```
#|
```



```
|#
```

EVENT: Start with the initial **nqthm** theory.

```
;;;;;;;;;;;;;;;;;;;;;;;;   mutex-molecular.ev    ;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;

;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;   com.ev   ;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;*sequence and finite set utilities

;;;The ith entry in l.
```

DEFINITION:
nth $(l,\ i)$
= **if** listp (l)
  then if $i = 1$ **then** car (l)
    else nth $(\mathrm{cdr}\,(l),\ i - 1)$ **endif**

elseif $l \in \mathbf{N}$
then if $i = \mathtt{1}$ **then** l
  else f endif
else f endif

Event: Disable nth.

```
;;;update ith entry of l to be k
```

Definition:
move $(l,\ i,\ k)$
$=$ **if** $i = \mathtt{0}$ **then** l
elseif $l \simeq$ **nil**
then if $i = \mathtt{1}$ **then** k
  else l **endif**
elseif $i = \mathtt{1}$ **then** cons $(k,\ \text{cdr}\,(l))$
else cons $(\text{car}\,(l),\ \text{move}\,(\text{cdr}\,(l),\ i - 1,\ k))$ **endif**

Event: Disable move.

Definition: at $(l,\ i,\ k) = (\text{nth}\,(l,\ i) = k)$

Event: Disable at.

Definition:
length (l)
$=$ **if** listp (l) **then** $1 + \text{length}\,(\text{cdr}\,(l))$
else zero **endif**

Event: Disable length.

```
;;;The nth entry in l is in the list i.
```

Definition: union-at-n $(l,\ n,\ i) = (\text{nth}\,(l,\ n) \in i)$

Event: Disable union-at-n.

```
;;;Any entry in l is in the list i.
```

Definition:
all-union $(l,\ n,\ i)$
$=$ **if** $n \simeq \mathtt{0}$ **then t**
else union-at-n $(l,\ n,\ i) \wedge$ all-union $(l,\ n - 1,\ i)$ **endif**

EVENT: Disable all-union.

```
;;;There exists an entry in l which belongs to
;;;the list i, moreover when exists, some such
;;;j is returned.
```

DEFINITION:
exist-union $(l,\ n,\ i)$
$=$ **if** $n \simeq 0$ **then f**
elseif union-at-n $(l,\ n,\ i)$ **then** n
else exist-union $(l,\ n-1,\ i)$ **endif**

EVENT: Disable exist-union.

```
;;;n is in the intersection of l8-12 and g34.
```

DEFINITION:
intersect-8-12-3-4-at-n $(n,\ l,\ g)$
$=$ (union-at-n $(l,\ n,$ `'(8 9 10 11 12)`$)$ $\land$ union-at-n $(g,\ n,$ `'(3 4)`$))$

EVENT: Disable intersect-8-12-3-4-at-n.

```
;;;There exists n in the intersection of l8-12 and g34.
```

DEFINITION:
exist-intersect-8-12-3-4 $(n,\ l,\ g)$
$=$ **if** $n \simeq 0$ **then f**
elseif intersect-8-12-3-4-at-n $(n,\ l,\ g)$ **then** n
else exist-intersect-8-12-3-4 $(n-1,\ l,\ g)$ **endif**

EVENT: Disable exist-intersect-8-12-3-4.

```
;*Flag invariant.
```

DEFINITION:
lg-1-at-n $(n,\ l,\ g)$
$=$ ((at $(l,\ n,\ 0)$ $\land$ at $(g,\ n,\ 0)$)
$\lor$ (at $(l,\ n,\ 1)$ $\land$ at $(g,\ n,\ 0)$)
$\lor$ (at $(l,\ n,\ 2)$ $\land$ at $(g,\ n,\ 0)$)
$\lor$ (at $(l,\ n,\ 3)$ $\land$ at $(g,\ n,\ 1)$)
$\lor$ (at $(l,\ n,\ 4)$ $\land$ at $(g,\ n,\ 1)$))

EVENT: Disable lg-1-at-n.

Definition:
lg-2-at-n $(n,\ l,\ g)$
$$
\begin{aligned}
= \quad & ((\text{at}\,(l,\ n,\ \texttt{5}) \land \text{at}\,(g,\ n,\ \texttt{3})) \\
& \lor \quad (\text{at}\,(l,\ n,\ \texttt{6}) \land \text{at}\,(g,\ n,\ \texttt{3})) \\
& \lor \quad (\text{at}\,(l,\ n,\ \texttt{7}) \land \text{at}\,(g,\ n,\ \texttt{2})) \\
& \lor \quad (\text{at}\,(l,\ n,\ \texttt{8}) \land \text{at}\,(g,\ n,\ \texttt{3})) \\
& \lor \quad (\text{at}\,(l,\ n,\ \texttt{8}) \land \text{at}\,(g,\ n,\ \texttt{2})))
\end{aligned}
$$

Event: Disable lg-2-at-n.

Definition:
lg-3-at-n $(n,\ l,\ g)$
$$
\begin{aligned}
= \quad & ((\text{at}\,(l,\ n,\ \texttt{9}) \land \text{at}\,(g,\ n,\ \texttt{4})) \\
& \lor \quad (\text{at}\,(l,\ n,\ \texttt{10}) \land \text{at}\,(g,\ n,\ \texttt{4})) \\
& \lor \quad (\text{at}\,(l,\ n,\ \texttt{11}) \land \text{at}\,(g,\ n,\ \texttt{4})) \\
& \lor \quad (\text{at}\,(l,\ n,\ \texttt{12}) \land \text{at}\,(g,\ n,\ \texttt{4})))
\end{aligned}
$$

Event: Disable lg-3-at-n.

Definition:
lg-at-n $(n,\ l,\ g)$
$= \quad (\text{lg-1-at-n}\,(n,\ l,\ g) \land \text{lg-2-at-n}\,(n,\ l,\ g) \land \text{lg-3-at-n}\,(n,\ l,\ g))$

Event: Disable lg-at-n.

Definition:
lg $(n,\ l,\ g)$
$= \quad$ **if** $n \simeq \texttt{0}$ **then t**
else lg-at-n $(n,\ l,\ g) \land$ lg $(n - 1,\ l,\ g)$ **endif**

Event: Disable lg.

```
;*The set {1...n}.
```

Definition:
nset (n)
$= \quad$ **if** $n \simeq \texttt{0}$ **then nil**
else cons $(n,\ \text{nset}\,(n - 1))$ **endif**

Event: Disable nset.

```
;;;n belongs to nset.
```

THEOREM: n-in-nset
$(n \not\simeq 0) \rightarrow (n \in \text{nset}(n))$

```
;;;Any element in nset is a number.
```

THEOREM: nset-number
$(k \in \text{nset}(n)) \rightarrow (k \in \mathbf{N})$

```
;;;If a nonzero number plus one belongs to nset,
;;;then so does the nonzero number itself.
```

THEOREM: add1-nset
$((k \not\simeq 0) \wedge ((1 + k) \in \text{nset}(n))) \rightarrow (k \in \text{nset}(n))$

```
;;;Any list has its length at least nonzero.
```

THEOREM: list-ln
$\text{listp}(l) \rightarrow (\text{length}(l) \neq 0)$

```
;;;(move l k i) is again a list if l is a list.
```

THEOREM: move-is-list
$\text{listp}(l) \rightarrow \text{listp}(\text{move}(l,\ k,\ i))$

EVENT: Enable length.

```
;;;(move l k i) has i as its kth entry.
;;;(enable length) is critical to prove this lemma.
```

THEOREM: move-nth
$(\text{listp}(l) \wedge (k \in \text{nset}(\text{length}(l)))) \rightarrow (\text{nth}(\text{move}(l,\ k,\ i),\ k) = i)$

THEOREM: zero-not-member-nset
$0 \notin \text{nset}(n)$

```
;;;Lists l and (move l k i) have the same length.
```

THEOREM: move-unchange-length
$(\text{listp}(l) \wedge (k \in \text{nset}(\text{length}(l))))$
$\rightarrow \quad (\text{length}(\text{move}(l,\ k,\ i)) = \text{length}(l))$

```
;;;Lists l and (move l k i) have the same entries
;;;except kth one.
```

THEOREM: move-unchange-other-than-nth
$(\text{listp}(l) \wedge (k \in \text{nset}(\text{length}(l))) \wedge (j \neq k))$
$\rightarrow \quad (\text{nth}(\text{move}(l,\ k,\ i),\ j) = \text{nth}(l,\ j))$

THEOREM: member-ex-union
exist-union $(l,\ n,\ i) \rightarrow (\text{exist-union}\,(l,\ n,\ i) \in \text{nset}\,(n))$

```
;;;(exist-union l n i) is a number.
```

THEOREM: number-ex-union
exist-union $(l,\ n,\ i) \rightarrow (\text{exist-union}\,(l,\ n,\ i) \in \mathbf{N})$

```
;;;(exist-intersect-8-12-3-4 n l g) belongs to nset.
```

THEOREM: member-intersect
exist-intersect-8-12-3-4 $(n,\ l,\ g)$
$\rightarrow \quad (\text{exist-intersect-8-12-3-4}\,(n,\ l,\ g) \in \text{nset}\,(n))$

```
;;;(exist-intersect-8-12-3-4 n l g) is a number.
```

THEOREM: number-intersect
exist-intersect-8-12-3-4 $(n,\ l,\ g) \rightarrow (\text{exist-intersect-8-12-3-4}\,(n,\ l,\ g) \in \mathbf{N})$

```
;;;any member of nset is nonzero.
```

THEOREM: k-not-0
$(k \in \text{nset}\,(n)) \rightarrow (k \not\simeq \texttt{0})$

```
;*lemmas for a0

;;;If j's entry in l is between 8..12 then
;;;(exist-union l n '(8 9 10 11 12)) holds.
```

THEOREM: j-ex-l8-12
$((j \in \text{nset}\,(n)) \wedge \text{union-at-n}\,(l,\ j,\ \texttt{'(8 9 10 11 12)}))$
$\rightarrow \quad \text{exist-union}\,(l,\ n,\ \texttt{'(8 9 10 11 12)})$

```
;;;Witness of (exist-union lp n '(8 9 10 11 12))
;;;has in lp its entry between 8...12.
```

THEOREM: ex-lp8-12-in-lp8-12
exist-union $(lp,\ n,\ \texttt{'(8 9 10 11 12)})$
$\rightarrow \quad \text{union-at-n}\,(lp,$
$\qquad \text{exist-union}\,(lp,\ n,\ \texttt{'(8 9 10 11 12)}),$
$\qquad \texttt{'(8 9 10 11 12)})$

```
;;;If (not (exist-union l n '(8 9 10 11 12)))
;;;holds, then (not (exist-union g n '(4))) by lg.
```

THEOREM: ex-if4
$((\neg\ \text{exist-union}\,(l,\ n,\ \texttt{'(8 9 10 11 12)})) \wedge \text{lg}\,(n,\ l,\ g))$
$\rightarrow \quad (\neg\ \text{exist-union}\,(g,\ n,\ \texttt{'(4)}))$

```
;;;If (not (exist-union g n '(1))) holds,
;;; then there is no entry either 3 or 4.
```

THEOREM: l34-empty
$((j \in \text{nset}(n)) \land \text{lg}(n, l, g) \land (\neg\ \text{exist-union}(g, n, \texttt{'(1)})))$
$\rightarrow \quad (\neg\ \text{union-at-n}(l, j, \texttt{'(3 4)}))$

```
;;;If j's entry in lp is 4, then (certainly)
;;;it is either 3 or 4.
```

THEOREM: lp4-then-un34
$\text{at}(lp, j, 4) \rightarrow \text{union-at-n}(lp, j, \texttt{'(3 4)})$

```
;;;If (exist-intersect-8-12-3-4 n l g) holds,
;;;then so does (exist-union g n '(3 4)).
```

THEOREM: int-8-12-3-4-then-un34
$\text{exist-intersect-8-12-3-4}(n, l, g) \rightarrow \text{exist-union}(g, n, \texttt{'(3 4)})$

```
;*lemmas for a1

;;;i is the witness of
;;;(exist-intersect-8-12-3-4 n lp gp).
```

THEOREM: int-wtn
$((j \in \text{nset}(n)) \land \text{intersect-8-12-3-4-at-n}(j, lp, gp))$
$\rightarrow \quad \text{exist-intersect-8-12-3-4}(n, lp, gp)$

```
;;;If there exists j such that j's entry in lp
;;;is between 8..12 and entry in gp is either 3 or 4
;;;then (intersect-8-12-3-4-at-n j lp gp) holds.
```

THEOREM: un8-12-and-un34-then-int
$(\text{union-at-n}(lp, j, \texttt{'(8 9 10 11 12)}) \land \text{union-at-n}(gp, j, \texttt{'(3 4)}))$
$\rightarrow \quad \text{intersect-8-12-3-4-at-n}(j, lp, gp)$

```
;;;By the two lemmas above,
;;;(exist-intersect-8-12-3-4 n lp gp) holds provided
;;;that there exists j such that j's entry in lp is
;;;between 8..12 and entry in gp is either 3 or 4.

;* ep-l8-12

;;;If the k's entry in l is 5, then the k's entry
;;;in g is 3 by lg.
```

THEOREM: lg-l5-g3
$((k \in \text{nset}(n)) \land \text{lg}(n, l, g) \land \text{at}(l, k, 5)) \rightarrow \text{at}(g, k, 3)$

```
;;;If the k's entry in gp is 3 then certainly
;;;it is either 3 or 4.
```

THEOREM: gp3-then-un34
$\text{at}(gp, k, 3) \rightarrow \text{union-at-n}(gp, k, \texttt{'(3 4)})$

```
;;;nep-l8-12

;;;If the k's entry in l is between 8..12 then
;;;it is either between 8..11 or equal to 12.
```

THEOREM: case-k
$(\text{union-at-n}(l, k, \texttt{'(8 9 10 11 12)})$
$\land \quad (\neg\, \text{union-at-n}(l, k, \texttt{'(8 9 10 11)})))$
$\rightarrow \quad \text{at}(l, k, 12)$

```
;;;;;k-not-l8-12

;;;If (exist-intersect-8-12-3-4 n l g) holds
;;;then the witness has its entry in g either equal
;;;to 3 or 4.
```

THEOREM: intersect-8-12-3-4-then-3-4
$\text{exist-intersect-8-12-3-4}(n, l, g)$
$\rightarrow \quad \text{union-at-n}(g, \text{exist-intersect-8-12-3-4}(n, l, g), \texttt{'(3 4)})$

```
;;;If (exist-intersect-8-12-3-4 n l g) holds,
;;; then the witness has its entry in g between 8 and 12.
```

THEOREM: intersect-8-12-3-4-then-8-12
$\text{exist-intersect-8-12-3-4}(n, l, g)$
$\rightarrow \quad \text{union-at-n}(l, \text{exist-intersect-8-12-3-4}(n, l, g), \texttt{'(8 9 10 11 12)})$

```
;;;k-in-l8-11

;;;If k's entry in lp is between 9 and 12,
;;;then it is certainly between 8 and 12.
```

THEOREM: un9-12-then-un8-12
$\text{union-at-n}(lp, k, \texttt{'(9 10 11 12)})$
$\rightarrow \quad \text{union-at-n}(lp, k, \texttt{'(8 9 10 11 12)})$

```
;;;If the i's entry in l is between 9 and 12,
;;;then the k's entry in g is 4.
```

THEOREM: if4
$((j \in \text{nset}(n)) \land \text{lg}(n, l, g) \land \text{union-at-n}(l, j, \texttt{'(9 10 11 12)}))$
$\rightarrow \quad \text{at}(g, j, 4)$

```
;;;k-in-l12

;;;If (exist-union lp n '(8 9 10 11 12)) holds then
;;;its witness does not have its entry in lp equal to 1.
```

THEOREM: ex-lp8-12-not-in-lp0
$\text{exist-union}(lp, n, \texttt{'(8 9 10 11 12)})$
$\rightarrow \quad (\neg\, \text{at}(lp, \text{exist-union}(lp, n, \texttt{'(8 9 10 11 12)}), 0))$

```
;;;If k's entry in lp is between 8 and 12,
;;; then it is either between 8 and 11 or 12.
```

THEOREM: k-in-lp9-12-or-lp8
$(\text{union-at-n}(lp, k, \texttt{'(8 9 10 11 12)})$
$\land \quad (\neg\, \text{union-at-n}(lp, k, \texttt{'(9 10 11 12)})))$
$\rightarrow \quad \text{at}(lp, k, 8)$

```
;;;If the k's entry is either 5 or 7,
;;;then it is between 5 and 7.
```

THEOREM: un57-then-un5-12
$\text{union-at-n}(l, k, \texttt{'(5 7)}) \rightarrow \text{union-at-n}(l, k, \texttt{'(5 6 7 8 9 10 11 12)})$

```
;;;If the k's entry in l is between 8 and 11,
;;;then it is between 5 and 12.
```

THEOREM: un8-11-then-un5-12
$\text{union-at-n}(l, k, \texttt{'(8 9 10 11)})$
$\rightarrow \quad \text{union-at-n}(l, k, \texttt{'(5 6 7 8 9 10 11 12)})$

```
;;;If the k's entry in l is between 8 and 12,
;;;then it is between 5 and 12.
```

THEOREM: un8-12-then-un5-12
$\text{union-at-n}(l, k, \texttt{'(8 9 10 11 12)})$
$\rightarrow \quad \text{union-at-n}(l, k, \texttt{'(5 6 7 8 9 10 11 12)})$

```
;*lemmas for a2

;;;i-eq-k-j-neq-k

;;;If the k's entry in l is either 10 or 11,
;;;then the k's entry in l is between 10 and 12.
```

THEOREM: un10-11-then-un10-12
union-at-n $(l,\ k,$ `'(10 11)`$) \rightarrow$ union-at-n $(l,\ k,$ `'(10 11 12)`$)$

```
;;;If the j's entry in g is either 0 or 1 then
;;;the j's entry in l is not between 5 and 12.
```

THEOREM: if1
$((j \in$ nset $(n)) \land$ lg $(n,\ l,\ g) \land$ union-at-n $(g,\ j,$ `'(0 1)`$))$
$\rightarrow \quad (\neg$ union-at-n $(l,\ j,$ `'(5 6 7 8 9 10 11 12)`$))$

```
;;;j-eq-k-i-neq-k

;;;If the k's entry in l is between 5 and 7,
;;;then it is certainly between 5 and 12.
```

THEOREM: un5-7-then-un5-11
union-at-n $(l,\ k,$ `'(5 6 7)`$) \rightarrow$ union-at-n $(l,\ k,$ `'(5 6 7 8 9 10 11)`$)$

```
;;;If the k's entry in lp is between 5 and 7 then
;;;it is certain between 5 and 11.
```

THEOREM: un57-then-un5-11
union-at-n $(l,\ k,$ `'(5 7)`$) \rightarrow$ union-at-n $(l,\ k,$ `'(5 6 7 8 9 10 11)`$)$

```
;;;If the k's entry in l is between 8 and 11,
;;;then it is certainly between 5 and 11.
```

THEOREM: un8-11-then-un5-11
union-at-n $(l,\ k,$ `'(8 9 10 11)`$)$
$\rightarrow \quad$ union-at-n $(l,\ k,$ `'(5 6 7 8 9 10 11)`$)$

```
;;;If the k's entry in lp is between 5 and 12 and
;;;the k's entry in lp is between 5 and 7, then
;;;the k's entry in lp in fact is between 9 and 12.
```

THEOREM: k-in-lp5-7-or-lp8-or-lp9-12
(union-at-n $(lp,\ k,$ `'(5 6 7 8 9 10 11 12)`$)$
$\land \quad (\neg$ union-at-n $(lp,\ k,$ `'(5 6 7)`$))$
$\land \quad (\neg$ at $(lp,\ k,\ 8)))$
$\rightarrow \quad$ union-at-n $(lp,\ k,$ `'(9 10 11 12)`$)$

```
;;;If the k's entry in l is between 5 and 11,
;;; then it is certainly between 5 and 12.
```

THEOREM: un5-11-then-un5-12
union-at-n $(l,\ k,$ `'(5 6 7 8 9 10 11)`$)$
$\rightarrow \quad$ union-at-n $(l,\ k,$ `'(5 6 7 8 9 10 11 12)`$)$

```
;;;If the k's entry in l is between 10 and 12,
;;; then it is certainly between 8 and 12.
```

