, lp, gp)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge (\neg \text{exist-union}(l, n, '(8\ 9\ 10\ 11\ 12))))$
 $\rightarrow \text{a1}(n, lp, gp)$

* nep-l8-12

;;;If (exist-intersect-8-12-3-4 n l g) holds and
 ;;;k is not its witness then
 ;;;(exist-intersect-8-12-3-4 n lp gp) holds.

THEOREM: int-k-not-ex-int

$(\text{ws}(n, l, g)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{rhoi}(n, k, l, g, lp, gp)$
 $\wedge (k \neq \text{exist-intersect-8-12-3-4}(n, l, g))$
 $\wedge \text{exist-intersect-8-12-3-4}(n, l, g)$
 $\rightarrow \text{exist-intersect-8-12-3-4}(n, lp, gp)$

;;;If (exist-union l n '(8 9 10 11 12)) holds and
 ;;;k's entry is not between 8 and 12 then
 ;;;(exist-intersect-8-12-3-4 n lp gp) holds.
 ;;; j \neq k

THEOREM: a1-k-not-in-l8-12-nep-l8-12

$(\text{ws}(n, l, g)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{rhoi}(n, k, l, g, lp, gp)$
 $\wedge \text{exist-union}(l, n, '(8\ 9\ 10\ 11\ 12))$
 $\wedge (\neg \text{union-at-n}(l, k, '(8\ 9\ 10\ 11\ 12)))$
 $\wedge \text{a1}(n, l, g)$
 $\rightarrow \text{exist-intersect-8-12-3-4}(n, lp, gp)$

* k-in-l8-11

;;;If the k's entry in l is between 8 and 11,
 ;;;then the k's entry in lp is between 9 and 12.
 ;;;We need rho-preserves-lg.

THEOREM: l8-11-k-in-lp9-12

(ws (n, l, g)
 \wedge ($k \in \text{nset}(n)$)
 \wedge rhoi (n, k, l, g, lp, gp)
 \wedge union-at-n ($l, k, '(8\ 9\ 10\ 11)$))
 \rightarrow union-at-n ($lp, k, '(9\ 10\ 11\ 12)$)

;;;If the k's entry in l is between 8 and 11,
;;;then the k's entry in lp is between 8 and 12
;;;and the entry in gp is either 3 or 4.
;;;l8-11-k-in-lp9-12, un9-12-then-un8-12 and
;;;rho-preserves-lg are used.

THEOREM: lm-a1-k-in-l8-11-nep-l8-12

(ws (n, l, g)
 \wedge ($k \in \text{nset}(n)$)
 \wedge rhoi (n, k, l, g, lp, gp)
 \wedge union-at-n ($l, k, '(8\ 9\ 10\ 11)$)
 \wedge lg (n, l, g)
 \rightarrow (union-at-n ($lp, k, '(8\ 9\ 10\ 11\ 12)$) \wedge union-at-n ($gp, k, '(3\ 4)$))

;;;If (exist-union lp n '(8 9 10 11 12)) holds,
;;;and the k's entry in l is between 8 and 11 then
;;;(exist-intersect-8-12-3-4 n lp gp) holds.
;;; j \leq k and k \in l8-11

THEOREM: a1-k-in-l8-11-nep-l8-12

(ws (n, l, g)
 \wedge ($k \in \text{nset}(n)$)
 \wedge rhoi (n, k, l, g, lp, gp)
 \wedge lg (n, l, g)
 \wedge exist-union ($lp, n, '(8\ 9\ 10\ 11\ 12)$)
 \wedge union-at-n ($l, k, '(8\ 9\ 10\ 11)$))
 \rightarrow exist-intersect-8-12-3-4 (n, lp, gp)

;;;If the k's entry in l is 12 then the k's entry in lp is 0.

THEOREM: k-in-lp0

(ws (n, l, g) \wedge ($k \in \text{nset}(n)$) \wedge rhoi (n, k, l, g, lp, gp) \wedge at ($l, k, 12$))
 \rightarrow at ($lp, k, 0$)

;;;If (exist-union lp n '(8 9 10 11 12)) holds
;;;and k's entry in l is 12, then k is not the
;;;witness of (exist-union lp n '(8 9 10 11 12)).

THEOREM: k-not-ex-lp8-12

$(ws(n, l, g)$
 $\wedge (k \in nset(n))$
 $\wedge rhoi(n, k, l, g, lp, gp)$
 $\wedge exist-union(lp, n, '(8 9 10 11 12))$
 $\wedge at(l, k, 12))$
 $\rightarrow (k \neq exist-union(lp, n, '(8 9 10 11 12)))$

;;;If the k's entry in lp is 8,
;;;then k's entry in l is either 5 or 7.

THEOREM: lp8-k-in-l57

$(ws(n, l, g) \wedge (k \in nset(n)) \wedge rhoi(n, k, l, g, lp, gp) \wedge at(lp, k, 8))$
 $\rightarrow union-at-n(l, k, '(5 7))$

;;;If the k's entry in lp is 8,
;;;then k's entry in l is between 5 and 12.