THEOREM: un10-12-then-un8-12
union-at-n$(l,\ i,$ `'(10 11 12)`$) \rightarrow$ union-at-n$(l,\ i,$ `'(8 9 10 11 12)`$)$

```
;;;j-eq-k-i-neq-k

;;;If (exist-union l n '(8 9 10 11 12)) does not hold,
;;;then the i's entry in l is not between 10 and 12.
```

THEOREM: i-not-l10-12
$((i \in \text{nset}(n)) \wedge (\neg$ exist-union$(l,\ n,$ `'(8 9 10 11 12)`$)))$
$\rightarrow\quad (\neg$ union-at-n$(l,\ i,$ `'(10 11 12)`$))$

```
;*lemmas for a3

;;;j-eq-k-i-neq-k

;;;If the k's entry in l is between 5 and 11,
;;;then the k's entry in l is between 9 and 11.
```

THEOREM: un5-11-eq-un58-or-un8-11
(union-at-n$(l,\ k,$ `'(5 6 7 8 9 10 11)`$)$
$\wedge\quad (\neg$ union-at-n$(l,\ k,$ `'(5 6 7 8)`$)))$
$\rightarrow\quad$ union-at-n$(l,\ k,$ `'(9 10 11)`$)$

```
;;;If the k's entry in g is 4,
;;;then the k's entry in l is between 5 and 8.
```

THEOREM: a3-if4
$((k \in \text{nset}(n)) \wedge \text{lg}(n,\ l,\ g) \wedge \text{at}(g,\ k,\ 4))$
$\rightarrow\quad (\neg$ union-at-n$(l,\ k,$ `'(5 6 7 8)`$))$

```
;;;If the k's entry in l is between 5 and 11,
;;;and the k's entry in l is between 5 and 12,
;;;then the k's entry in l is 9 and 11.
```

THEOREM: k-in-l5-11-g4-then-l9-11
$((k \in \text{nset}(n))$
$\wedge\quad \text{lg}(n,\ l,\ g)$
$\wedge\quad$ union-at-n$(l,\ k,$ `'(5 6 7 8 9 10 11)`$)$
$\wedge\quad \text{at}(g,\ k,\ 4))$
$\rightarrow\quad$ union-at-n$(l,\ k,$ `'(9 10 11)`$)$

```
;;;If the i's entry in l is 12,
;;;then the i's entry in l is between 8 and 12.
```

THEOREM: l12-then-un8-12
at $(l, i, 12) \rightarrow$ union-at-n $(l, i,$ '(8 9 10 11 12))

```
;;;If (exist-union l n '(8 9 10 11 12)) does not hold,
;;;then the i's entry in l is 12.
```

THEOREM: i-not-in-l12
$((i \in \text{nset}(n)) \wedge (\neg$ exist-union $(l, n,$ '(8 9 10 11 12))))
$\rightarrow$ $(\neg$ at $(l, i, 12))$

```
;;;j-neq-k-i-eq-k

;;;If the k's entry in l is 11,
;;; then the k's entry in l is between 10 and 12.
```

THEOREM: l11-then-un10-12
at $(l, k, 11) \rightarrow$ union-at-n $(l, k,$ '(10 11 12))

```
;;;If the j's entry in g is either 2 or 3,
;;;then the j's entry in l is between 5 and 8 by lg.
```

THEOREM: if3
$((j \in \text{nset}(n)) \wedge \text{lg}(n, l, g) \wedge (\neg$ union-at-n $(g, j,$ '(2 3))))
$\rightarrow$ $(\neg$ union-at-n $(l, j,$ '(5 6 7 8)))

```
;;;If the j's entry in l is between 5 and 12 and
;;;the j's entry in l is between 5 and 8, then
;;;the j's entry in l is 9 and 12.
```

THEOREM: l5-12-eq-l5-8-or-l9-12
(union-at-n $(l, j,$ '(5 6 7 8 9 10 11 12))
$\wedge$ $(\neg$ union-at-n $(l, j,$ '(5 6 7 8))))
$\rightarrow$ union-at-n $(l, j,$ '(9 10 11 12))

```
;;;i-j-eq-k

;;;If the k's entry in lp is 12,
;;;then it is certainly between 5 and 12.
```

THEOREM: l12-then-un9-12
at $(lp, k, 12) \rightarrow$ union-at-n $(lp, k,$ '(9 10 11 12))

```
;*lemmas for b1a

;;;If the u's entry in g is 4,
;;;then the u's entry in l is between 8 and 12 by lg.
```

THEOREM: b1a-if4
$((u \in \text{nset}(n)) \land \text{lg}(n, l, g) \land \text{at}(g, u, 4))$
$\rightarrow \quad \text{union-at-n}(l, u, \texttt{'(8 9 10 11 12)})$

```
;*lemmas for b1b


;;;If the k's entry in lp is between 9 and 12,
;;;then the k's entry in gp is iether 3 or 4 by lg.
```

THEOREM: lp9-12-then-k-in-g34
$((k \in \text{nset}(n)) \land \text{union-at-n}(lp, k, \texttt{'(9 10 11 12)}) \land \text{lg}(n, lp, gp))$
$\rightarrow \quad \text{union-at-n}(gp, k, \texttt{'(3 4)})$

```
;;;If the k's entry in lp is between 8 and 12, and
;;;it is not 8, then it is certainly between 9 and 12.
```

THEOREM: un8-12-then-l8-or-l9-12
$(\text{union-at-n}(lp, k, \texttt{'(8 9 10 11 12)}) \land (\neg\ \text{at}(lp, k, 8)))$
$\rightarrow \quad \text{union-at-n}(lp, k, \texttt{'(9 10 11 12)})$

```
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;; moldefn.ev ;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;* Well-formed states
```

DEFINITION:
$\text{molws}(n, l, g, h)$
$= \quad ((n \in \mathbf{N})$
$\land \quad \text{listp}(l)$
$\land \quad \text{listp}(g)$
$\land \quad \text{listp}(h)$
$\land \quad (\text{length}(l) = n)$
$\land \quad (\text{length}(g) = n)$
$\land \quad (\text{length}(h) = n)$
$\land \quad \text{all-union}(l, n, \texttt{'(0 1 2 3 4 5 6 7 8 9 10 11 12)})$
$\land \quad \text{all-union}(g, n, \texttt{'(0 1 2 3 4)})$
$\land \quad \text{all-union}(h, n, \text{nset}(1 + n)))$

EVENT: Disable molws.

```
;* Transitions
```

DEFINITION:
$\text{mrhoi0}(n, i, l, g, h, lp, gp, hp)$
$= \quad (\text{at}(l, i, 0) \land (gp = g) \land (lp = \text{move}(l, i, 1)) \land (hp = h))$

DEFINITION:
mrhoi1a $(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$= \quad (\text{at}\,(l,\ i,\ \texttt{1}) \land (gp = g) \land (lp = \text{move}\,(l,\ i,\ \texttt{2})) \land (hp = h))$

DEFINITION:
mrhoi1b $(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$= \quad (\text{at}\,(l,\ i,\ \texttt{1}) \land (gp = g) \land (lp = l) \land (hp = h))$

DEFINITION:
mrhoi2 $(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$= \quad (\text{at}\,(l,\ i,\ \texttt{2})$
$\quad \land \quad (lp = \text{move}\,(l,\ i,\ \texttt{3}))$
$\quad \land \quad (gp = \text{move}\,(g,\ i,\ \texttt{1}))$
$\quad \land \quad (hp = \text{move}\,(h,\ i,\ \texttt{1})))$

DEFINITION:
mrhoi3a $(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$= \quad (\text{at}\,(l,\ i,\ \texttt{3})$
$\quad \land \quad (gp = g)$
$\quad \land \quad (hp = h)$
$\quad \land \quad \text{at}\,(h,\ i,\ 1 + n)$
$\quad \land \quad (lp = \text{move}\,(l,\ i,\ \texttt{4})))$

DEFINITION:
mrhoi3b $(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$= \quad (\text{at}\,(l,\ i,\ \texttt{3})$
$\quad \land \quad (gp = g)$
$\quad \land \quad (lp = l)$
$\quad \land \quad (\text{nth}\,(h,\ i) < (1 + n))$
$\quad \land \quad (hp = \text{move}\,(h,\ i,\ 1 + \text{nth}\,(h,\ i)))$
$\quad \land \quad \text{union-at-n}\,(g,\ \text{nth}\,(h,\ i),\ \texttt{'(0 1 2)}))$

DEFINITION:
mrhoi4 $(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$= \quad (\text{at}\,(l,\ i,\ \texttt{4})$
$\quad \land \quad (gp = \text{move}\,(g,\ i,\ \texttt{3}))$
$\quad \land \quad (lp = \text{move}\,(l,\ i,\ \texttt{5}))$
$\quad \land \quad (hp = \text{move}\,(h,\ i,\ \texttt{1})))$

DEFINITION:
mrhoi5a $(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$= \quad (\text{at}\,(l,\ i,\ \texttt{5})$
$\quad \land \quad (gp = g)$
$\quad \land \quad (hp = h)$
$\quad \land \quad \text{at}\,(h,\ i,\ 1 + n)$
$\quad \land \quad (lp = \text{move}\,(l,\ i,\ \texttt{8})))$

DEFINITION:
mrhoi5b $(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$= \quad (\text{at}\,(l,\ i,\ \texttt{5})$
$\quad \wedge \quad (gp = g)$
$\quad \wedge \quad (hp = h)$
$\quad \wedge \quad (\text{nth}\,(h,\ i) < (1 + n))$
$\quad \wedge \quad \text{at}\,(g,\ \text{nth}\,(h,\ i),\ \texttt{1})$
$\quad \wedge \quad (lp = \text{move}\,(l,\ i,\ \texttt{6})))$

DEFINITION:
mrhoi5c $(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$= \quad (\text{at}\,(l,\ i,\ \texttt{5})$
$\quad \wedge \quad (gp = g)$
$\quad \wedge \quad (lp = l)$
$\quad \wedge \quad (\text{nth}\,(h,\ i) < (1 + n))$
$\quad \wedge \quad (\neg\ \text{at}\,(g,\ \text{nth}\,(h,\ i),\ \texttt{1}))$
$\quad \wedge \quad (hp = \text{move}\,(h,\ i,\ 1 + \text{nth}\,(h,\ i))))$

DEFINITION:
mrhoi6 $(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$= \quad (\text{at}\,(l,\ i,\ \texttt{6})$
$\quad \wedge \quad (gp = \text{move}\,(g,\ i,\ \texttt{2}))$
$\quad \wedge \quad (lp = \text{move}\,(l,\ i,\ \texttt{7}))$
$\quad \wedge \quad (hp = \text{move}\,(h,\ i,\ \texttt{1})))$

DEFINITION:
mrhoi7a $(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$= \quad (\text{at}\,(l,\ i,\ \texttt{7})$
$\quad \wedge \quad (lp = \text{move}\,(l,\ i,\ \texttt{8}))$
$\quad \wedge \quad \text{at}\,(g,\ \text{nth}\,(h,\ i),\ \texttt{4})$
$\quad \wedge \quad (gp = g)$
$\quad \wedge \quad (hp = h))$

DEFINITION:
mrhoi7b $(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$= \quad (\text{at}\,(l,\ i,\ \texttt{7})$
$\quad \wedge \quad (\neg\ \text{at}\,(g,\ \text{nth}\,(h,\ i),\ \texttt{4}))$
$\quad \wedge \quad (lp = l)$
$\quad \wedge \quad (gp = g)$
$\quad \wedge \quad (hp = \text{move}\,(h,\ i,\ 1 + ((\text{nth}\,(h,\ i) - 1)\ \mathbf{mod}\ n))))$

DEFINITION:
mrhoi8 $(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$= \quad (\text{at}\,(l,\ i,\ \texttt{8})$
$\quad \wedge \quad (gp = \text{move}\,(g,\ i,\ \texttt{4}))$

$\wedge \quad (lp = \text{move}\,(l,\ i,\ \texttt{9}))$
$\wedge \quad (hp = \text{move}\,(h,\ i,\ \texttt{1})))$

DEFINITION:
$\text{mrhoi9a}\,(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$= \quad (\text{at}\,(l,\ i,\ \texttt{9})$
$\wedge \quad \text{at}\,(h,\ i,\ i)$
$\wedge \quad (lp = \text{move}\,(l,\ i,\ \texttt{10}))$
$\wedge \quad (gp = g)$
$\wedge \quad (hp = h))$

DEFINITION:
$\text{mrhoi9b}\,(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$= \quad (\text{at}\,(l,\ i,\ \texttt{9})$
$\wedge \quad (\text{nth}\,(h,\ i) < i)$
$\wedge \quad \text{union-at-n}\,(g,\ \text{nth}\,(h,\ i),\ \texttt{'(0 1)})$
$\wedge \quad (hp = \text{move}\,(h,\ i,\ 1 + \text{nth}\,(h,\ i)))$
$\wedge \quad (gp = g)$
$\wedge \quad (lp = l))$

DEFINITION:
$\text{mrhoi10}\,(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$= \quad (\text{at}\,(l,\ i,\ \texttt{10})$
$\wedge \quad (lp = \text{move}\,(l,\ i,\ \texttt{11}))$
$\wedge \quad (gp = g)$
$\wedge \quad (hp = \text{move}\,(h,\ i,\ 1 + i)))$

DEFINITION:
$\text{mrhoi11a}\,(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$= \quad (\text{at}\,(l,\ i,\ \texttt{11})$
$\wedge \quad \text{at}\,(h,\ i,\ 1 + n)$
$\wedge \quad (lp = \text{move}\,(l,\ i,\ \texttt{12}))$
$\wedge \quad (gp = g)$
$\wedge \quad (hp = h))$

DEFINITION:
$\text{mrhoi11b}\,(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$= \quad (\text{at}\,(l,\ i,\ \texttt{11})$
$\wedge \quad (\text{nth}\,(h,\ i) < (1 + n))$
$\wedge \quad (\neg\ \text{union-at-n}\,(g,\ \text{nth}\,(h,\ i),\ \texttt{'(2 3)}))$
$\wedge \quad (hp = \text{move}\,(h,\ i,\ 1 + \text{nth}\,(h,\ i)))$
$\wedge \quad (gp = g)$
$\wedge \quad (lp = l))$

DEFINITION:

$\text{mrhoi12}\,(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$= \quad (\text{at}\,(l,\ i,\ \texttt{12})$
$\quad \land \quad (hp = h)$
$\quad \land \quad (gp = \text{move}\,(g,\ i,\ \texttt{0}))$
$\quad \land \quad (lp = \text{move}\,(l,\ i,\ \texttt{0})))$

DEFINITION:
$\text{mrhoi}\,(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$= \quad (\text{mrhoi0}\,(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\quad \lor \quad \text{mrhoi1a}\,(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\quad \lor \quad \text{mrhoi1b}\,(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\quad \lor \quad \text{mrhoi2}\,(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\quad \lor \quad \text{mrhoi3a}\,(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\quad \lor \quad \text{mrhoi3b}\,(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\quad \lor \quad \text{mrhoi4}\,(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\quad \lor \quad \text{mrhoi5a}\,(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\quad \lor \quad \text{mrhoi5b}\,(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\quad \lor \quad \text{mrhoi5c}\,(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\quad \lor \quad \text{mrhoi6}\,(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\quad \lor \quad \text{mrhoi7a}\,(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\quad \lor \quad \text{mrhoi7b}\,(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\quad \lor \quad \text{mrhoi8}\,(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\quad \lor \quad \text{mrhoi9a}\,(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\quad \lor \quad \text{mrhoi9b}\,(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\quad \lor \quad \text{mrhoi10}\,(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\quad \lor \quad \text{mrhoi11a}\,(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\quad \lor \quad \text{mrhoi11b}\,(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\quad \lor \quad \text{mrhoi12}\,(n,\ i,\ l,\ g,\ h,\ lp,\ gp,\ hp))$

EVENT: Disable mrhoi.

```
;*  Invariants
```

```
;;;b0
```

DEFINITION:
$\text{b0a}\,(n,\ l,\ h,\ i,\ j) = ((\text{at}\,(l,\ i,\ \texttt{5}) \land (j < \text{nth}\,(h,\ i))) \rightarrow (\neg\ \text{at}\,(l,\ j,\ \texttt{4})))$

EVENT: Disable b0a.

DEFINITION:
$\text{b0b}\,(n,\ l,\ h,\ i,\ j)$
$= \quad ((\text{at}\,(l,\ i,\ \texttt{5}) \land (j < \text{nth}\,(h,\ i)) \land \text{at}\,(l,\ j,\ \texttt{3})) \rightarrow (i \not< \text{nth}\,(h,\ j)))$

EVENT: Disable b0b.

```
;;;b1
```

DEFINITION:
$\text{b1a}(l, i, j) = (\text{union-at-n}(l, i, \texttt{'(8 9 10 11 12)}) \rightarrow (\neg\, \text{at}(l, j, \texttt{4})))$

EVENT: Disable b1a.

DEFINITION:
$\text{hint-8-12-3-4-at-n}(n, l, g, h, j)$
$= \quad (\text{intersect-8-12-3-4-at-n}(n, l, g) \wedge (n \not< \text{nth}(h, j)))$

EVENT: Disable hint-8-12-3-4-at-n.

DEFINITION:
$\text{exist-hint-8-12-3-4}(n, l, g, h, j)$
$= \quad$ **if** $n \simeq \texttt{0}$ **then f**
elseif $\text{hint-8-12-3-4-at-n}(n, l, g, h, j)$ **then** n
else $\text{exist-hint-8-12-3-4}(n - 1, l, g, h, j)$ **endif**

EVENT: Disable exist-hint-8-12-3-4.

DEFINITION:
$\text{b1b}(n, l, g, h, i, j)$
$= \quad ((\text{union-at-n}(l, i, \texttt{'(8 9 10 11 12)}) \wedge \text{at}(l, j, \texttt{3}))$
$\rightarrow \quad \text{exist-hint-8-12-3-4}(n, l, g, h, j))$

EVENT: Disable b1b.

DEFINITION:
$\text{b1c}(n, l, g, h, i)$
$= \quad ((\text{union-at-n}(l, i, \texttt{'(8 9 10 11 12)})$
$\wedge \quad (\neg\, \text{union-at-n}(g, i, \texttt{'(3 4)})))$
$\rightarrow \quad ((\text{nth}(h, i) \in \text{nset}(n)) \wedge \text{at}(g, \text{nth}(h, i), \texttt{4})))$

EVENT: Disable b1c.

DEFINITION:
$\text{b1d}(n, l, h, i) = (\text{at}(l, i, \texttt{7}) \rightarrow (\text{nth}(h, i) \in \text{nset}(n)))$

EVENT: Disable b1d.

```
;;;; b2
```

DEFINITION:
b2a (l, i, j)
$= \quad (((j < i) \wedge \text{union-at-n}\,(l, i, \texttt{'(10 11 12)}))$
$\quad \to \quad (\neg\, \text{union-at-n}\,(l, j, \texttt{'(5 6 7 8 9 10 11 12)})))$

EVENT: Disable b2a.

DEFINITION:
b2b (l, h, i, j)
$= \quad (((j < i) \wedge \text{at}\,(l, i, \texttt{9}) \wedge (j < \text{nth}\,(h, i)))$
$\quad \to \quad (\neg\, \text{union-at-n}\,(l, j, \texttt{'(5 6 7 8 9 10 11 12)})))$

EVENT: Disable b2b.

```
;;;b3
```

DEFINITION:
b3a (l, g, i, j)
$= \quad ((\text{at}\,(l, i, \texttt{12}) \wedge \text{union-at-n}\,(l, j, \texttt{'(5 6 7 8 9 10 11 12)}))$
$\quad \to \quad \text{at}\,(g, j, \texttt{4}))$

EVENT: Disable b3a.

DEFINITION:
b3b (l, g, h, i, j)
$= \quad ((\text{at}\,(l, i, \texttt{11})$
$\quad \wedge \quad (j < \text{nth}\,(h, i))$
$\quad \wedge \quad \text{union-at-n}\,(l, j, \texttt{'(5 6 7 8 9 10 11 12)}))$
$\quad \to \quad \text{at}\,(g, j, \texttt{4}))$

EVENT: Disable b3b.

```
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;; molbasic.ev ;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
```

THEOREM: hint-member
exist-hint-8-12-3-4 (n, l, g, h, j)
$\to \quad (\text{exist-hint-8-12-3-4}\,(n, l, g, h, j) \in \text{nset}\,(n))$

THEOREM: n-not-less-j
$(n < j) \to (j \notin \text{nset}\,(n))$

```
;;;molws implies that n is a number.
```

THEOREM: molws-num-n
molws $(n, l, g, h) \rightarrow (n \in \mathbf{N})$

```
;;;molws implies that l is a list.
```

THEOREM: molws-list-l
molws $(n, l, g, h) \rightarrow$ listp (l)

```
;;;molws implies that g is a list.
```

THEOREM: molws-list-g
molws $(n, l, g, h) \rightarrow$ listp (g)

```
;;;molws implies that h is a list.
```

THEOREM: molws-list-h
molws $(n, l, g, h) \rightarrow$ listp (h)

```
;;;molws implies that length of l is n.
```

THEOREM: molws-ln-l
molws $(n, l, g, h) \rightarrow (\text{length}(l) = n)$

```
;;;molws implies that length of g is n.
```

THEOREM: molws-ln-g
molws $(n, l, g, h) \rightarrow (\text{length}(g) = n)$

```
;;;molws implies that length of h is n.
```

THEOREM: molws-ln-h
molws $(n, l, g, h) \rightarrow (\text{length}(h) = n)$

```
;;;molws and mrho imply that lp is a list.
```

THEOREM: molws-ln-lp
$(\text{molws}(n, l, g, h) \wedge (k \in \text{nset}(n)) \wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp))$
$\rightarrow$ listp (lp)

```
;;;molws and mrho imply that gp is a list.
```

THEOREM: molws-ln-gp
$(\text{molws}(n, l, g, h) \wedge (k \in \text{nset}(n)) \wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp))$
$\rightarrow$ listp (gp)

```
;;;molws and mrho imply that hp is a list.
```

THEOREM: molws-ln-hp
$(\text{molws}(n, l, g, h) \land (k \in \text{nset}(n)) \land \text{mrhoi}(n, k, l, g, h, lp, gp, hp))$
$\rightarrow \quad \text{listp}(hp)$

```
;;;Another version of nset-number.
;;;This is available in the theorem
;;;where molws is disabled.
```

THEOREM: molws-num-k
$(\text{molws}(n, l, g, h) \land (k \in \text{nset}(n))) \rightarrow (k \in \mathbf{N})$

THEOREM: molws-union-h
$\text{molws}(n, l, g, h) \rightarrow \text{all-union}(h, n, \text{nset}(1 + n))$

THEOREM: lm-nth-numberp
$((i \in \mathbf{N}) \land \text{all-union}(h, n, \text{nset}(i)) \land (k \in \text{nset}(n))) \rightarrow (\text{nth}(h, k) \in \mathbf{N})$

THEOREM: nth-numberp
$(\text{molws}(n, l, g, h) \land (k \in \text{nset}(n))) \rightarrow (\text{nth}(h, k) \in \mathbf{N})$

```
;;;molws implies that n is nonzero.
```

THEOREM: molws-n-not-0
$\text{molws}(n, l, g, h) \rightarrow (n \neq \texttt{0})$

```
;;;Auxiliary lemma.
```

THEOREM: lm-l-mrholemma
$(\text{listp}(l)$
$\land \quad (j \in \text{nset}(\text{length}(l)))$
$\land \quad (k \in \text{nset}(\text{length}(l)))$
$\land \quad \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\land \quad (k \neq j))$
$\rightarrow \quad (\text{nth}(l, j) = \text{nth}(lp, j))$

EVENT: Disable lm-l-mrholemma.

```
;;;Mrholemma for list l.
```

THEOREM: l-mrholemma
$(\text{molws}(n, l, g, h)$
$\land \quad (j \in \text{nset}(n))$
$\land \quad (k \in \text{nset}(n))$
$\land \quad \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\land \quad (k \neq j))$
$\rightarrow \quad (\text{nth}(l, j) = \text{nth}(lp, j))$

```
;;;Auxiliary lemma.
```

THEOREM: lm-g-mrholemma
(listp (g)
 $\wedge$ $(j \in \text{nset}(\text{length}(g)))$
 $\wedge$ $(k \in \text{nset}(\text{length}(g)))$
 $\wedge$ $\text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge$ $(k \neq j))$
$\rightarrow$ $(\text{nth}(g, j) = \text{nth}(gp, j))$

EVENT: Disable lm-g-mrholemma.

```
;;;Mrholemma for list g.
```

THEOREM: g-mrholemma
(molws (n, l, g, h)
 $\wedge$ $(j \in \text{nset}(n))$
 $\wedge$ $(k \in \text{nset}(n))$
 $\wedge$ $\text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge$ $(k \neq j))$
$\rightarrow$ $(\text{nth}(g, j) = \text{nth}(gp, j))$

```
;;;Auxiliary lemma.
```

THEOREM: lm-h-mrholemma
(listp (h)
 $\wedge$ $(j \in \text{nset}(\text{length}(h)))$
 $\wedge$ $(k \in \text{nset}(\text{length}(h)))$
 $\wedge$ $\text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge$ $(k \neq j))$
$\rightarrow$ $(\text{nth}(h, j) = \text{nth}(hp, j))$

EVENT: Disable lm-h-mrholemma.

```
;;;Mrholemma for list g.
```

THEOREM: h-mrholemma
(molws (n, l, g, h)
 $\wedge$ $(j \in \text{nset}(n))$
 $\wedge$ $(k \in \text{nset}(n))$
 $\wedge$ $\text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge$ $(k \neq j))$
$\rightarrow$ $(\text{nth}(h, j) = \text{nth}(hp, j))$

```
;;; lp-gp-same-l-g

;;;Another version of Rholemma for l.
;;;It applies to (union-at-n l j m) in stead of
;;;(nth l j).
```

THEOREM: m-lp-same-l
(molws $(n,\ l,\ g,\ h)$
 ∧ listp (m)
 ∧ $(j \in$ nset $(n))$
 ∧ $(k \in$ nset $(n))$
 ∧ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 ∧ $(j \neq k)$
 ∧ union-at-n $(l,\ j,\ m))$
→ union-at-n $(lp,\ j,\ m)$

```
;;;Contrast to the one above,
;;;the order of l and lp is reversed.
```

THEOREM: m-l-same-lp
(molws $(n,\ l,\ g,\ h)$
 ∧ listp (m)
 ∧ $(j \in$ nset $(n))$
 ∧ $(k \in$ nset $(n))$
 ∧ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 ∧ $(j \neq k)$
 ∧ union-at-n $(lp,\ j,\ m))$
→ union-at-n $(l,\ j,\ m)$

THEOREM: m-lp-same-l-not
(molws $(n,\ l,\ g,\ h)$
 ∧ listp (m)
 ∧ $(j \in$ nset $(n))$
 ∧ $(k \in$ nset $(n))$
 ∧ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 ∧ $(j \neq k)$
 ∧ (¬ union-at-n $(lp,\ j,\ m)))$
→ (¬ union-at-n $(l,\ j,\ m))$

```
;;;Another version of Rholemma for g.
```

THEOREM: m-gp-same-g
(molws $(n,\ l,\ g,\ h)$
 ∧ listp (m)
 ∧ $(j \in$ nset $(n))$

$\land \quad (k \in \text{nset}(n))$
$\land \quad \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\land \quad (j \neq k)$
$\land \quad \text{union-at-n}(g, j, m))$
$\rightarrow \quad \text{union-at-n}(gp, j, m)$

```
;;;Contrast to the one above,
;;;the order of g and gp is reversed.
```

THEOREM: m-g-same-gp
$(\text{molws}(n, l, g, h)$
$\land \quad \text{listp}(m)$
$\land \quad (j \in \text{nset}(n))$
$\land \quad (k \in \text{nset}(n))$
$\land \quad \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\land \quad (j \neq k)$
$\land \quad \text{union-at-n}(gp, j, m))$
$\rightarrow \quad \text{union-at-n}(g, j, m)$

THEOREM: m-gp-same-g-not
$(\text{molws}(n, l, g, h)$
$\land \quad \text{listp}(m)$
$\land \quad (j \in \text{nset}(n))$
$\land \quad (k \in \text{nset}(n))$
$\land \quad \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\land \quad (j \neq k)$
$\land \quad (\neg \text{union-at-n}(gp, j, m)))$
$\rightarrow \quad (\neg \text{union-at-n}(g, j, m))$

```
;;;Another version of Rholemma for h.
```

THEOREM: m-hp-same-h
$(\text{molws}(n, l, g, h)$
$\land \quad \text{listp}(m)$
$\land \quad (j \in \text{nset}(n))$
$\land \quad (k \in \text{nset}(n))$
$\land \quad \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\land \quad (j \neq k)$
$\land \quad \text{union-at-n}(h, j, m))$
$\rightarrow \quad \text{union-at-n}(hp, j, m)$

```
;;;Contrast to the one above,
;;;the order of g and gp is reversed.
```

THEOREM: m-h-same-hp

(molws $(n,\ l,\ g,\ h)$
$\land$ listp (m)
$\land$ $(j \in \text{nset}(n))$
$\land$ $(k \in \text{nset}(n))$
$\land$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\land$ $(j \neq k)$
$\land$ union-at-n $(hp,\ j,\ m))$
$\rightarrow$ union-at-n $(h,\ j,\ m)$

```
;;;It applies to (at l j m) in stead of
;;;(nth l j).
```

THEOREM: m-l-same-lp-at
(molws $(n,\ l,\ g,\ h)$
$\land$ $(j \in \text{nset}(n))$
$\land$ $(k \in \text{nset}(n))$
$\land$ $(m \in \mathbf{N})$
$\land$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\land$ $(k \neq j)$
$\land$ at $(lp,\ j,\ m))$
$\rightarrow$ at $(l,\ j,\ m)$

THEOREM: m-gp-same-g-at
(molws $(n,\ l,\ g,\ h)$
$\land$ $(j \in \text{nset}(n))$
$\land$ $(k \in \text{nset}(n))$
$\land$ $(m \in \mathbf{N})$
$\land$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\land$ $(k \neq j)$
$\land$ at $(g,\ j,\ m))$
$\rightarrow$ at $(gp,\ j,\ m)$

THEOREM: m-l-same-lp-at-not
(molws $(n,\ l,\ g,\ h)$
$\land$ $(m \in \mathbf{N})$
$\land$ $(j \in \text{nset}(n))$
$\land$ $(k \in \text{nset}(n))$
$\land$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\land$ $(j \neq k)$
$\land$ $(\neg$ at $(l,\ j,\ m)))$
$\rightarrow$ $(\neg$ at $(lp,\ j,\ m))$

```
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;; mollg.ev ;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;mrhoi0
```

THEOREM: n-neq-k-mrhoi0
(listp (l)
$\land$ listp (g)
$\land$ $(n \in \mathbf{N})$
$\land$ $(k \in$ nset (length $(l)))$
$\land$ $(k \neq n)$
$\land$ at $(l,\ k,\ \mathtt{0})$
$\land$ lg-at-n $(n,\ l,\ g))$
$\rightarrow$ lg-at-n $(n,$ move $(l,\ k,\ \mathtt{1}),\ g)$

EVENT: Disable n-neq-k-mrhoi0.

THEOREM: n-eq-k-mrhoi0
(listp (l)
$\land$ listp (g)
$\land$ $(k \in$ nset (length $(l)))$
$\land$ at $(l,\ k,\ \mathtt{0})$
$\land$ lg-at-n $(k,\ l,\ g))$
$\rightarrow$ lg-at-n $(k,$ move $(l,\ k,\ \mathtt{1}),\ g)$

EVENT: Disable n-eq-k-mrhoi0.

THEOREM: lg-at-mrhoi0
(listp (l)
$\land$ listp (g)
$\land$ $(n \in \mathbf{N})$
$\land$ $(k \in$ nset (length $(l)))$
$\land$ at $(l,\ k,\ \mathtt{0})$
$\land$ lg-at-n $(n,\ l,\ g))$
$\rightarrow$ lg-at-n $(n,$ move $(l,\ k,\ \mathtt{1}),\ g)$

EVENT: Disable lg-at-mrhoi0.

THEOREM: lg-mrhoi0
(listp (l)
$\land$ listp (g)
$\land$ $(k \in$ nset (length $(l)))$
$\land$ $(n \in \mathbf{N})$
$\land$ at $(l,\ k,\ \mathtt{0})$
$\land$ lg $(n,\ l,\ g))$
$\rightarrow$ lg $(n,$ move $(l,\ k,\ \mathtt{1}),\ g)$

EVENT: Disable lg-mrhoi0.

THEOREM: mrhoi0-preserves-lg
(molws $(n,\ l,\ g,\ h)$
 $\land$ $(k \in$ nset $(n))$
 $\land$ mrhoi0 $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 $\land$ lg $(n,\ l,\ g))$
 $\rightarrow$ lg $(n,\ lp,\ gp)$

```
;;;mrhoi1a
```

THEOREM: n-neq-k-mrhoi1a
(listp (l)
 $\land$ listp (g)
 $\land$ $(n \in \mathbf{N})$
 $\land$ $(k \in$ nset (length $(l)))$
 $\land$ $(k \neq n)$
 $\land$ at $(l,\ k,\ 1)$
 $\land$ lg-at-n $(n,\ l,\ g))$
 $\rightarrow$ lg-at-n $(n,$ move $(l,\ k,\ 2),\ g)$

EVENT: Disable n-neq-k-mrhoi1a.

THEOREM: n-eq-k-mrhoi1a
(listp (l)
 $\land$ listp (g)
 $\land$ $(k \in$ nset (length $(l)))$
 $\land$ at $(l,\ k,\ 1)$
 $\land$ lg-at-n $(k,\ l,\ g))$
 $\rightarrow$ lg-at-n $(n,$ move $(l,\ k,\ 2),\ g)$

EVENT: Disable n-eq-k-mrhoi1a.

THEOREM: lg-at-mrhoi1a
(listp (l)
 $\land$ listp (g)
 $\land$ $(n \in \mathbf{N})$
 $\land$ $(k \in$ nset (length $(l)))$
 $\land$ at $(l,\ k,\ 1)$
 $\land$ lg-at-n $(n,\ l,\ g))$
 $\rightarrow$ lg-at-n $(n,$ move $(l,\ k,\ 2),\ g)$

EVENT: Disable lg-at-mrhoi1a.

THEOREM: lg-mrhoi1a

(listp (l)
 ∧ listp (g)
 ∧ ($k \in$ nset (length (l)))
 ∧ ($n \in \mathbf{N}$)
 ∧ at (l, k, `1`)
 ∧ lg (n, l, g))
→ lg (n, move (l, k, `2`), g)

Event: Disable lg-mrhoi1a.

Theorem: mrhoi1a-preserves-lg
(molws (n, l, g, h)
 ∧ ($k \in$ nset (n))
 ∧ mrhoi1a (n, k, l, g, h, lp, gp, hp)
 ∧ lg (n, l, g))
→ lg (n, lp, gp)

`;;;mrhoi1b`

Theorem: mrhoi1b-preserves-lg
(molws (n, l, g, h)
 ∧ ($k \in$ nset (n))
 ∧ mrhoi1b (n, k, l, g, h, lp, gp, hp)
 ∧ lg (n, l, g))
→ lg (n, lp, gp)

`;;;mrhoi2`

Theorem: n-neq-k-mrhoi2
(listp (l)
 ∧ listp (g)
 ∧ ($n \in \mathbf{N}$)
 ∧ ($k \in$ nset (length (l)))
 ∧ ($k \neq n$)
 ∧ at (l, k, `2`)
 ∧ lg-at-n (n, l, g))
→ lg-at-n (n, move (l, k, `3`), move (g, k, `1`))

Event: Disable n-neq-k-mrhoi2.

Theorem: n-eq-k-mrhoi2
(listp (l)
 ∧ listp (g)
 ∧ ($k \in$ nset (length (l)))

$\wedge$ at $(l, k, 2)$
$\wedge$ lg-at-n $(k, l, g))$
$\rightarrow$ lg-at-n $(n, \text{move}(l, k, 3), \text{move}(g, k, 1))$

Event: Disable n-eq-k-mrhoi2.

Theorem: lg-at-mrhoi2
(listp (l)
$\wedge$ listp (g)
$\wedge$ $(n \in \mathbf{N})$
$\wedge$ $(k \in \text{nset}(\text{length}(l)))$
$\wedge$ at $(l, k, 2)$
$\wedge$ lg-at-n $(n, l, g))$
$\rightarrow$ lg-at-n $(n, \text{move}(l, k, 3), \text{move}(g, k, 1))$

Event: Disable lg-at-mrhoi2.

Theorem: lg-mrhoi2
(listp (l)
$\wedge$ listp (g)
$\wedge$ $(k \in \text{nset}(\text{length}(l)))$
$\wedge$ $(n \in \mathbf{N})$
$\wedge$ at $(l, k, 2)$
$\wedge$ lg $(n, l, g))$
$\rightarrow$ lg $(n, \text{move}(l, k, 3), \text{move}(g, k, 1))$

Event: Disable lg-mrhoi2.

Theorem: mrhoi2-preserves-lg
(molws (n, l, g, h)
$\wedge$ $(k \in \text{nset}(n))$
$\wedge$ mrhoi2 $(n, k, l, g, h, lp, gp, hp)$
$\wedge$ lg $(n, l, g))$
$\rightarrow$ lg (n, lp, gp)

```
;;;mrhoi3a
```

Theorem: n-neq-k-mrhoi3a
(listp (l)
$\wedge$ listp (g)
$\wedge$ $(n \in \mathbf{N})$
$\wedge$ $(k \in \text{nset}(\text{length}(l)))$
$\wedge$ $(k \neq n)$

$\wedge$ at $(l,\ k,\ \mathtt{3})$
$\wedge$ lg-at-n $(n,\ l,\ g))$
$\rightarrow$ lg-at-n $(n,$ move $(l,\ k,\ \mathtt{4}),\ g)$

EVENT: Disable n-neq-k-mrhoi3a.

THEOREM: n-eq-k-mrhoi3a
(listp (l)
$\wedge$ listp (g)
$\wedge$ $(k \in$ nset (length $(l)))$
$\wedge$ at $(l,\ k,\ \mathtt{3})$
$\wedge$ lg-at-n $(k,\ l,\ g))$
$\rightarrow$ lg-at-n $(k,$ move $(l,\ k,\ \mathtt{4}),\ g)$

EVENT: Disable n-eq-k-mrhoi3a.

THEOREM: lg-at-mrhoi3a
(listp (l)
$\wedge$ listp (g)
$\wedge$ $(n \in \mathbf{N})$
$\wedge$ $(k \in$ nset (length $(l)))$
$\wedge$ at $(l,\ k,\ \mathtt{3})$
$\wedge$ lg-at-n $(n,\ l,\ g))$
$\rightarrow$ lg-at-n $(n,$ move $(l,\ k,\ \mathtt{4}),\ g)$

EVENT: Disable lg-at-mrhoi3a.

THEOREM: lg-mrhoi3a
(listp (l)
$\wedge$ listp (g)
$\wedge$ $(k \in$ nset (length $(l)))$
$\wedge$ $(n \in \mathbf{N})$
$\wedge$ at $(l,\ k,\ \mathtt{3})$
$\wedge$ lg $(n,\ l,\ g))$
$\rightarrow$ lg $(n,$ move $(l,\ k,\ \mathtt{4}),\ g)$

EVENT: Disable lg-mrhoi3a.

THEOREM: mrhoi3a-preserves-lg
(molws $(n,\ l,\ g,\ h)$
$\wedge$ $(k \in$ nset $(n))$
$\wedge$ mrhoi3a $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ lg $(n,\ l,\ g))$
$\rightarrow$ lg $(n,\ lp,\ gp)$

`;;;mrhoi3b`

Theorem: mrhoi3b-preserves-lg
$(\text{molws}(n, l, g, h)$
$\land\ (k \in \text{nset}(n))$
$\land\ \text{mrhoi3b}(n, k, l, g, h, lp, gp, hp)$
$\land\ \text{lg}(n, l, g))$
$\rightarrow\ \text{lg}(n, lp, gp)$

`;;;mrhoi4`

Theorem: n-neq-k-mrhoi4
$(\text{listp}(l)$
$\land\ \text{listp}(g)$
$\land\ (n \in \mathbf{N})$
$\land\ (k \in \text{nset}(\text{length}(l)))$
$\land\ (k \neq n)$
$\land\ \text{at}(l, k, \texttt{4})$
$\land\ \text{lg-at-n}(n, l, g))$
$\rightarrow\ \text{lg-at-n}(n, \text{move}(l, k, \texttt{5}), \text{move}(g, k, \texttt{3}))$

Event: Disable n-neq-k-mrhoi4.