THEOREM: k-in-lp8-then-l5-12

$(ws(n, l, g) \wedge rhoi(n, k, l, g, lp, gp) \wedge (k \in nset(n)) \wedge at(lp, k, 8))$
 $\rightarrow union-at-n(l, k, '(5 6 7 8 9 10 11 12))$

;;;If the k's entry in lp is between 9 and 12,
;;;then the k's entry in l is between 8 and 11.

THEOREM: lp9-12-k-in-l8-11

$(ws(n, l, g)$
 $\wedge (k \in nset(n))$
 $\wedge rhoi(n, k, l, g, lp, gp)$
 $\wedge union-at-n(lp, k, '(9 10 11 12)))$
 $\rightarrow union-at-n(l, k, '(8 9 10 11))$

;;;If the k's entry in lp is between 9 and 12,
;;;then the k's entry in l is between 5 and 12.

THEOREM: k-in-lp9-12-then-l5-12

$(ws(n, l, g)$
 $\wedge rhoi(n, k, l, g, lp, gp)$
 $\wedge (k \in nset(n))$
 $\wedge union-at-n(lp, k, '(9 10 11 12)))$
 $\rightarrow union-at-n(l, k, '(5 6 7 8 9 10 11 12))$

;;;If the k's entry in lp is between 8 and 12,
;;;then the k's entry in l is between 5 and 12.

THEOREM: k-in-l5-12

(ws (n, l, g)
 \wedge rhoi (n, k, l, g, lp, gp)
 \wedge ($k \in \text{nset}(n)$)
 \wedge union-at-n ($lp, k, '(8\ 9\ 10\ 11\ 12)$))
 \rightarrow union-at-n ($l, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)$)

;;;If (exist-union lp n '(8 9 10 11 12)) holds
;;;and k is not its witness, then the witness has
;;;its entry in l between 5 and 12.
;;;ex-lp8-12-in-lp8-12, member-ex-union used.

THEOREM: k-neq-ex-lp8-12-in-l5-12

(ws (n, l, g)
 \wedge rhoi (n, k, l, g, lp, gp)
 \wedge ($k \in \text{nset}(n)$)
 \wedge exist-union ($lp, n, '(8\ 9\ 10\ 11\ 12)$)
 \wedge ($k \neq \text{exist-union}(lp, n, '(8\ 9\ 10\ 11\ 12))$))
 \rightarrow union-at-n ($l,$
 exist-union ($lp, n, '(8\ 9\ 10\ 11\ 12)$),
 '(5 6 7 8 9 10 11 12))

;;;If (exist-union lp n '(8 9 10 11 12)) holds and
;;;the witness has its entry in lp between 8 and 12,
;;;then its entry in l is between 5 and 12.
;;;ex-lp8-12-in-lp8-12, member-ex-union used.

THEOREM: ex-lp8-12-then-l5-12

(ws (n, l, g)
 \wedge rhoi (n, k, l, g, lp, gp)
 \wedge ($k \in \text{nset}(n)$)
 \wedge exist-union ($lp, n, '(8\ 9\ 10\ 11\ 12)$))
 \rightarrow union-at-n ($l,$
 exist-union ($lp, n, '(8\ 9\ 10\ 11\ 12)$),
 '(5 6 7 8 9 10 11 12))

;;;If (exist-union lp n '(8 9 10 11 12)) holds
;;;and k is not the witness of
;;;(exist-union lp n '(8 9 10 11 12)), then
;;;the witness has its entry 4 in gp.

THEOREM: ex-lp8-12-in-gp4

(ws (n, l, g)
 \wedge ($k \in \text{nset}(n)$)
 \wedge rhoi (n, k, l, g, lp, gp)

\wedge at($l, k, 12$)
 \wedge a3-at-n1-n2($k, \text{exist-union}(lp, n, '(8\ 9\ 10\ 11\ 12)), l, g$)
 \wedge ($k \neq \text{exist-union}(lp, n, '(8\ 9\ 10\ 11\ 12))$)
 \wedge exist-union($lp, n, '(8\ 9\ 10\ 11\ 12)$)
 \rightarrow at($gp, \text{exist-union}(lp, n, '(8\ 9\ 10\ 11\ 12)), 4$)

;;;If (exist-union lp n '(8 9 10 11 12)) holds and
 ;;;k's entry in l is 12 then the witness has its
 ;;;entry in lp between 8 and 12 and in gp either 3 or 4.

THEOREM: lm-a1-k-in-l12-nep-l8-12

(ws(n, l, g)
 \wedge ($k \in \text{nset}(n)$)
 \wedge rhoi(n, k, l, g, lp, gp)
 \wedge exist-union($lp, n, '(8\ 9\ 10\ 11\ 12)$)
 \wedge at($l, k, 12$)
 \wedge a3-at-n1-n2($k, \text{exist-union}(lp, n, '(8\ 9\ 10\ 11\ 12)), l, g$)
 \rightarrow union-at-n($gp, \text{exist-union}(lp, n, '(8\ 9\ 10\ 11\ 12)), '(3\ 4)$)

;;;If (exist-union lp n '(8 9 10 11 12)) holds
 ;;;and k's entry in l is 12, then
 ;;;(exist-intersect-8-12-3-4 n lp gp) holds.