Theorem: n-eq-k-mrhoi4
$(\text{listp}(l)$
$\land\ \text{listp}(g)$
$\land\ (k \in \text{nset}(\text{length}(l)))$
$\land\ \text{at}(l, k, \texttt{4})$
$\land\ \text{lg-at-n}(k, l, g))$
$\rightarrow\ \text{lg-at-n}(n, \text{move}(l, k, \texttt{5}), \text{move}(g, k, \texttt{3}))$

Event: Disable n-eq-k-mrhoi4.

Theorem: lg-at-mrhoi4
$(\text{listp}(l)$
$\land\ \text{listp}(g)$
$\land\ (n \in \mathbf{N})$
$\land\ (k \in \text{nset}(\text{length}(l)))$
$\land\ \text{at}(l, k, \texttt{4})$
$\land\ \text{lg-at-n}(n, l, g))$
$\rightarrow\ \text{lg-at-n}(n, \text{move}(l, k, \texttt{5}), \text{move}(g, k, \texttt{3}))$

Event: Disable lg-at-mrhoi4.

THEOREM: lg-mrhoi4
(listp (l)
$\wedge$ listp (g)
$\wedge$ $(k \in \text{nset}(\text{length}(l)))$
$\wedge$ $(n \in \mathbf{N})$
$\wedge$ at $(l, k, \texttt{4})$
$\wedge$ lg $(n, l, g))$
$\rightarrow$ lg $(n, \text{move}(l, k, \texttt{5}), \text{move}(g, k, \texttt{3}))$

EVENT: Disable lg-mrhoi4.

THEOREM: mrhoi4-preserves-lg
(molws (n, l, g, h)
$\wedge$ $(k \in \text{nset}(n))$
$\wedge$ mrhoi4 $(n, k, l, g, h, lp, gp, hp)$
$\wedge$ lg $(n, l, g))$
$\rightarrow$ lg (n, lp, gp)

`;;;mrhoi5a`

THEOREM: n-neq-k-mrhoi5a
(listp (l)
$\wedge$ listp (g)
$\wedge$ $(n \in \mathbf{N})$
$\wedge$ $(k \in \text{nset}(\text{length}(l)))$
$\wedge$ $(k \neq n)$
$\wedge$ at $(l, k, \texttt{5})$
$\wedge$ lg-at-n $(n, l, g))$
$\rightarrow$ lg-at-n $(n, \text{move}(l, k, \texttt{8}), g)$

EVENT: Disable n-neq-k-mrhoi5a.

THEOREM: n-eq-k-mrhoi5a
(listp (l)
$\wedge$ listp (g)
$\wedge$ $(k \in \text{nset}(\text{length}(l)))$
$\wedge$ at $(l, k, \texttt{5})$
$\wedge$ lg-at-n $(k, l, g))$
$\rightarrow$ lg-at-n $(k, \text{move}(l, k, \texttt{8}), g)$

EVENT: Disable n-eq-k-mrhoi5a.

THEOREM: lg-at-mrhoi5a

$(\text{listp}(l)$
$\land \quad \text{listp}(g)$
$\land \quad (n \in \mathbf{N})$
$\land \quad (k \in \text{nset}(\text{length}(l)))$
$\land \quad \text{at}(l, k, \texttt{5})$
$\land \quad \text{lg-at-n}(n, l, g))$
$\rightarrow \quad \text{lg-at-n}(n, \text{move}(l, k, \texttt{8}), g)$

EVENT: Disable lg-at-mrhoi5a.

THEOREM: lg-mrhoi5a
$(\text{listp}(l)$
$\land \quad \text{listp}(g)$
$\land \quad (k \in \text{nset}(\text{length}(l)))$
$\land \quad (n \in \mathbf{N})$
$\land \quad \text{at}(l, k, \texttt{5})$
$\land \quad \text{lg}(n, l, g))$
$\rightarrow \quad \text{lg}(n, \text{move}(l, k, \texttt{8}), g)$

EVENT: Disable lg-mrhoi5a.

THEOREM: mrhoi5a-preserves-lg
$(\text{molws}(n, l, g, h)$
$\land \quad (k \in \text{nset}(n))$
$\land \quad \text{mrhoi5a}(n, k, l, g, h, lp, gp, hp)$
$\land \quad \text{lg}(n, l, g))$
$\rightarrow \quad \text{lg}(n, lp, gp)$

```
;;;mrhoi5b
```

THEOREM: n-neq-k-mrhoi5b
$(\text{listp}(l)$
$\land \quad \text{listp}(g)$
$\land \quad (n \in \mathbf{N})$
$\land \quad (k \in \text{nset}(\text{length}(l)))$
$\land \quad (k \neq n)$
$\land \quad \text{at}(l, k, \texttt{5})$
$\land \quad \text{lg-at-n}(n, l, g))$
$\rightarrow \quad \text{lg-at-n}(n, \text{move}(l, k, \texttt{6}), g)$

EVENT: Disable n-neq-k-mrhoi5b.

THEOREM: n-eq-k-mrhoi5b

(listp (l)
 ∧ listp (g)
 ∧ ($k \in$ nset (length (l)))
 ∧ at (l, k, 5)
 ∧ lg-at-n (k, l, g))
→ lg-at-n (k, move (l, k, 6), g)

EVENT: Disable n-eq-k-mrhoi5b.

THEOREM: lg-at-mrhoi5b
(listp (l)
 ∧ listp (g)
 ∧ ($n \in \mathbf{N}$)
 ∧ ($k \in$ nset (length (l)))
 ∧ at (l, k, 5)
 ∧ lg-at-n (n, l, g))
→ lg-at-n (n, move (l, k, 6), g)

EVENT: Disable lg-at-mrhoi5b.

THEOREM: lg-mrhoi5b
(listp (l)
 ∧ listp (g)
 ∧ ($k \in$ nset (length (l)))
 ∧ ($n \in \mathbf{N}$)
 ∧ at (l, k, 5)
 ∧ lg (n, l, g))
→ lg (n, move (l, k, 6), g)

EVENT: Disable lg-mrhoi5b.

THEOREM: mrhoi5b-preserves-lg
(molws (n, l, g, h)
 ∧ ($k \in$ nset (n))
 ∧ mrhoi5b (n, k, l, g, h, lp, gp, hp)
 ∧ lg (n, l, g))
→ lg (n, lp, gp)

`;;;mrhoi5c`

THEOREM: mrhoi5c-preserves-lg
(molws (n, l, g, h)
 ∧ ($k \in$ nset (n))

$\wedge$ mrhoi5c $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ lg $(n,\ l,\ g))$
$\rightarrow$ lg $(n,\ lp,\ gp)$

```
;;;mrhoi6
```

THEOREM: n-neq-k-mrhoi6
(listp (l)
$\wedge$ listp (g)
$\wedge$ $(n \in \mathbf{N})$
$\wedge$ $(k \in$ nset (length $(l)))$
$\wedge$ $(k \neq n)$
$\wedge$ at $(l,\ k,\ 6)$
$\wedge$ lg-at-n $(n,\ l,\ g))$
$\rightarrow$ lg-at-n $(n,$ move $(l,\ k,\ 7),$ move $(g,\ k,\ 2))$

EVENT: Disable n-neq-k-mrhoi6.

THEOREM: n-eq-k-mrhoi6
(listp (l)
$\wedge$ listp (g)
$\wedge$ $(k \in$ nset (length $(l)))$
$\wedge$ at $(l,\ k,\ 6)$
$\wedge$ lg-at-n $(k,\ l,\ g))$
$\rightarrow$ lg-at-n $(n,$ move $(l,\ k,\ 7),$ move $(g,\ k,\ 2))$

EVENT: Disable n-eq-k-mrhoi6.

THEOREM: lg-at-mrhoi6
(listp (l)
$\wedge$ listp (g)
$\wedge$ $(n \in \mathbf{N})$
$\wedge$ $(k \in$ nset (length $(l)))$
$\wedge$ at $(l,\ k,\ 6)$
$\wedge$ lg-at-n $(n,\ l,\ g))$
$\rightarrow$ lg-at-n $(n,$ move $(l,\ k,\ 7),$ move $(g,\ k,\ 2))$

EVENT: Disable lg-at-mrhoi6.

THEOREM: lg-mrhoi6
(listp (l)
$\wedge$ listp (g)
$\wedge$ $(k \in$ nset (length $(l)))$

$\wedge \quad (n \in \mathbf{N})$
$\wedge \quad \mathrm{at}\,(l,\ k,\ \mathtt{6})$
$\wedge \quad \mathrm{lg}\,(n,\ l,\ g))$
$\rightarrow \quad \mathrm{lg}\,(n,\ \mathrm{move}\,(l,\ k,\ \mathtt{7}),\ \mathrm{move}\,(g,\ k,\ \mathtt{2}))$

EVENT: Disable lg-mrhoi6.

THEOREM: mrhoi6-preserves-lg
$(\mathrm{molws}\,(n,\ l,\ g,\ h)$
$\wedge \quad (k \in \mathrm{nset}\,(n))$
$\wedge \quad \mathrm{mrhoi6}\,(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge \quad \mathrm{lg}\,(n,\ l,\ g))$
$\rightarrow \quad \mathrm{lg}\,(n,\ lp,\ gp)$

```
;;;mrhoi7a
```

THEOREM: n-neq-k-mrhoi7a
$(\mathrm{listp}\,(l)$
$\wedge \quad \mathrm{listp}\,(g)$
$\wedge \quad (n \in \mathbf{N})$
$\wedge \quad (k \in \mathrm{nset}\,(\mathrm{length}\,(l)))$
$\wedge \quad (k \neq n)$
$\wedge \quad \mathrm{at}\,(l,\ k,\ \mathtt{7})$
$\wedge \quad \mathrm{lg\text{-}at\text{-}n}\,(n,\ l,\ g))$
$\rightarrow \quad \mathrm{lg\text{-}at\text{-}n}\,(n,\ \mathrm{move}\,(l,\ k,\ \mathtt{8}),\ g)$

EVENT: Disable n-neq-k-mrhoi7a.

THEOREM: n-eq-k-mrhoi7a
$(\mathrm{listp}\,(l)$
$\wedge \quad \mathrm{listp}\,(g)$
$\wedge \quad (k \in \mathrm{nset}\,(\mathrm{length}\,(l)))$
$\wedge \quad \mathrm{at}\,(l,\ k,\ \mathtt{7})$
$\wedge \quad \mathrm{lg\text{-}at\text{-}n}\,(k,\ l,\ g))$
$\rightarrow \quad \mathrm{lg\text{-}at\text{-}n}\,(k,\ \mathrm{move}\,(l,\ k,\ \mathtt{8}),\ g)$

EVENT: Disable n-eq-k-mrhoi7a.

THEOREM: lg-at-mrhoi7a
$(\mathrm{listp}\,(l)$
$\wedge \quad \mathrm{listp}\,(g)$
$\wedge \quad (n \in \mathbf{N})$
$\wedge \quad (k \in \mathrm{nset}\,(\mathrm{length}\,(l)))$

$\wedge$ at $(l, k, 7)$
$\wedge$ lg-at-n $(n, l, g))$
$\rightarrow$ lg-at-n $(n, \text{move}(l, k, 8), g)$

EVENT: Disable lg-at-mrhoi7a.

THEOREM: lg-mrhoi7a
(listp (l)
$\wedge$ listp (g)
$\wedge$ $(k \in \text{nset}(\text{length}(l)))$
$\wedge$ $(n \in \mathbf{N})$
$\wedge$ at $(l, k, 7)$
$\wedge$ lg $(n, l, g))$
$\rightarrow$ lg $(n, \text{move}(l, k, 8), g)$

EVENT: Disable lg-mrhoi7a.

THEOREM: mrhoi7a-preserves-lg
(molws (n, l, g, h)
$\wedge$ $(k \in \text{nset}(n))$
$\wedge$ mrhoi7a $(n, k, l, g, h, lp, gp, hp)$
$\wedge$ lg $(n, l, g))$
$\rightarrow$ lg (n, lp, gp)

```
;;;mrhoi7b
```

THEOREM: mrhoi7b-preserves-lg
(molws (n, l, g, h)
$\wedge$ $(k \in \text{nset}(n))$
$\wedge$ mrhoi7b $(n, k, l, g, h, lp, gp, hp)$
$\wedge$ lg $(n, l, g))$
$\rightarrow$ lg (n, lp, gp)

```
;;;mrhoi8
```

THEOREM: n-neq-k-mrhoi8
(listp (l)
$\wedge$ listp (g)
$\wedge$ $(n \in \mathbf{N})$
$\wedge$ $(k \in \text{nset}(\text{length}(l)))$
$\wedge$ $(k \neq n)$
$\wedge$ at $(l, k, 8)$
$\wedge$ lg-at-n $(n, l, g))$
$\rightarrow$ lg-at-n $(n, \text{move}(l, k, 9), \text{move}(g, k, 4))$

EVENT: Disable n-neq-k-mrhoi8.

THEOREM: n-eq-k-mrhoi8
(listp (l)
 $\wedge$ listp (g)
 $\wedge$ $(k \in$ nset (length $(l)))$
 $\wedge$ at $(l,\ k,\ \texttt{8})$
 $\wedge$ lg-at-n $(k,\ l,\ g))$
$\rightarrow$ lg-at-n $(n,$ move $(l,\ k,\ \texttt{9}),$ move $(g,\ k,\ \texttt{4}))$

EVENT: Disable n-eq-k-mrhoi8.

THEOREM: lg-at-mrhoi8
(listp (l)
 $\wedge$ listp (g)
 $\wedge$ $(n \in \mathbf{N})$
 $\wedge$ $(k \in$ nset (length $(l)))$
 $\wedge$ at $(l,\ k,\ \texttt{8})$
 $\wedge$ lg-at-n $(n,\ l,\ g))$
$\rightarrow$ lg-at-n $(n,$ move $(l,\ k,\ \texttt{9}),$ move $(g,\ k,\ \texttt{4}))$

EVENT: Disable lg-at-mrhoi8.

THEOREM: lg-mrhoi8
(listp (l)
 $\wedge$ listp (g)
 $\wedge$ $(k \in$ nset (length $(l)))$
 $\wedge$ $(n \in \mathbf{N})$
 $\wedge$ at $(l,\ k,\ \texttt{8})$
 $\wedge$ lg $(n,\ l,\ g))$
$\rightarrow$ lg $(n,$ move $(l,\ k,\ \texttt{9}),$ move $(g,\ k,\ \texttt{4}))$

EVENT: Disable lg-mrhoi8.

THEOREM: mrhoi8-preserves-lg
(molws $(n,\ l,\ g,\ h)$
 $\wedge$ $(k \in$ nset $(n))$
 $\wedge$ mrhoi8 $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 $\wedge$ lg $(n,\ l,\ g))$
$\rightarrow$ lg $(n,\ lp,\ gp)$

`;;;mrhoi9a`

THEOREM: n-neq-k-mrhoi9a
(listp (l)
 $\land$ listp (g)
 $\land$ $(n \in \mathbf{N})$
 $\land$ $(k \in$ nset (length (l)))
 $\land$ $(k \neq n)$
 $\land$ at $(l,\ k,\ \mathtt{9})$
 $\land$ lg-at-n $(n,\ l,\ g)$)
$\to$ lg-at-n $(n,$ move $(l,\ k,\ \mathtt{10}),\ g)$

EVENT: Disable n-neq-k-mrhoi9a.

THEOREM: n-eq-k-mrhoi9a
(listp (l)
 $\land$ listp (g)
 $\land$ $(k \in$ nset (length (l)))
 $\land$ at $(l,\ k,\ \mathtt{9})$
 $\land$ lg-at-n $(k,\ l,\ g)$)
$\to$ lg-at-n $(k,$ move $(l,\ k,\ \mathtt{10}),\ g)$

EVENT: Disable n-eq-k-mrhoi9a.

THEOREM: lg-at-mrhoi9a
(listp (l)
 $\land$ listp (g)
 $\land$ $(n \in \mathbf{N})$
 $\land$ $(k \in$ nset (length (l)))
 $\land$ at $(l,\ k,\ \mathtt{9})$
 $\land$ lg-at-n $(n,\ l,\ g)$)
$\to$ lg-at-n $(n,$ move $(l,\ k,\ \mathtt{10}),\ g)$

EVENT: Disable lg-at-mrhoi9a.

THEOREM: lg-mrhoi9a
(listp (l)
 $\land$ listp (g)
 $\land$ $(k \in$ nset (length (l)))
 $\land$ $(n \in \mathbf{N})$
 $\land$ at $(l,\ k,\ \mathtt{9})$
 $\land$ lg $(n,\ l,\ g)$)
$\to$ lg $(n,$ move $(l,\ k,\ \mathtt{10}),\ g)$

EVENT: Disable lg-mrhoi9a.

THEOREM: mrhoi9a-preserves-lg
(molws $(n,\ l,\ g,\ h)$
 $\land$ $(k \in$ nset $(n))$
 $\land$ mrhoi9a $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 $\land$ lg $(n,\ l,\ g))$
$\rightarrow$ lg $(n,\ lp,\ gp)$

`;;;mrhoi9b`

THEOREM: mrhoi9b-preserves-lg
(molws $(n,\ l,\ g,\ h)$
 $\land$ $(k \in$ nset $(n))$
 $\land$ mrhoi9b $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 $\land$ lg $(n,\ l,\ g))$
$\rightarrow$ lg $(n,\ lp,\ gp)$

`;;;mrhoi10`

THEOREM: n-neq-k-mrhoi10
(listp (l)
 $\land$ listp (g)
 $\land$ $(n \in \mathbf{N})$
 $\land$ $(k \in$ nset (length $(l)))$
 $\land$ $(k \neq n)$
 $\land$ at $(l,\ k,\ \texttt{10})$
 $\land$ lg-at-n $(n,\ l,\ g))$
$\rightarrow$ lg-at-n $(n,$ move $(l,\ k,\ \texttt{11}),\ g)$

EVENT: Disable n-neq-k-mrhoi10.

THEOREM: n-eq-k-mrhoi10
(listp (l)
 $\land$ listp (g)
 $\land$ $(k \in$ nset (length $(l)))$
 $\land$ at $(l,\ k,\ \texttt{10})$
 $\land$ lg-at-n $(k,\ l,\ g))$
$\rightarrow$ lg-at-n $(k,$ move $(l,\ k,\ \texttt{11}),\ g)$

EVENT: Disable n-eq-k-mrhoi10.

THEOREM: lg-at-mrhoi10
(listp (l)
 $\land$ listp (g)
 $\land$ $(n \in \mathbf{N})$

$\wedge$ $(k \in \text{nset}(\text{length}(l)))$
$\wedge$ $\text{at}(l,\ k,\ \texttt{10})$
$\wedge$ $\text{lg-at-n}(n,\ l,\ g))$
$\rightarrow$ $\text{lg-at-n}(n,\ \text{move}(l,\ k,\ \texttt{11}),\ g)$

EVENT: Disable lg-at-mrhoi10.

THEOREM: lg-mrhoi10
$(\text{listp}(l)$
$\wedge$ $\text{listp}(g)$
$\wedge$ $(k \in \text{nset}(\text{length}(l)))$
$\wedge$ $(n \in \mathbf{N})$
$\wedge$ $\text{at}(l,\ k,\ \texttt{10})$
$\wedge$ $\text{lg}(n,\ l,\ g))$
$\rightarrow$ $\text{lg}(n,\ \text{move}(l,\ k,\ \texttt{11}),\ g)$

EVENT: Disable lg-mrhoi10.

THEOREM: mrhoi10-preserves-lg
$(\text{molws}(n,\ l,\ g,\ h)$
$\wedge$ $(k \in \text{nset}(n))$
$\wedge$ $\text{mrhoi10}(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ $\text{lg}(n,\ l,\ g))$
$\rightarrow$ $\text{lg}(n,\ lp,\ gp)$

```
;;;mrhoi11a
```

THEOREM: n-neq-k-mrhoi11a
$(\text{listp}(l)$
$\wedge$ $\text{listp}(g)$
$\wedge$ $(n \in \mathbf{N})$
$\wedge$ $(k \in \text{nset}(\text{length}(l)))$
$\wedge$ $(k \neq n)$
$\wedge$ $\text{at}(l,\ k,\ \texttt{11})$
$\wedge$ $\text{lg-at-n}(n,\ l,\ g))$
$\rightarrow$ $\text{lg-at-n}(n,\ \text{move}(l,\ k,\ \texttt{12}),\ g)$

EVENT: Disable n-neq-k-mrhoi11a.

THEOREM: n-eq-k-mrhoi11a
$(\text{listp}(l)$
$\wedge$ $\text{listp}(g)$
$\wedge$ $(k \in \text{nset}(\text{length}(l)))$

$\wedge$ at (l, k, 11)
$\wedge$ lg-at-n (k, l, g))
$\rightarrow$ lg-at-n (k, move (l, k, 12), g)

EVENT: Disable n-eq-k-mrhoi11a.

THEOREM: lg-at-mrhoi11a
(listp (l)
$\wedge$ listp (g)
$\wedge$ ($n \in \mathbf{N}$)
$\wedge$ ($k \in$ nset (length (l)))
$\wedge$ at (l, k, 11)
$\wedge$ lg-at-n (n, l, g))
$\rightarrow$ lg-at-n (n, move (l, k, 12), g)

EVENT: Disable lg-at-mrhoi11a.

THEOREM: lg-mrhoi11a
(listp (l)
$\wedge$ listp (g)
$\wedge$ ($k \in$ nset (length (l)))
$\wedge$ ($n \in \mathbf{N}$)
$\wedge$ at (l, k, 11)
$\wedge$ lg (n, l, g))
$\rightarrow$ lg (n, move (l, k, 12), g)

EVENT: Disable lg-mrhoi11a.

THEOREM: mrhoi11a-preserves-lg
(molws (n, l, g, h)
$\wedge$ ($k \in$ nset (n))
$\wedge$ mrhoi11a (n, k, l, g, h, lp, gp, hp)
$\wedge$ lg (n, l, g))
$\rightarrow$ lg (n, lp, gp)

```
;;;mrhoi11b
```

THEOREM: mrhoi11b-preserves-lg
(molws (n, l, g, h)
$\wedge$ ($k \in$ nset (n))
$\wedge$ mrhoi11b (n, k, l, g, h, lp, gp, hp)
$\wedge$ lg (n, l, g))
$\rightarrow$ lg (n, lp, gp)

```
;;;mrhoi12
```

THEOREM: n-neq-k-mrhoi12

$$
\begin{array}{ll}
 & (\text{listp}\,(l) \\
\land & \text{listp}\,(g) \\
\land & (n \in \mathbf{N}) \\
\land & (k \in \text{nset}\,(\text{length}\,(l))) \\
\land & (k \neq n) \\
\land & \text{at}\,(l,\ k,\ \texttt{12}) \\
\land & \text{lg-at-n}\,(n,\ l,\ g)) \\
\rightarrow & \text{lg-at-n}\,(n,\ \text{move}\,(l,\ k,\ \texttt{0}),\ \text{move}\,(g,\ k,\ \texttt{0}))
\end{array}
$$

EVENT: Disable n-neq-k-mrhoi12.

THEOREM: n-eq-k-mrhoi12

$$
\begin{array}{ll}
 & (\text{listp}\,(l) \\
\land & \text{listp}\,(g) \\
\land & (k \in \text{nset}\,(\text{length}\,(l))) \\
\land & \text{at}\,(l,\ k,\ \texttt{12}) \\
\land & \text{lg-at-n}\,(k,\ l,\ g)) \\
\rightarrow & \text{lg-at-n}\,(n,\ \text{move}\,(l,\ k,\ \texttt{0}),\ \text{move}\,(g,\ k,\ \texttt{0}))
\end{array}
$$

EVENT: Disable n-eq-k-mrhoi12.

THEOREM: lg-at-mrhoi12

$$
\begin{array}{ll}
 & (\text{listp}\,(l) \\
\land & \text{listp}\,(g) \\
\land & (n \in \mathbf{N}) \\
\land & (k \in \text{nset}\,(\text{length}\,(l))) \\
\land & \text{at}\,(l,\ k,\ \texttt{12}) \\
\land & \text{lg-at-n}\,(n,\ l,\ g)) \\
\rightarrow & \text{lg-at-n}\,(n,\ \text{move}\,(l,\ k,\ \texttt{0}),\ \text{move}\,(g,\ k,\ \texttt{0}))
\end{array}
$$

EVENT: Disable lg-at-mrhoi12.

THEOREM: lg-mrhoi12

$$
\begin{array}{ll}
 & (\text{listp}\,(l) \\
\land & \text{listp}\,(g) \\
\land & (k \in \text{nset}\,(\text{length}\,(l))) \\
\land & (n \in \mathbf{N}) \\
\land & \text{at}\,(l,\ k,\ \texttt{12}) \\
\land & \text{lg}\,(n,\ l,\ g)) \\
\rightarrow & \text{lg}\,(n,\ \text{move}\,(l,\ k,\ \texttt{0}),\ \text{move}\,(g,\ k,\ \texttt{0}))
\end{array}
$$

Event: Disable lg-mrhoi12.

Theorem: mrhoi12-preserves-lg
(molws (n, l, g, h)
 $\land$ $(k \in \text{nset}(n))$
 $\land$ mrhoi12 $(n, k, l, g, h, lp, gp, hp)$
 $\land$ lg $(n, l, g))$
$\rightarrow$ lg (n, lp, gp)

Theorem: mrho-preserves-lg
(molws (n, l, g, h)
 $\land$ $(k \in \text{nset}(n))$
 $\land$ mrhoi $(n, k, l, g, h, lp, gp, hp)$
 $\land$ lg $(n, l, g))$
$\rightarrow$ lg (n, lp, gp)

Event: Disable mrhoi0-preserves-lg.

Event: Disable mrhoi1a-preserves-lg.

Event: Disable mrhoi1b-preserves-lg.

Event: Disable mrhoi2-preserves-lg.

Event: Disable mrhoi3a-preserves-lg.

Event: Disable mrhoi3b-preserves-lg.

Event: Disable mrhoi4-preserves-lg.

Event: Disable mrhoi5a-preserves-lg.

Event: Disable mrhoi5b-preserves-lg.

Event: Disable mrhoi5c-preserves-lg.

Event: Disable mrhoi6-preserves-lg.

Event: Disable mrhoi7a-preserves-lg.

EVENT: Disable mrhoi7b-preserves-lg.

EVENT: Disable mrhoi8-preserves-lg.

EVENT: Disable mrhoi9a-preserves-lg.

EVENT: Disable mrhoi9b-preserves-lg.

EVENT: Disable mrhoi10-preserves-lg.

EVENT: Disable mrhoi11a-preserves-lg.

EVENT: Disable mrhoi11b-preserves-lg.

EVENT: Disable mrhoi12-preserves-lg.

```
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;   b0.ev   ;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;;;;;;;;;;;   b0a   ;;;;;;;;;;;;;;;;

;;;;;;;;;;;;;Common in mole and atom.
```

THEOREM: b0a-if1
$((j \in \text{nset}(n)) \land \text{lg}(n, l, g) \land (\neg \text{ at}(g, j, 1))) \rightarrow (\neg \text{ at}(l, j, 4))$

```
;;;;;;;;;;;;;comm end.
```

THEOREM: if1-nth-h-k
$(\text{molws}(n, l, g, h)$
$\land \quad (k \in \text{nset}(n))$
$\land \quad (j \in \text{nset}(n))$
$\land \quad \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\land \quad \text{lg}(n, l, g)$
$\land \quad \text{at}(h, k, j)$
$\land \quad (\neg \text{ at}(g, \text{nth}(h, k), 1)))$
$\rightarrow \quad (\neg \text{ at}(l, j, 4))$

THEOREM: l5-not-g1
$(\text{molws}(n, l, g, h)$
$\land \quad (k \in \text{nset}(n))$
$\land \quad (j \in \text{nset}(n))$

$\wedge$ mrhoi $(n, k, l, g, h, lp, gp, hp)$
$\wedge$ at (h, k, j)
$\wedge$ at $(l, k, \mathtt{5})$
$\wedge$ at $(lp, k, \mathtt{5}))$
$\rightarrow$ $(\neg$ at $(g, \text{nth}(h, k), \mathtt{1}))$

THEOREM: l5-nth-h-k-eq-j
(at (h, k, j)
$\wedge$ molws (n, l, g, h)
$\wedge$ $(k \in \text{nset}(n))$
$\wedge$ $(j \in \text{nset}(n))$
$\wedge$ mrhoi $(n, k, l, g, h, lp, gp, hp)$
$\wedge$ lg (n, l, g)
$\wedge$ at $(l, k, \mathtt{5})$
$\wedge$ at $(lp, k, \mathtt{5}))$
$\rightarrow$ $(\neg$ at $(l, j, \mathtt{4}))$

THEOREM: l5-j-lt-nth-k
$((j < \text{nth}(h, k))$
$\wedge$ molws (n, l, g, h)
$\wedge$ $(k \in \text{nset}(n))$
$\wedge$ $(j \in \text{nset}(n))$
$\wedge$ mrhoi $(n, k, l, g, h, lp, gp, hp)$
$\wedge$ b0a (n, l, h, k, j)
$\wedge$ at $(l, k, \mathtt{5}))$
$\rightarrow$ $(\neg$ at $(l, j, \mathtt{4}))$

THEOREM: nth-k-lt-j-or-eq-j
(molws (n, l, g, h)
$\wedge$ $(k \in \text{nset}(n))$
$\wedge$ $(j \in \text{nset}(n))$
$\wedge$ $((j - 1) < \text{nth}(h, k))$
$\wedge$ $(j \not< \text{nth}(h, k)))$
$\rightarrow$ at (h, k, j)

THEOREM: lm-j-not-in-l4
(molws (n, l, g, h)
$\wedge$ $(j \in \text{nset}(n))$
$\wedge$ $(k \in \text{nset}(n))$
$\wedge$ mrhoi $(n, k, l, g, h, lp, gp, hp)$
$\wedge$ lg (n, l, g)
$\wedge$ b0a (n, l, h, k, j)
$\wedge$ at $(l, k, \mathtt{5})$
$\wedge$ at $(lp, k, \mathtt{5})$
$\wedge$ $((j - 1) < \text{nth}(h, k)))$
$\rightarrow$ $(\neg$ at $(l, j, \mathtt{4}))$

THEOREM: cond-l5
(molws(n, l, g, h)
$\land$ $(k \in \text{nset}(n))$
$\land$ $(j \in \text{nset}(n))$
$\land$ mrhoi$(n, k, l, g, h, lp, gp, hp)$
$\land$ at$(l, k, \texttt{5})$
$\land$ $(j < \text{nth}(hp, k)))$
$\rightarrow$ $((j - 1) < \text{nth}(h, k))$

THEOREM: j-not-in-l4
(molws(n, l, g, h)
$\land$ $(j \in \text{nset}(n))$
$\land$ $(k \in \text{nset}(n))$
$\land$ mrhoi$(n, k, l, g, h, lp, gp, hp)$
$\land$ lg(n, l, g)
$\land$ b0a(n, l, h, k, j)
$\land$ at$(l, k, \texttt{5})$
$\land$ at$(lp, k, \texttt{5})$
$\land$ $(j < \text{nth}(hp, k)))$
$\rightarrow$ $(\neg$ at$(l, j, \texttt{4}))$

THEOREM: k-in-l5
(molws(n, l, g, h)
$\land$ $(j \in \text{nset}(n))$
$\land$ $(k \in \text{nset}(n))$
$\land$ mrhoi$(n, k, l, g, h, lp, gp, hp)$
$\land$ at$(lp, k, \texttt{5})$
$\land$ $(j < \text{nth}(hp, k)))$
$\rightarrow$ at$(l, k, \texttt{5})$

```
;;;The order of the hints is crucial.
```

THEOREM: lm-b0a-i-eq-k-j-neq-k
(molws(n, l, g, h)
$\land$ $(j \in \text{nset}(n))$
$\land$ $(k \in \text{nset}(n))$
$\land$ mrhoi$(n, k, l, g, h, lp, gp, hp)$
$\land$ lg(n, l, g)
$\land$ b0a(n, l, h, k, j)
$\land$ at$(lp, k, \texttt{5})$
$\land$ $(j < \text{nth}(hp, k)))$
$\rightarrow$ $(\neg$ at$(l, j, \texttt{4}))$

THEOREM: b0a-i-eq-k-j-neq-k
(molws(n, l, g, h)

$\wedge \quad (j \in \text{nset}\,(n))$
$\wedge \quad (k \in \text{nset}\,(n))$
$\wedge \quad \text{mrhoi}\,(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge \quad (j \neq k)$
$\wedge \quad \text{lg}\,(n,\ l,\ g)$
$\wedge \quad \text{b0a}\,(n,\ l,\ h,\ k,\ j))$
$\rightarrow \quad \text{b0a}\,(n,\ lp,\ hp,\ k,\ j)$

Theorem: b0a-i-j-eq-k
$(\text{molws}\,(n,\ l,\ g,\ h)$
$\wedge \quad (k \in \text{nset}\,(n))$
$\wedge \quad \text{mrhoi}\,(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge \quad \text{b0a}\,(n,\ l,\ h,\ k,\ k))$
$\rightarrow \quad \text{b0a}\,(n,\ lp,\ hp,\ k,\ k)$

Theorem: b0a-i-eq-k
$(\text{molws}\,(n,\ l,\ g,\ h)$
$\wedge \quad (j \in \text{nset}\,(n))$
$\wedge \quad (k \in \text{nset}\,(n))$
$\wedge \quad \text{mrhoi}\,(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge \quad \text{lg}\,(n,\ l,\ g)$
$\wedge \quad \text{b0a}\,(n,\ l,\ h,\ k,\ j))$
$\rightarrow \quad \text{b0a}\,(n,\ lp,\ hp,\ k,\ j)$

```
;;;n-not-less-j is necessary.
```

Theorem: cond-lp4
$(\text{molws}\,(n,\ l,\ g,\ h)$
$\wedge \quad (i \in \text{nset}\,(n))$
$\wedge \quad (k \in \text{nset}\,(n))$
$\wedge \quad \text{mrhoi}\,(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge \quad \text{at}\,(l,\ k,\ \mathtt{3})$
$\wedge \quad (i \not< \text{nth}\,(h,\ k)))$
$\rightarrow \quad (\neg\ \text{at}\,(lp,\ k,\ \mathtt{4}))$

Theorem: not-l3-then-lp4
$(\text{molws}\,(n,\ l,\ g,\ h)$
$\wedge \quad (k \in \text{nset}\,(n))$
$\wedge \quad \text{mrhoi}\,(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge \quad (\neg\ \text{at}\,(l,\ k,\ \mathtt{3})))$
$\rightarrow \quad (\neg\ \text{at}\,(lp,\ k,\ \mathtt{4}))$

Theorem: i-in-l5
$(\text{molws}\,(n,\ l,\ g,\ h)$
$\wedge \quad (i \in \text{nset}\,(n))$

$\wedge \quad (k \in \text{nset}(n))$
$\wedge \quad \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\wedge \quad \text{b0b}(n, l, h, i, k)$
$\wedge \quad \text{at}(l, i, \texttt{5})$
$\wedge \quad (k < \text{nth}(h, i)))$
$\rightarrow \quad (\neg \text{ at}(lp, k, \texttt{4}))$

THEOREM: lm-b0a-i-neq-k-j-eq-k
$(\text{molws}(n, l, g, h)$
$\wedge \quad (i \in \text{nset}(n))$
$\wedge \quad (k \in \text{nset}(n))$
$\wedge \quad \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\wedge \quad (i \neq k)$
$\wedge \quad \text{b0b}(n, l, h, i, k)$
$\wedge \quad \text{at}(lp, i, \texttt{5})$
$\wedge \quad (k < \text{nth}(h, i)))$
$\rightarrow \quad (\neg \text{ at}(lp, k, \texttt{4}))$

THEOREM: b0a-i-neq-k-j-eq-k
$(\text{molws}(n, l, g, h)$
$\wedge \quad (i \in \text{nset}(n))$
$\wedge \quad (k \in \text{nset}(n))$
$\wedge \quad \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\wedge \quad (i \neq k)$
$\wedge \quad \text{b0a}(n, l, h, i, k)$
$\wedge \quad \text{b0b}(n, l, h, i, k))$
$\rightarrow \quad \text{b0a}(n, lp, hp, i, k)$

THEOREM: b0a-i-j-neq-k
$(\text{molws}(n, l, g, h)$
$\wedge \quad (i \in \text{nset}(n))$
$\wedge \quad (j \in \text{nset}(n))$
$\wedge \quad (k \in \text{nset}(n))$
$\wedge \quad \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\wedge \quad (i \neq k)$
$\wedge \quad (j \neq k)$
$\wedge \quad \text{b0a}(n, l, h, i, j)$
$\wedge \quad \text{b0b}(n, l, h, i, j))$
$\rightarrow \quad \text{b0a}(n, lp, hp, i, j)$

THEOREM: b0a-i-neq-k
$(\text{molws}(n, l, g, h)$
$\wedge \quad (i \in \text{nset}(n))$
$\wedge \quad (j \in \text{nset}(n))$
$\wedge \quad (k \in \text{nset}(n))$

$\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ $(i \neq k)$
$\wedge$ b0a $(n,\ l,\ h,\ i,\ j)$
$\wedge$ b0b $(n,\ l,\ h,\ i,\ j))$
$\rightarrow$ b0a $(n,\ lp,\ hp,\ i,\ j)$

THEOREM: rho-preserves-b0a
(molws $(n,\ l,\ g,\ h)$
$\wedge$ $(i \in \text{nset}\,(n))$
$\wedge$ $(j \in \text{nset}\,(n))$
$\wedge$ $(k \in \text{nset}\,(n))$
$\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ lg $(n,\ l,\ g)$
$\wedge$ b0a $(n,\ l,\ h,\ i,\ j)$
$\wedge$ b0b $(n,\ l,\ h,\ i,\ j))$
$\rightarrow$ b0a $(n,\ lp,\ hp,\ i,\ j)$

```
;;;;;;;;;;;;;   b0b   ;;;;;;;;;;;;;;;;

;;;;;;;;;;;;;;;;;Common in mole and atom.
```

THEOREM: b0b-if1
$((j \in \text{nset}\,(n)) \wedge \text{lg}\,(n,\ l,\ g) \wedge \text{at}\,(l,\ j,\ \texttt{3})) \rightarrow \text{at}\,(g,\ j,\ \texttt{1})$

THEOREM: b0b-if3
$((i \in \text{nset}\,(n)) \wedge \text{lg}\,(n,\ l,\ g) \wedge \text{at}\,(l,\ i,\ \texttt{5}))$
$\rightarrow$ $(\neg$ union-at-n $(g,\ i,$ `'(0 1 2)`$))$

```
;;;;;;;;;;;;;;;;;Common in mole and atom end.
```

THEOREM: lm-j-neq-h-k
(molws $(n,\ l,\ g,\ h)$
$\wedge$ $(k \in \text{nset}\,(n))$
$\wedge$ $(j \in \text{nset}\,(n))$
$\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ lg $(n,\ l,\ g)$
$\wedge$ at $(l,\ j,\ \texttt{3})$
$\wedge$ $(\neg\ \text{at}\,(g,\ \text{nth}\,(h,\ k),\ \texttt{1})))$
$\rightarrow$ $(\neg\ \text{at}\,(h,\ k,\ j))$

THEOREM: h-k-not-g1
(molws $(n,\ l,\ g,\ h)$
$\wedge$ $(k \in \text{nset}\,(n))$
$\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$

$\wedge$ at $(l,\ k,\ \mathtt{5})$
$\wedge$ at $(lp,\ k,\ \mathtt{5}))$
$\rightarrow$ $(\neg$ at $(g,\ \mathrm{nth}\,(h,\ k),\ \mathtt{1}))$