THEOREM: a1-k-in-l12-nep-l8-12

(ws(n, l, g)
 \wedge ($k \in \text{nset}(n)$)
 \wedge rhoi(n, k, l, g, lp, gp)
 \wedge exist-union($lp, n, '(8\ 9\ 10\ 11\ 12)$)
 \wedge at($l, k, 12$)
 \wedge a3-at-n1-n2($k, \text{exist-union}(lp, n, '(8\ 9\ 10\ 11\ 12)), l, g$)
 \rightarrow exist-intersect-8-12-3-4(n, lp, gp)

;;;Auxiliary lemma for a1-nep-l8-12.
 ;;;We have an instance of a3 in the lemma.

THEOREM: lm1-a1-nep-l8-12

(ws(n, l, g)
 \wedge ($k \in \text{nset}(n)$)
 \wedge rhoi(n, k, l, g, lp, gp)
 \wedge lg(n, l, g)
 \wedge a1(n, l, g)
 \wedge a3-at-n1-n2($k, \text{exist-union}(lp, n, '(8\ 9\ 10\ 11\ 12)), l, g$)
 \wedge exist-union($lp, n, '(8\ 9\ 10\ 11\ 12)$)
 \wedge exist-union($l, n, '(8\ 9\ 10\ 11\ 12)$)
 \rightarrow exist-intersect-8-12-3-4(n, lp, gp)

THEOREM: lm-a1-nep-l8-12

(ws (n, l, g)
 ^ (k ∈ nset (n))
 ^ rhoi (n, k, l, g, lp, gp)
 ^ lg (n, l, g)
 ^ a1 (n, l, g)
 ^ a3 (n, n, l, g)
 ^ exist-union (l, n, '(8 9 10 11 12))
 ^ exist-union (lp, n, '(8 9 10 11 12)))
 → exist-intersect-8-12-3-4 (n, lp, gp)

;;;If (exist-union lp n '(8 9 10 11 12)) and
;;;(exist-union l n '(8 9 10 11 12)) hold,
;;;then so does (exist-intersect-8-12-3-4 n lp gp).

THEOREM: a1-nep-l8-12

(ws (n, l, g)
 ^ (k ∈ nset (n))
 ^ rhoi (n, k, l, g, lp, gp)
 ^ lg (n, l, g)
 ^ a1 (n, l, g)
 ^ a3 (n, n, l, g)
 ^ exist-union (l, n, '(8 9 10 11 12)))
 → a1 (n, lp, gp)

;;;If (exist-union lp n '(8 9 10 11 12)) holds,
;;;then so does (exist-intersect-8-12-3-4 n lp gp).

THEOREM: rho-preserves-a1

(ws (n, l, g)
 ^ (k ∈ nset (n))
 ^ rhoi (n, k, l, g, lp, gp)
 ^ lg (n, l, g)
 ^ a1 (n, l, g)
 ^ a3 (n, n, l, g))
 → a1 (n, lp, gp)

;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;; a2.ev ;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;* i-eq-k-j-neq-k

;;;If the k's entry in lp is between 10 and 12
;;;and the k's entry in l is between 10 and 12,
;;; then (phi9 k n g) holds.

THEOREM: k-in-l10-11-or-phi9

```

(ws (n, l, g)
  ∧ (k ∈ nset (n))
  ∧ rhoi (n, k, l, g, lp, gp)
  ∧ union-at-n (lp, k, '(10 11 12))
  ∧ (¬ union-at-n (l, k, '(10 11))))
→ phi9 (k, n, g)

```

```

;;;If j is less than k and (phi9 k n g) holds,
;;;then the j's entry in g is either 0 or 1.

```

```

THEOREM: phi9-j-in-g01
((j ∈ nset (n)) ∧ (j < k) ∧ phi9 (k, n, g))
→ union-at-n (g, j, '(0 1))

```

```

;;;If j is less than k and (phi9 k n g) holds,
;;;then the j's entry in lp is not between 5 and 12.
;;;lp-same-l-not is used.

```

```

THEOREM: case-k-in-phi9
(ws (n, l, g)
  ∧ (j ∈ nset (n))
  ∧ (k ∈ nset (n))
  ∧ rhoi (n, k, l, g, lp, gp)
  ∧ (j ≠ k)
  ∧ (j < k)
  ∧ lg (n, l, g)
  ∧ phi9 (k, n, g))
→ (¬ union-at-n (lp, j, '(5 6 7 8 9 10 11 12)))

```

```

;;;If j is not equal to k and the k's entry in l is
;;;either 10 or 11, then the j's entry in lp is not
;;;between 5 and 12.