THEOREM: j-neq-h-k
(molws $(n,\ l,\ g,\ h)$
$\wedge$ $(j \in \mathrm{nset}\,(n))$
$\wedge$ $(k \in \mathrm{nset}\,(n))$
$\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ lg $(n,\ l,\ g)$
$\wedge$ at $(lp,\ k,\ \mathtt{5})$
$\wedge$ at $(l,\ k,\ \mathtt{5})$
$\wedge$ at $(l,\ j,\ \mathtt{3}))$
$\rightarrow$ $(\neg$ at $(h,\ k,\ j))$

THEOREM: n-k-leq-sub1-i
(molws $(n,\ l,\ g,\ h)$
$\wedge$ $(k \in \mathrm{nset}\,(n))$
$\wedge$ $(i \in \mathrm{nset}\,(n))$
$\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ $(\neg$ at $(h,\ k,\ i))$
$\wedge$ $(i \not< \mathrm{nth}\,(h,\ k)))$
$\rightarrow$ $((i - 1) \not< \mathrm{nth}\,(h,\ k))$

```
;;;This is proved with help of n-k-leq-sub1-i.
```

THEOREM: lm1-j-in-l3
(molws $(n,\ l,\ g,\ h)$
$\wedge$ $(j \in \mathrm{nset}\,(n))$
$\wedge$ $(k \in \mathrm{nset}\,(n))$
$\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ lg $(n,\ l,\ g)$
$\wedge$ b0b $(n,\ l,\ h,\ k,\ j)$
$\wedge$ at $(l,\ k,\ \mathtt{5})$
$\wedge$ at $(lp,\ k,\ \mathtt{5})$
$\wedge$ at $(l,\ j,\ \mathtt{3})$
$\wedge$ $((j - 1) < \mathrm{nth}\,(h,\ k)))$
$\rightarrow$ $(k \not< \mathrm{nth}\,(h,\ j))$

THEOREM: lm-j-in-l3
(molws $(n,\ l,\ g,\ h)$
$\wedge$ $(j \in \mathrm{nset}\,(n))$
$\wedge$ $(k \in \mathrm{nset}\,(n))$
$\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ lg $(n,\ l,\ g)$

$\wedge$ b0b (n, l, h, k, j)
$\wedge$ at $(l, k, \mathtt{5})$
$\wedge$ at $(lp, k, \mathtt{5})$
$\wedge$ at $(l, j, \mathtt{3})$
$\wedge$ $(j < \text{nth}\,(hp, k)))$
$\rightarrow$ $(k \not< \text{nth}\,(h, j))$

`;;;The order of hints is crucial.`

THEOREM: j-in-l3
(molws (n, l, g, h)
$\wedge$ $(j \in \text{nset}\,(n))$
$\wedge$ $(k \in \text{nset}\,(n))$
$\wedge$ mrhoi $(n, k, l, g, h, lp, gp, hp)$
$\wedge$ lg (n, l, g)
$\wedge$ b0b (n, l, h, k, j)
$\wedge$ at $(lp, k, \mathtt{5})$
$\wedge$ at $(l, j, \mathtt{3})$
$\wedge$ $(j < \text{nth}\,(hp, k)))$
$\rightarrow$ $(k \not< \text{nth}\,(h, j))$

THEOREM: lm-b0b-i-eq-k-j-neq-k
(molws (n, l, g, h)
$\wedge$ $(j \in \text{nset}\,(n))$
$\wedge$ $(k \in \text{nset}\,(n))$
$\wedge$ mrhoi $(n, k, l, g, h, lp, gp, hp)$
$\wedge$ $(j \neq k)$
$\wedge$ lg (n, l, g)
$\wedge$ b0b (n, l, h, k, j)
$\wedge$ at $(lp, k, \mathtt{5})$
$\wedge$ at $(lp, j, \mathtt{3})$
$\wedge$ $(j < \text{nth}\,(hp, k)))$
$\rightarrow$ $(k \not< \text{nth}\,(h, j))$

THEOREM: b0b-i-eq-k-j-neq-k
(molws (n, l, g, h)
$\wedge$ $(j \in \text{nset}\,(n))$
$\wedge$ $(k \in \text{nset}\,(n))$
$\wedge$ mrhoi $(n, k, l, g, h, lp, gp, hp)$
$\wedge$ $(j \neq k)$
$\wedge$ lg (n, l, g)
$\wedge$ b0b $(n, l, h, k, j))$
$\rightarrow$ b0b (n, lp, hp, k, j)

THEOREM: b0b-i-j-eq-k

$$
\begin{array}{ll}
(\text{molws}\,(n,\ l,\ g,\ h) & \\
\ \wedge & (k \in \text{nset}\,(n)) \\
\ \wedge & \text{mrhoi}\,(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp) \\
\ \wedge & \text{lg}\,(n,\ l,\ g)) \\
\rightarrow & \text{b0b}\,(n,\ lp,\ hp,\ k,\ k)
\end{array}
$$

Theorem: b0b-i-eq-k

$$
\begin{array}{ll}
(\text{molws}\,(n,\ l,\ g,\ h) & \\
\ \wedge & (j \in \text{nset}\,(n)) \\
\ \wedge & (k \in \text{nset}\,(n)) \\
\ \wedge & \text{mrhoi}\,(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp) \\
\ \wedge & \text{lg}\,(n,\ l,\ g) \\
\ \wedge & \text{b0b}\,(n,\ l,\ h,\ k,\ j)) \\
\rightarrow & \text{b0b}\,(n,\ lp,\ hp,\ k,\ j)
\end{array}
$$

Theorem: lm-i-neq-h-k

$$
\begin{array}{ll}
(\text{molws}\,(n,\ l,\ g,\ h) & \\
\ \wedge & (k \in \text{nset}\,(n)) \\
\ \wedge & (i \in \text{nset}\,(n)) \\
\ \wedge & \text{mrhoi}\,(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp) \\
\ \wedge & \text{lg}\,(n,\ l,\ g) \\
\ \wedge & \text{at}\,(l,\ i,\ \texttt{5}) \\
\ \wedge & \text{union-at-n}\,(g,\ \text{nth}\,(h,\ k),\ \texttt{'(0 1 2)})) \\
\rightarrow & (\neg\ \text{at}\,(h,\ k,\ i))
\end{array}
$$

Theorem: h-k-g02

$$
\begin{array}{ll}
(\text{molws}\,(n,\ l,\ g,\ h) & \\
\ \wedge & (k \in \text{nset}\,(n)) \\
\ \wedge & \text{mrhoi}\,(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp) \\
\ \wedge & \text{at}\,(l,\ k,\ \texttt{3}) \\
\ \wedge & \text{at}\,(lp,\ k,\ \texttt{3})) \\
\rightarrow & \text{union-at-n}\,(g,\ \text{nth}\,(h,\ k),\ \texttt{'(0 1 2)})
\end{array}
$$

Theorem: i-neq-h-k

$$
\begin{array}{ll}
(\text{molws}\,(n,\ l,\ g,\ h) & \\
\ \wedge & (k \in \text{nset}\,(n)) \\
\ \wedge & (i \in \text{nset}\,(n)) \\
\ \wedge & \text{mrhoi}\,(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp) \\
\ \wedge & \text{lg}\,(n,\ l,\ g) \\
\ \wedge & \text{at}\,(l,\ i,\ \texttt{5}) \\
\ \wedge & \text{at}\,(l,\ k,\ \texttt{3}) \\
\ \wedge & \text{at}\,(lp,\ k,\ \texttt{3})) \\
\rightarrow & (\neg\ \text{at}\,(h,\ k,\ i))
\end{array}
$$

Theorem: lm1-k-in-l3-imp

$(\mathrm{molws}(n,\ l,\ g,\ h)$
$\wedge\quad (k \in \mathrm{nset}(n))$
$\wedge\quad (i \in \mathrm{nset}(n))$
$\wedge\quad \mathrm{mrhoi}(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge\quad \mathrm{lg}(n,\ l,\ g)$
$\wedge\quad \mathrm{at}(l,\ i,\ \mathtt{5})$
$\wedge\quad \mathrm{at}(l,\ k,\ \mathtt{3})$
$\wedge\quad \mathrm{at}(lp,\ k,\ \mathtt{3})$
$\wedge\quad (i \not< \mathrm{nth}(h,\ k)))$
$\rightarrow\quad ((i - 1) \not< \mathrm{nth}(h,\ k))$

THEOREM: lm-k-in-l3-imp
$(\mathrm{molws}(n,\ l,\ g,\ h)$
$\wedge\quad (k \in \mathrm{nset}(n))$
$\wedge\quad (i \in \mathrm{nset}(n))$
$\wedge\quad \mathrm{mrhoi}(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge\quad \mathrm{lg}(n,\ l,\ g)$
$\wedge\quad \mathrm{b0b}(n,\ l,\ h,\ i,\ k)$
$\wedge\quad \mathrm{at}(l,\ i,\ \mathtt{5})$
$\wedge\quad \mathrm{at}(l,\ k,\ \mathtt{3})$
$\wedge\quad \mathrm{at}(lp,\ k,\ \mathtt{3})$
$\wedge\quad (k < \mathrm{nth}(h,\ i)))$
$\rightarrow\quad ((i - 1) \not< \mathrm{nth}(h,\ k))$

THEOREM: cond-l3
$(\mathrm{molws}(n,\ l,\ g,\ h)$
$\wedge\quad (k \in \mathrm{nset}(n))$
$\wedge\quad (i \in \mathrm{nset}(n))$
$\wedge\quad \mathrm{mrhoi}(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge\quad \mathrm{at}(l,\ k,\ \mathtt{3})$
$\wedge\quad (i < \mathrm{nth}(hp,\ k)))$
$\rightarrow\quad ((i - 1) < \mathrm{nth}(h,\ k))$

THEOREM: k-in-l3-imp
$(\mathrm{molws}(n,\ l,\ g,\ h)$
$\wedge\quad (i \in \mathrm{nset}(n))$
$\wedge\quad (k \in \mathrm{nset}(n))$
$\wedge\quad \mathrm{mrhoi}(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge\quad \mathrm{lg}(n,\ l,\ g)$
$\wedge\quad \mathrm{b0b}(n,\ l,\ h,\ i,\ k)$
$\wedge\quad \mathrm{at}(l,\ i,\ \mathtt{5})$
$\wedge\quad \mathrm{at}(l,\ k,\ \mathtt{3})$
$\wedge\quad \mathrm{at}(lp,\ k,\ \mathtt{3})$
$\wedge\quad (k < \mathrm{nth}(h,\ i)))$
$\rightarrow\quad (i \not< \mathrm{nth}(hp,\ k))$

Theorem: k-in-l2-imp
(molws $(n,\ l,\ g,\ h)$
∧ $(i \in \text{nset}\,(n))$
∧ $(k \in \text{nset}\,(n))$
∧ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
∧ at $(lp,\ k,\ 3)$
∧ at $(l,\ k,\ 2))$
→ $(i \not< \text{nth}\,(hp,\ k))$

Theorem: lp3-then-l2-or-l3
(molws $(n,\ l,\ g,\ h)$
∧ $(i \in \text{nset}\,(n))$
∧ $(k \in \text{nset}\,(n))$
∧ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
∧ at $(lp,\ k,\ 3)$
∧ $(\neg$ at $(l,\ k,\ 2)))$
→ at $(l,\ k,\ 3)$

Theorem: b0b-i-in-l5
(molws $(n,\ l,\ g,\ h)$
∧ $(i \in \text{nset}\,(n))$
∧ $(k \in \text{nset}\,(n))$
∧ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
∧ lg $(n,\ l,\ g)$
∧ b0b $(n,\ l,\ h,\ i,\ k)$
∧ at $(l,\ i,\ 5)$
∧ at $(lp,\ k,\ 3)$
∧ $(k < \text{nth}\,(h,\ i)))$
→ $(i \not< \text{nth}\,(hp,\ k))$

Theorem: lm-b0b-i-neq-k-j-eq-k
(molws $(n,\ l,\ g,\ h)$
∧ $(i \in \text{nset}\,(n))$
∧ $(k \in \text{nset}\,(n))$
∧ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
∧ $(i \neq k)$
∧ b0b $(n,\ l,\ h,\ i,\ k)$
∧ lg $(n,\ l,\ g)$
∧ at $(lp,\ i,\ 5)$
∧ at $(lp,\ k,\ 3)$
∧ $(k < \text{nth}\,(h,\ i)))$
→ $(i \not< \text{nth}\,(hp,\ k))$

Theorem: b0b-i-neq-k-j-eq-k
(molws $(n,\ l,\ g,\ h)$

$\wedge$ $(i \in \text{nset}(n))$
$\wedge$ $(k \in \text{nset}(n))$
$\wedge$ $\text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\wedge$ $(i \neq k)$
$\wedge$ $\text{b0b}(n, l, h, i, k)$
$\wedge$ $\text{lg}(n, l, g))$
$\rightarrow$ $\text{b0b}(n, lp, hp, i, k)$

THEOREM: b0b-i-j-neq-k
$(\text{molws}(n, l, g, h)$
$\wedge$ $(i \in \text{nset}(n))$
$\wedge$ $(j \in \text{nset}(n))$
$\wedge$ $(k \in \text{nset}(n))$
$\wedge$ $\text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\wedge$ $(i \neq k)$
$\wedge$ $(j \neq k)$
$\wedge$ $\text{b0b}(n, l, h, i, j))$
$\rightarrow$ $\text{b0b}(n, lp, hp, i, j)$

THEOREM: b0b-i-neq-k
$(\text{molws}(n, l, g, h)$
$\wedge$ $(i \in \text{nset}(n))$
$\wedge$ $(j \in \text{nset}(n))$
$\wedge$ $(k \in \text{nset}(n))$
$\wedge$ $\text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\wedge$ $(i \neq k)$
$\wedge$ $\text{lg}(n, l, g)$
$\wedge$ $\text{b0b}(n, l, h, i, j))$
$\rightarrow$ $\text{b0b}(n, lp, hp, i, j)$

THEOREM: rho-preserves-b0b
$(\text{molws}(n, l, g, h)$
$\wedge$ $(i \in \text{nset}(n))$
$\wedge$ $(j \in \text{nset}(n))$
$\wedge$ $(k \in \text{nset}(n))$
$\wedge$ $\text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\wedge$ $\text{lg}(n, l, g)$
$\wedge$ $\text{b0b}(n, l, h, i, j))$
$\rightarrow$ $\text{b0b}(n, lp, hp, i, j)$

```
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;   b1.ev   ;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;;;;;;;;;;;   b1a   ;;;;;;;;;;;;;;;;
```

THEOREM: lm-h-k-eq-add1-n-nex-hint

$((n \not\simeq 0) \wedge \text{listp}(h) \wedge (n \not< i) \wedge (k \in \text{nset}(n)) \wedge (n < \text{nth}(h, k)))$
$\rightarrow \quad (\neg\ \text{exist-hint-8-12-3-4}(i, l, g, h, k))$

THEOREM: h-k-eq-add1-n-nex-hint
$(\text{molws}(n, l, g, h) \wedge (k \in \text{nset}(n)) \wedge \text{at}(h, k, 1 + n))$
$\rightarrow \quad (\neg\ \text{exist-hint-8-12-3-4}(n, l, g, h, k))$

THEOREM: h-k-eq-add1-n-k-not-in-l3
$(\text{molws}(n, l, g, h)$
$\wedge \quad (i \in \text{nset}(n))$
$\wedge \quad (k \in \text{nset}(n))$
$\wedge \quad \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\wedge \quad \text{b1b}(n, l, g, h, i, k)$
$\wedge \quad \text{at}(h, k, 1 + n)$
$\wedge \quad \text{union-at-n}(l, i, \texttt{'(8 9 10 11 12)}))$
$\rightarrow \quad (\neg\ \text{at}(l, k, \texttt{3}))$

THEOREM: not-l3-then-not-lp4
$(\text{molws}(n, l, g, h)$
$\wedge \quad (k \in \text{nset}(n))$
$\wedge \quad \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\wedge \quad (\neg\ \text{at}(l, k, \texttt{3})))$
$\rightarrow \quad (\neg\ \text{at}(lp, k, \texttt{4}))$

THEOREM: h-k-eq-add1-n
$(\text{molws}(n, l, g, h)$
$\wedge \quad (i \in \text{nset}(n))$
$\wedge \quad (k \in \text{nset}(n))$
$\wedge \quad \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\wedge \quad \text{b1b}(n, l, g, h, i, k)$
$\wedge \quad \text{at}(h, k, 1 + n)$
$\wedge \quad \text{union-at-n}(l, i, \texttt{'(8 9 10 11 12)}))$
$\rightarrow \quad (\neg\ \text{at}(lp, k, \texttt{4}))$

THEOREM: h-k-neq-add1-n
$(\text{molws}(n, l, g, h)$
$\wedge \quad (k \in \text{nset}(n))$
$\wedge \quad \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\wedge \quad (\neg\ \text{at}(h, k, 1 + n)))$
$\rightarrow \quad (\neg\ \text{at}(lp, k, \texttt{4}))$

```
;;;The order of the hints is crucial.
```

THEOREM: lm-b1a-i-neq-k-j-eq-k
$(\text{molws}(n, l, g, h)$

$\wedge$ $(i \in \mathrm{nset}\,(n))$
$\wedge$ $(k \in \mathrm{nset}\,(n))$
$\wedge$ $\mathrm{mrhoi}\,(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ $\mathrm{b1b}\,(n,\ l,\ g,\ h,\ i,\ k)$
$\wedge$ union-at-n $(l,\ i,$ `'(8 9 10 11 12)`$))$
$\rightarrow$ $(\neg\ \mathrm{at}\,(lp,\ k,\ 4))$

`;;;need m-l-same-lp.`

Theorem: b1a-i-neq-k-j-eq-k
$(\mathrm{molws}\,(n,\ l,\ g,\ h)$
$\wedge$ $(i \in \mathrm{nset}\,(n))$
$\wedge$ $(k \in \mathrm{nset}\,(n))$
$\wedge$ $\mathrm{mrhoi}\,(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ $(i \neq k)$
$\wedge$ $\mathrm{b1b}\,(n,\ l,\ g,\ h,\ i,\ k))$
$\rightarrow$ $\mathrm{b1a}\,(lp,\ i,\ k)$

Theorem: b1a-i-j-neq-k
$(\mathrm{molws}\,(n,\ l,\ g,\ h)$
$\wedge$ $(i \in \mathrm{nset}\,(n))$
$\wedge$ $(j \in \mathrm{nset}\,(n))$
$\wedge$ $(k \in \mathrm{nset}\,(n))$
$\wedge$ $\mathrm{mrhoi}\,(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ $(i \neq k)$
$\wedge$ $(j \neq k)$
$\wedge$ $\mathrm{b1a}\,(l,\ i,\ j))$
$\rightarrow$ $\mathrm{b1a}\,(lp,\ i,\ j)$

Theorem: b1a-i-neq-k
$(\mathrm{molws}\,(n,\ l,\ g,\ h)$
$\wedge$ $(i \in \mathrm{nset}\,(n))$
$\wedge$ $(j \in \mathrm{nset}\,(n))$
$\wedge$ $(k \in \mathrm{nset}\,(n))$
$\wedge$ $\mathrm{mrhoi}\,(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ $(i \neq k)$
$\wedge$ $\mathrm{b1a}\,(l,\ i,\ j)$
$\wedge$ $\mathrm{b1b}\,(n,\ l,\ g,\ h,\ i,\ j))$
$\rightarrow$ $\mathrm{b1a}\,(lp,\ i,\ j)$

Theorem: cond-l7
$(\mathrm{molws}\,(n,\ l,\ g,\ h)$
$\wedge$ $(k \in \mathrm{nset}\,(n))$
$\wedge$ $\mathrm{mrhoi}\,(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ $\mathrm{at}\,(l,\ k,\ 7)$

$\wedge$ union-at-n $(lp,\ k,\ \texttt{'(8 9 10 11 12)}))$
$\rightarrow$ at $(g,\ \text{nth}\,(h,\ k),\ \texttt{4})$