```

```

THEOREM: case-k-in-l10-11
(ws (n, l, g)
  ∧ (j ∈ nset (n))
  ∧ (k ∈ nset (n))
  ∧ rhoi (n, k, l, g, lp, gp)
  ∧ a2-at-n1-n2 (k, j, l)
  ∧ (j ≠ k)
  ∧ union-at-n (l, k, '(10 11)))
→ (¬ union-at-n (lp, j, '(5 6 7 8 9 10 11 12)))

```

```

;;;Auxiliary lemma for lm-i-eq-k-j-neq-k with
;;;(a2-at-n1-n2 k j l).

```

THEOREM: lm1-i-eq-k-j-neq-k

(ws (n, l, g)
 ^ (j ∈ nset (n))
 ^ (k ∈ nset (n))
 ^ rhoi (n, k, l, g, lp, gp)
 ^ (j ≠ k)
 ^ (j < k)
 ^ lg (n, l, g)
 ^ a2-at-n1-n2 (k, j, l)
 ^ union-at-n (lp, k, '(10 11 12)))
 → (¬ union-at-n (lp, j, '(5 6 7 8 9 10 11 12)))

;;;If j is less than k and the k's entry in lp is
;;;between 10 and 12, then the j's entry in lp is
;;;not between 5 and 12.

THEOREM: lm-i-eq-k-j-neq-k

(ws (n, l, g)
 ^ (j ∈ nset (n))
 ^ (k ∈ nset (n))
 ^ rhoi (n, k, l, g, lp, gp)
 ^ (j ≠ k)
 ^ (j < k)
 ^ lg (n, l, g)
 ^ a2-at-n1-n2 (k, j, l))
 → a2-at-n1-n2 (k, j, lp)

;;;If j is less than k,
;;;then (a2-at-n1-n2 k j lp) holds.

THEOREM: i-eq-k-j-neq-k

(ws (n, l, g)
 ^ (j ∈ nset (n))
 ^ (k ∈ nset (n))
 ^ rhoi (n, k, l, g, lp, gp)
 ^ (j ≠ k)
 ^ (j < k)
 ^ lg (n, l, g)
 ^ a2 (n, n, l))
 → a2-at-n1-n2 (k, j, lp)

;* j-eq-k-i-neq-k

;;;If the k's entry in l is not 4 and the k's entry in lp
;;;is between 5 and 7, then the k's entry in l is
;;;between 5 and 7.

THEOREM: k-in-lp5-7-not-l4-then-l5-7

(ws (n, l, g)
 \wedge ($k \in \text{nset}(n)$)
 \wedge rhoi(n, k, l, g, lp, gp)
 \wedge (\neg at($l, k, 4$))
 \wedge union-at-n($lp, k, '(5\ 6\ 7)$))
 \rightarrow union-at-n($l, k, '(5\ 6\ 7)$)

;;;If the k's entry in lp is between 5 and 7 then
;;;the k's entry in l is certainly between 5 and 12.

THEOREM: k-in-lp5-7-then-l5-11

(ws (n, l, g)
 \wedge ($k \in \text{nset}(n)$)
 \wedge rhoi(n, k, l, g, lp, gp)
 \wedge (\neg at($l, k, 4$))
 \wedge union-at-n($lp, k, '(5\ 6\ 7)$))
 \rightarrow union-at-n($l, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11)$)

;;;If the k's entry in lp is 8,
;;;then the k's entry in l is between 5 and 11.

THEOREM: k-in-lp8-then-l5-11

(ws (n, l, g) \wedge ($k \in \text{nset}(n)$) \wedge rhoi(n, k, l, g, lp, gp) \wedge at($lp, k, 8$))
 \rightarrow union-at-n($l, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11)$)

;;;If the k's entry in lp is between 9 and 12,
;;;then the k's entry in l is between 5 and 12.

THEOREM: k-in-lp9-12-then-l5-11

(ws (n, l, g)
 \wedge ($k \in \text{nset}(n)$)
 \wedge rhoi(n, k, l, g, lp, gp)
 \wedge union-at-n($lp, k, '(9\ 10\ 11\ 12)$))
 \rightarrow union-at-n($l, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11)$)

;;;If the k's entry in l is not 4 an the k's entry in lp is
;;;between 5 and 12, then the k's entry in l is
;;;between 5 and 11.

THEOREM: k-in-l5-11

(ws (n, l, g)
 \wedge ($k \in \text{nset}(n)$)
 \wedge rhoi(n, k, l, g, lp, gp)
 \wedge (\neg at($l, k, 4$))
 \wedge union-at-n($lp, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)$))
 \rightarrow union-at-n($l, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11)$)

;;;If the k's entry in l is not 4, and the k's entry
 ;;;in lp is not between 5 and 12, then the k's entry
 ;;;in lp is not between 5 and 12.

THEOREM: k-not-in-l4

(ws (n, l, g)
 ∧ (k ∈ nset (n))
 ∧ rhoi (n, k, l, g, lp, gp)
 ∧ (¬ at (l, k, 4))
 ∧ (¬ union-at-n (l, k, '(5 6 7 8 9 10 11 12))))
 → (¬ union-at-n (lp, k, '(5 6 7 8 9 10 11 12))))

;;;If a0 holds, and the k's entry in l is not
 ;;;between 5 and 12, then the k's entry in lp is not
 ;;;between 5 and 12.

THEOREM: k-not-in-lp5-12

(ws (n, l, g)
 ∧ (i ∈ nset (n))
 ∧ (k ∈ nset (n))
 ∧ rhoi (n, k, l, g, lp, gp)
 ∧ a0 (n, l, k)
 ∧ union-at-n (l, i, '(10 11 12))
 ∧ (¬ union-at-n (l, k, '(5 6 7 8 9 10 11 12))))
 → (¬ union-at-n (lp, k, '(5 6 7 8 9 10 11 12))))

;;;Auxiliary lemma for lm-i-neq-k-j-eq-k.
 ;;;There is (a2-at-n1-n2 i k l) in the lemma.

THEOREM: lm1-i-neq-k-j-eq-k

(ws (n, l, g)
 ∧ (i ∈ nset (n))
 ∧ (k ∈ nset (n))
 ∧ rhoi (n, k, l, g, lp, gp)
 ∧ (i ≠ k)
 ∧ (k < i)
 ∧ a0 (n, l, k)
 ∧ a2-at-n1-n2 (i, k, l)
 ∧ union-at-n (lp, i, '(10 11 12))
 → (¬ union-at-n (lp, k, '(5 6 7 8 9 10 11 12))))

;;;If k is less than i and the i's entry in lp is
 ;;;between 10 and 12, then the k's entry in lp is
 ;;;between 5 and 12.

THEOREM: lm-i-neq-k-j-eq-k

(ws (n, l, g)
 \wedge ($i \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge rhoi(n, k, l, g, lp, gp)
 \wedge ($i \neq k$)
 \wedge ($k < i$)
 \wedge a0(n, l, k)
 \wedge a2-at-n1-n2(i, k, l)
 \rightarrow a2-at-n1-n2(i, k, lp)

;;;If k is less than i then (a2-at-n1-n2 i k lp) holds.

THEOREM: i-neq-k-j-eq-k

(ws (n, l, g)
 \wedge ($k \in \text{nset}(n)$)
 \wedge ($i \in \text{nset}(n)$)
 \wedge rhoi(n, k, l, g, lp, gp)
 \wedge ($i \neq k$)
 \wedge ($k < i$)
 \wedge a0(n, l, k)
 \wedge a2(n, n, l)
 \rightarrow a2-at-n1-n2(i, k, lp)

;* i-j-neq-k

;;;If i and j are not equal to k and the i 's entry in lp is
;;;between 10 and 12, then the j 's entry in lp is
;;;between 5 and 12.

THEOREM: lm-i-j-neq-k

(ws (n, l, g)
 \wedge ($i \in \text{nset}(n)$)
 \wedge ($j \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge rhoi(n, k, l, g, lp, gp)
 \wedge ($i \neq k$)
 \wedge ($j \neq k$)
 \wedge ($j < i$)
 \wedge a2-at-n1-n2(i, j, l)
 \rightarrow a2-at-n1-n2(i, j, lp)

;;;If i and j are not equal to k ,
;;; then (a2-at-n1-n2 i j lp) holds.

THEOREM: i-j-neq-k

(ws (n, l, g)
 \wedge ($i \in \text{nset}(n)$)
 \wedge ($j \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge rhoi (n, k, l, g, lp, gp)
 \wedge ($i \neq k$)
 \wedge ($j \neq k$)
 \wedge ($j < i$)
 \wedge a2 (n, n, l)
 \rightarrow a2-at-n1-n2 (i, j, lp)

;;;If i is not equal to k and j is less than i ,
;;;then (a2-at-n1-n2 $i j lp$) holds.
;;;The order of the hints is crucial.

THEOREM: i-neq-k

(ws (n, l, g)
 \wedge ($k \in \text{nset}(n)$)
 \wedge ($i \in \text{nset}(n)$)
 \wedge ($j \in \text{nset}(n)$)
 \wedge rhoi (n, k, l, g, lp, gp)
 \wedge ($i \neq k$)
 \wedge ($j < i$)
 \wedge a0 (n, l, k)
 \wedge a2 (n, n, l)
 \rightarrow a2-at-n1-n2 (i, j, lp)

;;;If j is less than k then (a2-at-n1-n2 $k j lp$) holds.

THEOREM: i-eq-k

(ws (n, l, g)
 \wedge ($j \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge rhoi (n, k, l, g, lp, gp)
 \wedge ($j < k$)
 \wedge lg (n, l, g)
 \wedge a2 (n, n, l)
 \rightarrow a2-at-n1-n2 (k, j, lp)

;;;If i is less than j then (a2-at-n1-n2 $k j lp$) holds.
;;;Again the order of the hints is crucial.