THEOREM: k-in-l7-imp
(molws $(n,\ l,\ g,\ h)$
$\wedge$ $(j \in \text{nset}\,(n))$
$\wedge$ $(k \in \text{nset}\,(n))$
$\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ b1d $(n,\ l,\ h,\ k)$
$\wedge$ lg $(n,\ l,\ g)$
$\wedge$ at $(l,\ k,\ \texttt{7})$
$\wedge$ b1a $(l,\ \text{nth}\,(h,\ k),\ j)$
$\wedge$ union-at-n $(lp,\ k,\ \texttt{'(8 9 10 11 12)}))$
$\rightarrow$ $(\neg\ \text{at}\,(l,\ j,\ \texttt{4}))$

THEOREM: l5-j-lt-h-k
(molws $(n,\ l,\ g,\ h)$
$\wedge$ $(j \in \text{nset}\,(n))$
$\wedge$ $(k \in \text{nset}\,(n))$
$\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ at $(l,\ k,\ \texttt{5})$
$\wedge$ union-at-n $(lp,\ k,\ \texttt{'(8 9 10 11 12)}))$
$\rightarrow$ $(j < \text{nth}\,(h,\ k))$

THEOREM: k-in-l5-then-j-not-l4
(molws $(n,\ l,\ g,\ h)$
$\wedge$ $(j \in \text{nset}\,(n))$
$\wedge$ $(k \in \text{nset}\,(n))$
$\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ at $(l,\ k,\ \texttt{5})$
$\wedge$ union-at-n $(lp,\ k,\ \texttt{'(8 9 10 11 12)})$
$\wedge$ b0a $(n,\ l,\ h,\ k,\ j))$
$\rightarrow$ $(\neg\ \text{at}\,(l,\ j,\ \texttt{4}))$

THEOREM: lp9-12-k-in-l8-12
(molws $(n,\ l,\ g,\ h)$
$\wedge$ $(k \in \text{nset}\,(n))$
$\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ union-at-n $(lp,\ k,\ \texttt{'(9 10 11 12)}))$
$\rightarrow$ union-at-n $(l,\ k,\ \texttt{'(8 9 10 11 12)})$

THEOREM: k-in-lp9-12-then-j-not-l4
(molws $(n,\ l,\ g,\ h)$
$\wedge$ $(j \in \text{nset}\,(n))$
$\wedge$ $(k \in \text{nset}\,(n))$

$\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ b1a $(l,\ k,\ j)$
$\wedge$ union-at-n $(lp,\ k,$ `'(9 10 11 12)`$))$
$\rightarrow$ $(\neg$ at $(l,\ j,\ 4))$

THEOREM: k-not-in-l7-then-lp9-12-or-l5
(molws $(n,\ l,\ g,\ h)$
$\wedge$ $(k \in$ nset $(n))$
$\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ union-at-n $(lp,\ k,$ `'(8 9 10 11 12)`$)$
$\wedge$ $(\neg$ at $(l,\ k,\ 7))$
$\wedge$ $(\neg$ union-at-n $(lp,\ k,$ `'(9 10 11 12)`$)))$
$\rightarrow$ at $(l,\ k,\ 5)$

THEOREM: k-in-not-l7-imp
(molws $(n,\ l,\ g,\ h)$
$\wedge$ $(j \in$ nset $(n))$
$\wedge$ $(k \in$ nset $(n))$
$\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ $(\neg$ at $(l,\ k,\ 7))$
$\wedge$ b0a $(n,\ l,\ h,\ k,\ j)$
$\wedge$ b1a $(l,\ k,\ j)$
$\wedge$ union-at-n $(lp,\ k,$ `'(8 9 10 11 12)`$))$
$\rightarrow$ $(\neg$ at $(l,\ j,\ 4))$

```
;;;I wonder why the following two do not imply
;;;lm-b1a-i-eq-k-j-neq-k although those without
;;;(member u (nset n)) are perfectly able to
;;;imply it.
;;;(prove-lemma k-in-lp9-12-then-j-not-l4 (rewrite)
;;;   (implies (and (molws n l g h)
;;;                 (member j (nset n))
;;;                 (member k (nset n))
;;;                 (mrhoi n k l g h lp gp hp)
;;;                 (b1a l k j)
;;;                 (union-at-n lp k '(9 10 11 12)))
;;;            (not (at l j 4))))
;;;
;;;(prove-lemma k-in-lp8-then-j-not-l4 (rewrite)
;;;   (implies (and (at lp k 8)
;;;                 (molws n l g h)
;;;                 (member j (nset n))
;;;                 (member u (nset n))
;;;                 (member k (nset n))
;;;                 (mrhoi n k l g h lp gp hp)
```

```
;;;                    (lg n l g)
;;;                    (b0a n l h k j)
;;;                    (b1a l (nth h k) j))
;;;               (not (at l j 4))))
```

THEOREM: lm-b1a-i-eq-k-j-neq-k
(molws $(n,\ l,\ g,\ h)$
$\land$ $(j \in \text{nset}\,(n))$
$\land$ $(k \in \text{nset}\,(n))$
$\land$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\land$ b1d $(n,\ l,\ h,\ k)$
$\land$ lg $(n,\ l,\ g)$
$\land$ b0a $(n,\ l,\ h,\ k,\ j)$
$\land$ b1a $(l,\ k,\ j)$
$\land$ b1a $(l,\ \text{nth}\,(h,\ k),\ j)$
$\land$ union-at-n $(lp,\ k,$ '(8 9 10 11 12)))
$\rightarrow$ $(\neg\ \text{at}\,(l,\ j,\ 4))$

```
;;;m-l-same-lp-at-not is used.
```

THEOREM: b1a-i-eq-k-j-neq-k
(molws $(n,\ l,\ g,\ h)$
$\land$ $(j \in \text{nset}\,(n))$
$\land$ $(k \in \text{nset}\,(n))$
$\land$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\land$ $(j \neq k)$
$\land$ b1d $(n,\ l,\ h,\ k)$
$\land$ lg $(n,\ l,\ g)$
$\land$ b0a $(n,\ l,\ h,\ k,\ j)$
$\land$ b1a $(l,\ k,\ j)$
$\land$ b1a $(l,\ \text{nth}\,(h,\ k),\ j))$
$\rightarrow$ b1a $(lp,\ k,\ j)$

THEOREM: b1a-i-j-eq-k
(molws $(n,\ l,\ g,\ h)$
$\land$ $(k \in \text{nset}\,(n))$
$\land$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\land$ b1a $(l,\ k,\ k))$
$\rightarrow$ b1a $(lp,\ k,\ k)$

THEOREM: b1a-i-eq-k
(molws $(n,\ l,\ g,\ h)$
$\land$ $(j \in \text{nset}\,(n))$
$\land$ $(k \in \text{nset}\,(n))$

$\wedge$ mrhoi $(n, k, l, g, h, lp, gp, hp)$
$\wedge$ b1d (n, l, h, k)
$\wedge$ lg (n, l, g)
$\wedge$ b0a (n, l, h, k, j)
$\wedge$ b1a (l, k, j)
$\wedge$ b1a $(l, \text{nth}(h, k), j))$
$\rightarrow$ b1a (lp, k, j)

THEOREM: mrho-preserves-b1a
(molws (n, l, g, h)
$\wedge$ $(i \in \text{nset}(n))$
$\wedge$ $(j \in \text{nset}(n))$
$\wedge$ $(k \in \text{nset}(n))$
$\wedge$ mrhoi $(n, k, l, g, h, lp, gp, hp)$
$\wedge$ b1d (n, l, h, i)
$\wedge$ lg (n, l, g)
$\wedge$ b0a (n, l, h, i, j)
$\wedge$ b1a (l, i, j)
$\wedge$ b1a $(l, \text{nth}(h, i), j)$
$\wedge$ b1b $(n, l, g, h, i, j))$
$\rightarrow$ b1a (lp, i, j)

```
;;;;;;;;;;;;;   b1b   ;;;;;;;;;;;;;;;;

;;;;;;;;;;;;;;;;;common in mole and atom.
```

THEOREM: un8-11-then-un8-12
union-at-n $(lp, r,$ `'(8 9 10 11)`$)$ $\rightarrow$ union-at-n $(lp, r,$ `'(8 9 10 11 12)`$)$

THEOREM: l8-11-k-in-gp34
$((r \in \text{nset}(n)) \wedge \text{lg}(n, l, g) \wedge$ union-at-n $(l, r,$ `'(8 9 10 11)`$))$
$\rightarrow$ union-at-n $(gp, r,$ `'(3 4)`$)$

THEOREM: u-if4
$((u \in \text{nset}(n)) \wedge \text{lg}(n, l, g) \wedge \text{at}(g, u, 4)) \rightarrow (\neg\, \text{at}(l, u, 2))$

THEOREM: l12-then-un10-12
at $(l, u, 12)$ $\rightarrow$ union-at-n $(l, u,$ `'(10 11 12)`$)$

THEOREM: r-neq-k
(union-at-n $(l, k,$ `'(8 9 10 11)`$)$ $\wedge$ at $(l, r, 12)) \rightarrow (k \neq r)$

```
;;;;;;;;;;;;;;;;;common in mole and atom end.
;;;;;;;;;;;;;;;;;Lemmas on hints.
```

THEOREM: ex-hint-in-l8-12
exist-hint-8-12-3-4 (n, l, g, h, j)
$\rightarrow$ union-at-n $(l$, exist-hint-8-12-3-4 (n, l, g, h, j), `'(8 9 10 11 12)`)

THEOREM: ex-hint-in-g34
exist-hint-8-12-3-4 (n, l, g, h, k)
$\rightarrow$ union-at-n $(g$, exist-hint-8-12-3-4 (n, l, g, h, k), `'(3 4)`)

THEOREM: ex-hint-l-g-h
exist-hint-8-12-3-4 (n, l, g, h, j)
$\rightarrow$ (exist-hint-8-12-3-4 $(n, l, g, h, j) \not< \text{nth}(h, j))$

THEOREM: ex-hint-lp-gp-h-in-int-8-12-3-4
exist-hint-8-12-3-4 (n, lp, gp, h, j)
$\rightarrow$ intersect-8-12-3-4-at-n (exist-hint-8-12-3-4 $(n, lp, gp, h, j), lp, gp)$

THEOREM: ex-hint-lp-gp-h-leq-h-j
exist-hint-8-12-3-4 (n, lp, gp, h, j)
$\rightarrow$ (exist-hint-8-12-3-4 $(n, lp, gp, h, j) \not< \text{nth}(h, j))$

THEOREM: ex-hint-not-in-g02
exist-hint-8-12-3-4 (n, l, g, h, k)
$\rightarrow$ ($\neg$ union-at-n $(g$, exist-hint-8-12-3-4 (n, l, g, h, k), `'(0 1 2)`))

THEOREM: hint-wtn
$((r \in \text{nset}(n)) \wedge$ intersect-8-12-3-4-at-n $(r, lp, gp) \wedge (r \not< \text{nth}(h, j)))$
$\rightarrow$ exist-hint-8-12-3-4 (n, lp, gp, h, j)

```
;;;;;;;;;;;;;;;;Lemmas on hints end.
```

THEOREM: lm-k-in-l7-imp
(molws (n, l, g, h)
$\wedge$ $(k \in \text{nset}(n))$
$\wedge$ mrhoi $(n, k, l, g, h, lp, gp, hp)$
$\wedge$ b1d (n, l, h, k)
$\wedge$ lg (n, l, g)
$\wedge$ at $(l, k, 7)$
$\wedge$ b1b $(n, l, g, h, \text{nth}(h, k), j)$
$\wedge$ at $(l, j, 3)$
$\wedge$ union-at-n $(lp, k$, `'(8 9 10 11 12)`))
$\rightarrow$ exist-hint-8-12-3-4 (n, l, g, h, j)

THEOREM: ex-hint-neq-k-imp
(molws (n, l, g, h)
$\wedge$ $(k \in \text{nset}(n))$

$\wedge$ mrhoi $(n, k, l, g, h, lp, gp, hp)$
$\wedge$ exist-hint-8-12-3-4 (n, l, g, h, j)
$\wedge$ $(k \neq$ exist-hint-8-12-3-4 $(n, l, g, h, j)))$
$\rightarrow$ intersect-8-12-3-4-at-n (exist-hint-8-12-3-4 $(n, l, g, h, j), lp, gp)$

THEOREM: ex-hint-neq-k-in-l7
(at $(l, k, \texttt{7})$ $\wedge$ exist-hint-8-12-3-4 $(n, l, g, h, j))$
$\rightarrow$ $(k \neq$ exist-hint-8-12-3-4 $(n, l, g, h, j))$

THEOREM: ex-hint-in-int-8-12-3-4-l7
(molws (n, l, g, h)
$\wedge$ $(k \in$ nset $(n))$
$\wedge$ mrhoi $(n, k, l, g, h, lp, gp, hp)$
$\wedge$ at $(l, k, \texttt{7})$
$\wedge$ exist-hint-8-12-3-4 $(n, l, g, h, j))$
$\rightarrow$ intersect-8-12-3-4-at-n (exist-hint-8-12-3-4 $(n, l, g, h, j), lp, gp)$

THEOREM: ex-hint-wtn-l7
(molws (n, l, g, h)
$\wedge$ $(k \in$ nset $(n))$
$\wedge$ mrhoi $(n, k, l, g, h, lp, gp, hp)$
$\wedge$ at $(l, k, \texttt{7})$
$\wedge$ exist-hint-8-12-3-4 $(n, l, g, h, j))$
$\rightarrow$ exist-hint-8-12-3-4 (n, lp, gp, h, j)

THEOREM: b1b-k-in-l7-imp
(molws (n, l, g, h)
$\wedge$ $(k \in$ nset $(n))$
$\wedge$ mrhoi $(n, k, l, g, h, lp, gp, hp)$
$\wedge$ b1d (n, l, h, k)
$\wedge$ lg (n, l, g)
$\wedge$ at $(l, k, \texttt{7})$
$\wedge$ b1b $(n, l, g, h,$ nth $(h, k), j)$
$\wedge$ at $(l, j, \texttt{3})$
$\wedge$ union-at-n $(lp, k,$ `'(8 9 10 11 12)`$))$
$\rightarrow$ exist-hint-8-12-3-4 (n, lp, gp, h, j)

THEOREM: h-j-leq-k
(molws (n, l, g, h)
$\wedge$ $(j \in$ nset $(n))$
$\wedge$ $(k \in$ nset $(n))$
$\wedge$ mrhoi $(n, k, l, g, h, lp, gp, hp)$
$\wedge$ at $(l, j, \texttt{3})$
$\wedge$ at $(l, k, \texttt{5})$
$\wedge$ b0b (n, l, h, k, j)

$\wedge$ union-at-n $(lp,\ k,\ \texttt{'(8 9 10 11 12)}))$
$\rightarrow$ $(k \not< \text{nth}\,(h,\ j))$

THEOREM: lm-lp8-then-k-in-g34
(listp (l)
$\wedge$ $(n \in \mathbf{N})$
$\wedge$ (length $(l) = n)$
$\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ $(k \in \text{nset}\,(n))$
$\wedge$ lg $(n,\ l,\ g)$
$\wedge$ at $(l,\ k,\ \texttt{5})$
$\wedge$ at $(lp,\ k,\ \texttt{8}))$
$\rightarrow$ union-at-n $(gp,\ k,\ \texttt{'(3 4)})$

EVENT: Disable lm-lp8-then-k-in-g34.

THEOREM: lp8-then-k-in-g34
(molws $(n,\ l,\ g,\ h)$
$\wedge$ $(k \in \text{nset}\,(n))$
$\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ lg $(n,\ l,\ g)$
$\wedge$ at $(l,\ k,\ \texttt{5})$
$\wedge$ at $(lp,\ k,\ \texttt{8}))$
$\rightarrow$ union-at-n $(gp,\ k,\ \texttt{'(3 4)})$

THEOREM: lm-k-in-g34
(molws $(n,\ l,\ g,\ h)$
$\wedge$ $(k \in \text{nset}\,(n))$
$\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ lg $(n,\ l,\ g)$
$\wedge$ lg $(n,\ lp,\ gp)$
$\wedge$ at $(l,\ k,\ \texttt{5})$
$\wedge$ union-at-n $(lp,\ k,\ \texttt{'(8 9 10 11 12)}))$
$\rightarrow$ union-at-n $(gp,\ k,\ \texttt{'(3 4)})$

THEOREM: k-in-g34
(molws $(n,\ l,\ g,\ h)$
$\wedge$ $(k \in \text{nset}\,(n))$
$\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ lg $(n,\ l,\ g)$
$\wedge$ at $(l,\ k,\ \texttt{5})$
$\wedge$ union-at-n $(lp,\ k,\ \texttt{'(8 9 10 11 12)}))$
$\rightarrow$ union-at-n $(gp,\ k,\ \texttt{'(3 4)})$

THEOREM: k-in-int
(molws $(n,\ l,\ g,\ h)$
 ∧ $(k \in \text{nset}\,(n))$
 ∧ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 ∧ lg $(n,\ l,\ g)$
 ∧ at $(l,\ k,\ \texttt{5})$
 ∧ union-at-n $(lp,\ k,\ \texttt{'(8 9 10 11 12)}))$
→ intersect-8-12-3-4-at-n $(k,\ lp,\ gp)$

THEOREM: k-in-l5-imp
(molws $(n,\ l,\ g,\ h)$
 ∧ $(j \in \text{nset}\,(n))$
 ∧ $(k \in \text{nset}\,(n))$
 ∧ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 ∧ lg $(n,\ l,\ g)$
 ∧ at $(l,\ j,\ \texttt{3})$
 ∧ at $(l,\ k,\ \texttt{5})$
 ∧ b0b $(n,\ l,\ h,\ k,\ j)$
 ∧ union-at-n $(lp,\ k,\ \texttt{'(8 9 10 11 12)}))$
→ exist-hint-8-12-3-4 $(n,\ lp,\ gp,\ h,\ j)$

```
;;;This is slow.
```

THEOREM: ex-hint-in-l12
(molws $(n,\ l,\ g,\ h)$
 ∧ $(k \in \text{nset}\,(n))$
 ∧ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 ∧ union-at-n $(l,\ k,\ \texttt{'(8 9 10 11)})$
 ∧ exist-hint-8-12-3-4 $(n,\ l,\ g,\ h,\ j)$
 ∧ at $(l,$ exist-hint-8-12-3-4 $(n,\ l,\ g,\ h,\ j),\ \texttt{12}))$
→ intersect-8-12-3-4-at-n (exist-hint-8-12-3-4 $(n,\ l,\ g,\ h,\ j),\ lp,\ gp)$

THEOREM: r-neq-k-l8-11-k-in-lp8-12
(molws $(n,\ l,\ g,\ h)$
 ∧ $(r \in \text{nset}\,(n))$
 ∧ $(k \in \text{nset}\,(n))$
 ∧ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 ∧ $(r \neq k)$
 ∧ union-at-n $(l,\ r,\ \texttt{'(8 9 10 11)}))$
→ union-at-n $(lp,\ r,\ \texttt{'(8 9 10 11)})$

THEOREM: r-eq-k-l8-11-k-in-lp8-12
(molws $(n,\ l,\ g,\ h)$
 ∧ $(k \in \text{nset}\,(n))$
 ∧ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$

$\wedge$ union-at-n (l, k, `'(8 9 10 11)`))
$\rightarrow$ union-at-n (lp, k, `'(8 9 10 11 12)`)

THEOREM: l8-11-k-in-lp8-12
(molws (n, l, g, h)
$\wedge$ ($r \in$ nset (n))
$\wedge$ ($k \in$ nset (n))
$\wedge$ mrhoi (n, k, l, g, h, lp, gp, hp)
$\wedge$ union-at-n (l, r, `'(8 9 10 11)`))
$\rightarrow$ union-at-n (lp, r, `'(8 9 10 11 12)`)

THEOREM: hint-in-l8-11
(molws (n, l, g, h)
$\wedge$ ($k \in$ nset (n))
$\wedge$ mrhoi (n, k, l, g, h, lp, gp, hp)
$\wedge$ lg (n, l, g)
$\wedge$ exist-hint-8-12-3-4 (n, l, g, h, j)
$\wedge$ union-at-n (l, exist-hint-8-12-3-4 (n, l, g, h, j), `'(8 9 10 11)`))
$\rightarrow$ intersect-8-12-3-4-at-n (exist-hint-8-12-3-4 (n, l, g, h, j), lp, gp)