THEOREM: rho-preserves-a2

(ws (n, l, g)

$\wedge (k \in \text{nset}(n))$
 $\wedge (i \in \text{nset}(n))$
 $\wedge (j \in \text{nset}(n))$
 $\wedge \text{rhoi}(n, k, l, g, lp, gp)$
 $\wedge (j < i)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{a0}(n, l, k)$
 $\wedge \text{a2}(n, n, l)$
 $\rightarrow \text{a2-at-n1-n2}(i, j, lp)$

;;; a3.ev ;;;
 ;* j-eq-k-i-neq-k

;;;If the i's entry in l is 12 and the k's entry in lp is
 ;;;between 5 and 12 then the k's entry in l is between 9
 ;;;and 11.

THEOREM: lm-k-in-l9-11

$(\text{ws}(n, l, g)$
 $\wedge (i \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{rhoi}(n, k, l, g, lp, gp)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{a0}(n, l, k)$
 $\wedge \text{a3-at-n1-n2}(i, k, l, g)$
 $\wedge \text{at}(l, i, 12)$
 $\wedge \text{union-at-n}(l, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11)))$
 $\rightarrow \text{union-at-n}(l, k, '(9\ 10\ 11))$

;;;If i is not equal to k, the i's entry in l is 12,
 ;;;and the k's entry in lp is between 5 and 12,
 ;;;then the k's entry in l is between 9 and 11.

THEOREM: k-in-l9-11

$(\text{ws}(n, l, g)$
 $\wedge (i \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{rhoi}(n, k, l, g, lp, gp)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{a0}(n, l, k)$
 $\wedge \text{a3-at-n1-n2}(i, k, l, g)$
 $\wedge \text{at}(l, i, 12)$
 $\wedge \text{union-at-n}(lp, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)))$
 $\rightarrow \text{union-at-n}(l, k, '(9\ 10\ 11))$

;;;If the k's entry in lp is between 9 and 11
 ;;;then the k's entry in lp is between 9 and 12.

THEOREM: k-in-lp9-12

(ws (n, l, g)
 ∧ (k ∈ nset (n))
 ∧ rhoi (n, k, l, g, lp, gp)
 ∧ union-at-n (l, k, '(9 10 11)))
 → union-at-n (lp, k, '(9 10 11 12))

;;;Auxiliary lemma for lm-a3-i-neq-k-j-eq-k.
 ;;;There is (a3-at-n1-n2 i k l g) in the lemma.

THEOREM: lm1-a3-i-neq-k-j-eq-k

(ws (n, l, g)
 ∧ (i ∈ nset (n))
 ∧ (k ∈ nset (n))
 ∧ rhoi (n, k, l, g, lp, gp)
 ∧ (i ≠ k)
 ∧ lg (n, l, g)
 ∧ lg (n, lp, gp)
 ∧ a0 (n, l, k)
 ∧ a3-at-n1-n2 (i, k, l, g)
 ∧ at (l, i, 12)
 ∧ union-at-n (lp, k, '(5 6 7 8 9 10 11 12)))
 → at (gp, k, 4)

;;;If i is not equal to k, the i's entry in lp is 12,
 ;;;and the k's entry in lp is between 5 and 12,
 ;;;then the k's entry in gp is 4.

THEOREM: lm-a3-i-neq-k-j-eq-k

(ws (n, l, g)
 ∧ (i ∈ nset (n))
 ∧ (k ∈ nset (n))
 ∧ rhoi (n, k, l, g, lp, gp)
 ∧ (i ≠ k)
 ∧ lg (n, l, g)
 ∧ a0 (n, l, k)
 ∧ a3-at-n1-n2 (i, k, l, g))
 → a3-at-n1-n2 (i, k, lp, gp)

;;;If i is not equal to k,
 ;;;then (a3-at-n1-n2 i k lp gp) holds.
 ;;;The order of the hypotheses is crucial.

THEOREM: a3-i-neq-k-j-eq-k

$(ws(n, l, g)$
 $\wedge (i \in nset(n))$
 $\wedge (k \in nset(n))$
 $\wedge rhoi(n, k, l, g, lp, gp)$
 $\wedge lg(n, l, g)$
 $\wedge a0(n, l, k)$
 $\wedge a3(n, n, l, g)$
 $\wedge (i \neq k))$
 $\rightarrow a3-at-n1-n2(i, k, lp, gp)$

;* i-eq-k-j-neq-k

;;;If the k's entry in lp is 12 then (phi11 k n g) holds.

THEOREM: cond-rhoi1

$(ws(n, l, g) \wedge (k \in nset(n)) \wedge rhoi(n, k, l, g, lp, gp) \wedge at(lp, k, 12))$
 $\rightarrow phi11(k, n, g)$

;;;If the k's entry in l is between 10 and 12,
;;;the j's entry in l is between 5 and 12, and
;;;(a2-at-n2 k n l) holds, then k is less than j.
;;;Because Bmp does not rewrite the clause
;;;(lessp k j), we take its contrapositive.

THEOREM: k-lt-j

$((j \in nset(n))$
 $\wedge (j \neq k)$
 $\wedge union-at-n(l, k, '(10 11 12))$
 $\wedge union-at-n(l, j, '(5 6 7 8 9 10 11 12))$
 $\wedge (k \not< j))$
 $\rightarrow (\neg a2-at-n2(k, n, l))$

;;;If k is less than j and (phi11 k n g) holds,
;;;then the j's entry in g is either 2 or 3.

THEOREM: phi11-j-not-in-g23

$((j \in nset(n)) \wedge (k < j) \wedge phi11(k, n, g))$
 $\rightarrow (\neg union-at-n(g, j, '(2 3)))$

;;;If j is not equal to k, (a2-at-n2 k n l), (phi11 k n g)
;;;the k's entry in l is between 10 and 12 and
;;;the j's entry in l is between 5 and 12,
;;;then the j's entry in g is either 2 or 3.

THEOREM: lm1-j-not-in-g23

$((j \in \text{nset}(n))$
 $\wedge (j \neq k)$
 $\wedge \text{a2-at-n2}(k, n, l)$
 $\wedge \text{phil1}(k, n, g)$
 $\wedge \text{union-at-n}(l, k, '(10\ 11\ 12))$
 $\wedge \text{union-at-n}(l, j, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)))$
 $\rightarrow (\neg \text{union-at-n}(g, j, '(2\ 3)))$

;;;If j is not equal to k, the k's entry in l is
;;;between 10 and 12, the k's entry in lp is 12 and
;;;the j's entry in l is between 5 and 12,
;;;then the j's entry in g is either 2 or 3.

THEOREM: lm2-j-not-in-g23

$(\text{ws}(n, l, g)$
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{rhoi}(n, k, l, g, lp, gp)$
 $\wedge (j \neq k)$
 $\wedge \text{a2-at-n2}(k, n, l)$
 $\wedge \text{at}(lp, k, 12)$
 $\wedge \text{at}(l, k, 11)$
 $\wedge \text{union-at-n}(l, j, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)))$
 $\rightarrow (\neg \text{union-at-n}(g, j, '(2\ 3)))$

;;;If j is not equal to k, the k's entry in lp is 12,
;;;and the j's entry in l is between 5 and 12,
;;;then the j's entry in g is either 2 or 3.

THEOREM: j-not-in-g23

$(\text{ws}(n, l, g)$
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{rhoi}(n, k, l, g, lp, gp)$
 $\wedge (j \neq k)$
 $\wedge \text{a2}(n, n, l)$
 $\wedge \text{at}(l, k, 11)$
 $\wedge \text{at}(lp, k, 12)$
 $\wedge \text{union-at-n}(l, j, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)))$
 $\rightarrow (\neg \text{union-at-n}(g, j, '(2\ 3)))$

THEOREM: j-in-g4

$((j \in \text{nset}(n))$
 $\wedge \text{lg}(n, l, g)$

$\wedge (\neg \text{union-at-n}(g, j, '(2\ 3)))$
 $\wedge \text{union-at-n}(l, j, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12))$
 $\rightarrow \text{at}(g, j, 4)$

;;;un9-12-then-un8-12, if3, j-not-in-g23,
 ;;;l5-12-eq-l5-8-or-l9-12, un8-12-then-un5-12,
 ;;;and j-in-g4 are used.
 ;;;If j is not equal to k, the k's entry in lp is 12,
 ;;;the j's entry in l is between 5 and 12,
 ;;;then the j's entry in g is 4.

THEOREM: a3-j-in-l5-12

$(\text{ws}(n, l, g)$
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{rhoi}(n, k, l, g, lp, gp)$
 $\wedge (j \neq k)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{a2}(n, n, l)$
 $\wedge \text{at}(l, k, 11)$
 $\wedge \text{at}(lp, k, 12)$
 $\wedge \text{union-at-n}(l, j, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)))$
 $\rightarrow \text{at}(g, j, 4)$

;;;If the k's entry in lp is 12,
 ;;;then the k's entry in l is 11.

THEOREM: k-in-l11

$(\text{ws}(n, l, g) \wedge (k \in \text{nset}(n)) \wedge \text{rhoi}(n, k, l, g, lp, gp) \wedge \text{at}(lp, k, 12))$
 $\rightarrow \text{at}(l, k, 11)$

;;;If k is not equal to j and the j's entry in g is 4,
 ;;;then the j's entry in gp is 4.

THEOREM: lm1-a3-i-eq-k-j-neq-k

$(\text{ws}(n, l, g)$
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{rhoi}(n, k, l, g, lp, gp)$
 $\wedge (j \neq k)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{a2}(n, n, l)$
 $\wedge \text{at}(lp, k, 12)$
 $\wedge \text{union-at-n}(lp, j, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)))$
 $\rightarrow \text{at}(g, j, 4)$

;;;If j is not equal to k, the k's entry in lp is 12,
 ;;;and the j's entry in lp is between 5 and 12,
 ;;;then the j's entry in gp is 4.

THEOREM: lm-a3-i-eq-k-j-neq-k

(ws (n, l, g)
 ^ (j ∈ nset (n))
 ^ (k ∈ nset (n))
 ^ rhoi (n, k, l, g, lp, gp)
 ^ (j ≠ k)
 ^ lg (n, l, g)
 ^ a2 (n, n, l)
 ^ a3-at-n1-n2 (k, j, l, g))
 → a3-at-n1-n2 (k, j, lp, gp)

;;;If j is not equal to k then (a3-at-n1-n2 k j lp gp).

THEOREM: a3-i-eq-k-j-neq-k

(ws (n, l, g)
 ^ (j ∈ nset (n))
 ^ (k ∈ nset (n))
 ^ rhoi (n, k, l, g, lp, gp)
 ^ (j ≠ k)
 ^ lg (n, l, g)
 ^ a2 (n, n, l)
 ^ a3 (n, n, l, g))
 → a3-at-n1-n2 (k, j, lp, gp)

;* i-j-neq-k

;;;If i,j are not equal to k, the i's entry in lp is 12
 ;;;and the j's entry in lp between 5 and 12
 ;;;then the j's entry in gp is 4.

THEOREM: lm-a3-i-j-neq-k

(ws (n, l, g)
 ^ (i ∈ nset (n))
 ^ (j ∈ nset (n))
 ^ (k ∈ nset (n))
 ^ rhoi (n, k, l, g, lp, gp)
 ^ (i ≠ k)
 ^ (j ≠ k)
 ^ a3-at-n1-n2 (i, j, l, g))
 → a3-at-n1-n2 (i, j, lp, gp)

;;;If i, j are not equal to k ,
 ;;;then $(a3-at-n1-n2\ i\ j\ lp\ gp)$.

THEOREM: a3-i-j-neq-k

$(ws\ (n, l, g)$
 $\wedge\ (i \in nset\ (n))$
 $\wedge\ (j \in nset\ (n))$
 $\wedge\ (k \in nset\ (n))$
 $\wedge\ rhoi\ (n, k, l, g, lp, gp)$
 $\wedge\ a3\ (n, n, l, g)$
 $\wedge\ (i \neq k)$
 $\wedge\ (j \neq k)$
 $\rightarrow\ a3-at-n1-n2\ (i, j, lp, gp)$

;* i-j-eq-k

THEOREM: lm-a3-i-j-eq-k

$(ws\ (n, l, g)$
 $\wedge\ (k \in nset\ (n))$
 $\wedge\ rhoi\ (n, k, l, g, lp, gp)$
 $\wedge\ lg\ (n, l, g)$
 $\wedge\ a2\ (n, n, l)$
 $\wedge\ a3-at-n1-n2\ (k, k, l, g)$
 $\rightarrow\ a3-at-n1-n2\ (k, k, lp, gp)$

;;;($a3-at-n1-n2\ k\ k\ lp\ gp$) holds by lg.

THEOREM: a3-i-j-eq-k

$(ws\ (n, l, g)$
 $\wedge\ (k \in nset\ (n))$
 $\wedge\ rhoi\ (n, k, l, g, lp, gp)$
 $\wedge\ lg\ (n, l, g)$
 $\wedge\ a2\ (n, n, l)$
 $\wedge\ a3\ (n, n, l, g)$
 $\rightarrow\ a3-at-n1-n2\ (k, k, lp, gp)$

;;;($a3-at-n1-n2\ k\ j\ lp\ gp$) holds.

THEOREM: a3-i-eq-k

$(ws\ (n, l, g)$
 $\wedge\ (j \in nset\ (n))$
 $\wedge\ (k \in nset\ (n))$
 $\wedge\ rhoi\ (n, k, l, g, lp, gp)$
 $\wedge\ lg\ (n, l, g)$

\wedge a2(n, n, l)
 \wedge a3(n, n, l, g)
 \rightarrow a3-at-n1-n2(k, j, lp, gp)

;;;If i is not equal to k ,
 ;;;then (a3-at-n1-n2 $i j lp gp$) holds.

THEOREM: a3-i-neq-k

(ws(n, l, g)
 \wedge ($k \in \text{nset}(n)$)
 \wedge ($i \in \text{nset}(n)$)
 \wedge ($j \in \text{nset}(n)$)
 \wedge rhoi(n, k, l, g, lp, gp)
 \wedge lg(n, l, g)
 \wedge a0(n, l, k)
 \wedge a2(n, n, l)
 \wedge a3(n, n, l, g)
 \wedge ($i \neq k$)
 \rightarrow a3-at-n1-n2(i, j, lp, gp)

;;;(a3-at-n1-n2 $i j lp gp$) holds.

THEOREM: rho-preserves-a3

(ws(n, l, g)
 \wedge ($k \in \text{nset}(n)$)
 \wedge ($i \in \text{nset}(n)$)
 \wedge ($j \in \text{nset}(n)$)
 \wedge rhoi(n, k, l, g, lp, gp)
 \wedge lg(n, l, g)
 \wedge a0(n, l, k)
 \wedge a2(n, n, l)
 \wedge a3(n, n, l, g)
 \rightarrow a3-at-n1-n2(i, j, lp, gp)

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