THEOREM: ex-hint-not-in-l12
(molws (n, l, g, h)
$\wedge$ ($k \in$ nset (n))
$\wedge$ mrhoi (n, k, l, g, h, lp, gp, hp)
$\wedge$ lg (n, l, g)
$\wedge$ exist-hint-8-12-3-4 (n, l, g, h, j)
$\wedge$ ($\neg$ at (l, exist-hint-8-12-3-4 (n, l, g, h, j), `12`)))
$\rightarrow$ intersect-8-12-3-4-at-n (exist-hint-8-12-3-4 (n, l, g, h, j), lp, gp)

THEOREM: ex-hint-in-int-8-12-3-4-l8-11
(molws (n, l, g, h)
$\wedge$ ($k \in$ nset (n))
$\wedge$ mrhoi (n, k, l, g, h, lp, gp, hp)
$\wedge$ lg (n, l, g)
$\wedge$ union-at-n (l, k, `'(8 9 10 11)`)
$\wedge$ exist-hint-8-12-3-4 (n, l, g, h, j))
$\rightarrow$ intersect-8-12-3-4-at-n (exist-hint-8-12-3-4 (n, l, g, h, j), lp, gp)

THEOREM: ex-hint-wtn-l8-11
(molws (n, l, g, h)
$\wedge$ ($k \in$ nset (n))
$\wedge$ mrhoi (n, k, l, g, h, lp, gp, hp)
$\wedge$ lg (n, l, g)
$\wedge$ union-at-n (l, k, `'(8 9 10 11)`)
$\wedge$ exist-hint-8-12-3-4 (n, l, g, h, j))
$\rightarrow$ exist-hint-8-12-3-4 (n, lp, gp, h, j)

THEOREM: k-in-l8-11-imp
(molws $(n,\ l,\ g,\ h)$
 $\wedge$ $(k \in \text{nset}\,(n))$
 $\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 $\wedge$ lg $(n,\ l,\ g)$
 $\wedge$ at $(l,\ j,\ \texttt{3})$
 $\wedge$ b1b $(n,\ l,\ g,\ h,\ k,\ j)$
 $\wedge$ union-at-n $(l,\ k,\ \texttt{'(8 9 10 11)}))$
$\rightarrow$ exist-hint-8-12-3-4 $(n,\ lp,\ gp,\ h,\ j)$

THEOREM: m-lp9-12-k-in-l8-11
(molws $(n,\ l,\ g,\ h)$
 $\wedge$ $(k \in \text{nset}\,(n))$
 $\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 $\wedge$ union-at-n $(lp,\ k,\ \texttt{'(9 10 11 12)}))$
$\rightarrow$ union-at-n $(l,\ k,\ \texttt{'(8 9 10 11)})$

THEOREM: k-in-lp9-12-imp
(molws $(n,\ l,\ g,\ h)$
 $\wedge$ $(k \in \text{nset}\,(n))$
 $\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 $\wedge$ lg $(n,\ l,\ g)$
 $\wedge$ at $(l,\ j,\ \texttt{3})$
 $\wedge$ b1b $(n,\ l,\ g,\ h,\ k,\ j)$
 $\wedge$ union-at-n $(lp,\ k,\ \texttt{'(9 10 11 12)}))$
$\rightarrow$ exist-hint-8-12-3-4 $(n,\ lp,\ gp,\ h,\ j)$

THEOREM: k-not-in-l7-imp
(molws $(n,\ l,\ g,\ h)$
 $\wedge$ $(j \in \text{nset}\,(n))$
 $\wedge$ $(k \in \text{nset}\,(n))$
 $\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 $\wedge$ lg $(n,\ l,\ g)$
 $\wedge$ at $(l,\ j,\ \texttt{3})$
 $\wedge$ $(\neg$ at $(l,\ k,\ \texttt{7}))$
 $\wedge$ b0b $(n,\ l,\ h,\ k,\ j)$
 $\wedge$ b1b $(n,\ l,\ g,\ h,\ k,\ j)$
 $\wedge$ union-at-n $(lp,\ k,\ \texttt{'(8 9 10 11 12)}))$
$\rightarrow$ exist-hint-8-12-3-4 $(n,\ lp,\ gp,\ h,\ j)$

THEOREM: lm1-b1b-i-eq-k-j-neq-k
(molws $(n,\ l,\ g,\ h)$
 $\wedge$ $(j \in \text{nset}\,(n))$
 $\wedge$ $(k \in \text{nset}\,(n))$
 $\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$

$\wedge$ b1d (n, l, h, k)
$\wedge$ lg (n, l, g)
$\wedge$ at $(l, j, \mathtt{3})$
$\wedge$ b0b (n, l, h, k, j)
$\wedge$ b1b (n, l, g, h, k, j)
$\wedge$ b1b $(n, l, g, h, \mathrm{nth}(h, k), j)$
$\wedge$ union-at-n $(lp, k, \mathtt{'(8\ 9\ 10\ 11\ 12)}))$
$\rightarrow$ exist-hint-8-12-3-4 (n, lp, gp, h, j)

THEOREM: ex-hint-lp-gp-h-leq-hp-j
(molws (n, l, g, h)
$\wedge$ $(j \in \mathrm{nset}(n))$
$\wedge$ $(k \in \mathrm{nset}(n))$
$\wedge$ mrhoi $(n, k, l, g, h, lp, gp, hp)$
$\wedge$ $(j \neq k)$
$\wedge$ exist-hint-8-12-3-4 $(n, lp, gp, h, j))$
$\rightarrow$ (exist-hint-8-12-3-4 $(n, lp, gp, h, j) \not< \mathrm{nth}(hp, j))$

THEOREM: j-neq-k-then-hp-eq-h
(molws (n, l, g, h)
$\wedge$ $(j \in \mathrm{nset}(n))$
$\wedge$ $(k \in \mathrm{nset}(n))$
$\wedge$ mrhoi $(n, k, l, g, h, lp, gp, hp)$
$\wedge$ $(j \neq k)$
$\wedge$ exist-hint-8-12-3-4 $(n, lp, gp, h, j))$
$\rightarrow$ exist-hint-8-12-3-4 (n, lp, gp, hp, j)

THEOREM: lm-b1b-i-eq-k-j-neq-k
(molws (n, l, g, h)
$\wedge$ $(j \in \mathrm{nset}(n))$
$\wedge$ $(k \in \mathrm{nset}(n))$
$\wedge$ mrhoi $(n, k, l, g, h, lp, gp, hp)$
$\wedge$ b1d (n, l, h, k)
$\wedge$ $(j \neq k)$
$\wedge$ lg (n, l, g)
$\wedge$ b0b (n, l, h, k, j)
$\wedge$ b1b (n, l, g, h, k, j)
$\wedge$ b1b $(n, l, g, h, \mathrm{nth}(h, k), j)$
$\wedge$ at $(l, j, \mathtt{3})$
$\wedge$ union-at-n $(lp, k, \mathtt{'(8\ 9\ 10\ 11\ 12)}))$
$\rightarrow$ exist-hint-8-12-3-4 (n, lp, gp, hp, j)

THEOREM: b1b-i-eq-k-j-neq-k
(molws (n, l, g, h)
$\wedge$ $(j \in \mathrm{nset}(n))$

$\land$ $(k \in \text{nset}(n))$
$\land$ $\text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\land$ $\text{b1d}(n, l, h, k)$
$\land$ $(j \neq k)$
$\land$ $\text{lg}(n, l, g)$
$\land$ $\text{b0b}(n, l, h, k, j)$
$\land$ $\text{b1b}(n, l, g, h, k, j)$
$\land$ $\text{b1b}(n, l, g, h, \text{nth}(h, k), j))$
$\rightarrow$ $\text{b1b}(n, lp, gp, hp, k, j)$

THEOREM: b1b-i-j-eq-k
$(\text{molws}(n, l, g, h)$
$\land$ $(k \in \text{nset}(n))$
$\land$ $\text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\land$ $\text{b1b}(n, l, g, h, k, k))$
$\rightarrow$ $\text{b1b}(n, lp, gp, hp, k, k)$

```
;;;I wonder if (b1d n l h i) is
;;;better than (b1d n l h k).
```

THEOREM: b1b-i-eq-k
$(\text{molws}(n, l, g, h)$
$\land$ $(j \in \text{nset}(n))$
$\land$ $(k \in \text{nset}(n))$
$\land$ $\text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\land$ $\text{b1d}(n, l, h, k)$
$\land$ $\text{lg}(n, l, g)$
$\land$ $\text{b0b}(n, l, h, k, j)$
$\land$ $\text{b1b}(n, l, g, h, k, j)$
$\land$ $\text{b1b}(n, l, g, h, \text{nth}(h, k), j))$
$\rightarrow$ $\text{b1b}(n, lp, gp, hp, k, j)$

```
;;;I wonder why the following two do not imply
;;;lm-b1b-i-neq-k-j-eq-k.
;;;(prove-lemma k-not-in-l3 (rewrite)
;;;   (implies (and (molws n l g h)
;;;                 (member i (nset n))
;;;                 (member k (nset n))
;;;                 (mrhoi n k l g h lp gp hp)
;;;                 (not (equal i k))
;;;                 (lg n l g)
;;;                 (at lp k 3)
;;;                 (not (at l k 3))
;;;                 (union-at-n l i '(8 9 10 11 12)))
;;;         (exist-hint-8-12-3-4 n lp gp hp k)))
```

```
;;;
;;;(prove-lemma k-in-l3 (rewrite)
;;;   (implies (and (molws n l g h)
;;;                 (member i (nset n))
;;;                 (member k (nset n))
;;;                 (mrhoi n k l g h lp gp hp)
;;;                 (at l k 3)
;;;                 (at lp k 3)
;;;                 (union-at-n l i '(8 9 10 11 12)))
;;;          (exist-hint-8-12-3-4 n lp gp hp k)))
;;;
;;;(prove-lemma lm-b1b-i-neq-k-j-eq-k (rewrite)
;;;   (implies (and (molws n l g h)
;;;                 (member i (nset n))
;;;                 (member k (nset n))
;;;                 (mrhoi n k l g h lp gp hp)
;;;                 (not (equal i k))
;;;                 (lg n l g)
;;;                 (at lp k 3)
;;;                 (union-at-n l i '(8 9 10 11 12)))
;;;          (exist-hint-8-12-3-4 n lp gp hp k)))
```

THEOREM: ex-hint-leq-h-k
exist-hint-8-12-3-4 (n, l, g, h, k)
$\rightarrow$ (exist-hint-8-12-3-4 $(n, l, g, h, k) \not< \text{nth}(h, k))$

THEOREM: h-k-leq-sub1-ex-hint
(exist-hint-8-12-3-4 (n, l, g, h, k)
$\wedge$ $(\text{nth}(h, k) \neq$ exist-hint-8-12-3-4 $(n, l, g, h, k)))$
$\rightarrow$ ((exist-hint-8-12-3-4 $(n, l, g, h, k) - 1) \not< \text{nth}(h, k))$

THEOREM: ex-hint-neq-h-k
(molws (n, l, g, h)
$\wedge$ $(k \in \text{nset}(n))$
$\wedge$ mrhoi $(n, k, l, g, h, lp, gp, hp)$
$\wedge$ exist-hint-8-12-3-4 (n, l, g, h, k)
$\wedge$ at $(l, k, 3)$
$\wedge$ at $(lp, k, 3))$
$\rightarrow$ $(\text{nth}(h, k) \neq$ exist-hint-8-12-3-4 $(n, l, g, h, k))$

THEOREM: lm-hp-k-leq-ex-l-g-h
(molws (n, l, g, h)
$\wedge$ $(k \in \text{nset}(n))$
$\wedge$ mrhoi $(n, k, l, g, h, lp, gp, hp)$

$\wedge$ at $(l,\ k,\ \texttt{3})$
$\wedge$ at $(lp,\ k,\ \texttt{3})$
$\wedge$ exist-hint-8-12-3-4 $(n,\ l,\ g,\ h,\ k))$
$\rightarrow$ ((exist-hint-8-12-3-4 $(n,\ l,\ g,\ h,\ k) - 1) \not< \mathrm{nth}\,(h,\ k))$

THEOREM: ex-cond-l3
(molws $(n,\ l,\ g,\ h)$
$\wedge$ $(k \in \mathrm{nset}\,(n))$
$\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ at $(l,\ k,\ \texttt{3})$
$\wedge$ exist-hint-8-12-3-4 $(n,\ l,\ g,\ h,\ k)$
$\wedge$ ((exist-hint-8-12-3-4 $(n,\ l,\ g,\ h,\ k) - 1) \not< \mathrm{nth}\,(h,\ k)))$
$\rightarrow$ (exist-hint-8-12-3-4 $(n,\ l,\ g,\ h,\ k) \not< \mathrm{nth}\,(hp,\ k))$

THEOREM: hp-k-leq-ex-l-g-h
(molws $(n,\ l,\ g,\ h)$
$\wedge$ $(k \in \mathrm{nset}\,(n))$
$\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ at $(l,\ k,\ \texttt{3})$
$\wedge$ at $(lp,\ k,\ \texttt{3})$
$\wedge$ exist-hint-8-12-3-4 $(n,\ l,\ g,\ h,\ k))$
$\rightarrow$ (exist-hint-8-12-3-4 $(n,\ l,\ g,\ h,\ k) \not< \mathrm{nth}\,(hp,\ k))$

THEOREM: ex-hint-neq-k-in-l3
(at $(l,\ k,\ \texttt{3})$
$\wedge$ union-at-n $(l,$ exist-hint-8-12-3-4 $(n,\ l,\ g,\ h,\ k),$ `'(8 9 10 11 12)`))
$\rightarrow$ $(k \neq$ exist-hint-8-12-3-4 $(n,\ l,\ g,\ h,\ k))$

```
;;;This is successfully proved
;;;by m-gp-same-g and m-lp-same-l.
;;;This is successfully proved by ex-hint-neq-k-imp,
;;;ex-hint-neq-k-in-l3 and ex-hint-in-l8-12.
```

THEOREM: ex-hint-l-g-h-in-int-8-12-3-4
(molws $(n,\ l,\ g,\ h)$
$\wedge$ $(k \in \mathrm{nset}\,(n))$
$\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ at $(l,\ k,\ \texttt{3})$
$\wedge$ exist-hint-8-12-3-4 $(n,\ l,\ g,\ h,\ k))$
$\rightarrow$ intersect-8-12-3-4-at-n (exist-hint-8-12-3-4 $(n,\ l,\ g,\ h,\ k),\ lp,\ gp)$

THEOREM: ex-l-g-h-k-in-l3
(molws $(n,\ l,\ g,\ h)$
$\wedge$ $(k \in \mathrm{nset}\,(n))$
$\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$

$\land$ at $(l,\ k,\ 3)$
$\land$ at $(lp,\ k,\ 3)$
$\land$ exist-hint-8-12-3-4 $(n,\ l,\ g,\ h,\ k))$
$\rightarrow$ exist-hint-8-12-3-4 $(n,\ lp,\ gp,\ hp,\ k)$

THEOREM: lm-k-in-l3
(molws $(n,\ l,\ g,\ h)$
$\land$ $(i \in$ nset $(n))$
$\land$ $(k \in$ nset $(n))$
$\land$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\land$ at $(l,\ k,\ 3)$
$\land$ at $(lp,\ k,\ 3)$
$\land$ union-at-n $(l,\ i,$ `'(8 9 10 11 12)`$)$
$\land$ b1b $(n,\ l,\ g,\ h,\ i,\ k))$
$\rightarrow$ exist-hint-8-12-3-4 $(n,\ lp,\ gp,\ hp,\ k)$

THEOREM: k-in-l3
(molws $(n,\ l,\ g,\ h)$
$\land$ $(i \in$ nset $(n))$
$\land$ $(k \in$ nset $(n))$
$\land$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\land$ $(i \neq k)$
$\land$ at $(l,\ k,\ 3)$
$\land$ b1b $(n,\ l,\ g,\ h,\ i,\ k))$
$\rightarrow$ b1b $(n,\ lp,\ gp,\ hp,\ i,\ k)$

THEOREM: hp-k-leq-i
(molws $(n,\ l,\ g,\ h)$
$\land$ $(i \in$ nset $(n))$
$\land$ $(k \in$ nset $(n))$
$\land$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\land$ at $(l,\ k,\ 2)$
$\land$ at $(lp,\ k,\ 3))$
$\rightarrow$ $(i \not< \text{nth}\,(hp,\ k))$

THEOREM: b1b-u-neq-k
$((u \in$ nset $(n))$
$\land$ $(k \in$ nset $(n))$
$\land$ lg $(n,\ l,\ g)$
$\land$ at $(g,\ u,\ 4)$
$\land$ at $(l,\ k,\ 2))$
$\rightarrow$ $(k \neq u)$

THEOREM: lm-u-in-int-8-12-3-4
(molws $(n,\ l,\ g,\ h)$

$\wedge$ $(u \in \text{nset}(n))$
$\wedge$ $(k \in \text{nset}(n))$
$\wedge$ $\text{mrhoi}(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ $\text{lg}(n,\ l,\ g)$
$\wedge$ $\text{at}(l,\ k,\ 2)$
$\wedge$ $\text{at}(g,\ u,\ 4)$
$\wedge$ $\text{union-at-n}(lp,\ u,\ \texttt{'(8 9 10 11 12)}))$
$\rightarrow$ $\text{intersect-8-12-3-4-at-n}(u,\ lp,\ gp)$

THEOREM: k-neq-u-in-lp8-12
$(\text{molws}(n,\ l,\ g,\ h)$
$\wedge$ $(u \in \text{nset}(n))$
$\wedge$ $(k \in \text{nset}(n))$
$\wedge$ $\text{mrhoi}(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ $(u \neq k)$
$\wedge$ $\text{lg}(n,\ l,\ g)$
$\wedge$ $\text{at}(g,\ u,\ 4))$
$\rightarrow$ $\text{union-at-n}(lp,\ u,\ \texttt{'(8 9 10 11 12)})$

THEOREM: lm1-u-in-int-8-12-3-4
$(\text{molws}(n,\ l,\ g,\ h)$
$\wedge$ $(u \in \text{nset}(n))$
$\wedge$ $(k \in \text{nset}(n))$
$\wedge$ $\text{mrhoi}(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ $\text{lg}(n,\ l,\ g)$
$\wedge$ $\text{at}(l,\ k,\ 2)$
$\wedge$ $\text{at}(g,\ u,\ 4))$
$\rightarrow$ $\text{union-at-n}(lp,\ u,\ \texttt{'(8 9 10 11 12)})$

THEOREM: u-in-int-8-12-3-4
$(\text{molws}(n,\ l,\ g,\ h)$
$\wedge$ $(u \in \text{nset}(n))$
$\wedge$ $(k \in \text{nset}(n))$
$\wedge$ $\text{mrhoi}(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ $\text{lg}(n,\ l,\ g)$
$\wedge$ $\text{at}(l,\ k,\ 2)$
$\wedge$ $\text{at}(g,\ u,\ 4))$
$\rightarrow$ $\text{intersect-8-12-3-4-at-n}(u,\ lp,\ gp)$

```
;;;I wonder why the following does not trigger
;;;molws-ln-lp, molws-ln-gp.
;;;(prove-lemma h-i-in-g34-imp (rewrite)
;;;   (implies (and (molws n l g h)
;;;                 (member k (nset n))
;;;                 (mrhoi n k l g h lp gp hp)
```

```
;;;                 (member (nth h i) (nset n))
;;;                 (lg n l g)
;;;                 (at l k 2)
;;;                 (at lp k 3)
;;;                 (at g (nth h i) 4))
;;;            (exist-hint-8-12-3-4 n lp gp hp k)))
;;;although
;;;(prove-lemma h-i-in-g34-imp (rewrite)
;;;   (implies (and (member (nth h i) (nset n))
;;;                 (molws n l g h)
;;;                 (member k (nset n))
;;;                 (mrhoi n k l g h lp gp hp)
;;;                 (lg n l g)
;;;                 (at l k 2)
;;;                 (at lp k 3)
;;;                 (at g (nth h i) 4))
;;;            (exist-hint-8-12-3-4 n lp gp hp k)))
;;;does.
```

THEOREM: h-i-in-g34-imp

$((\text{nth}\,(h,\,i) \in \text{nset}\,(n))$
$\land\quad \text{molws}\,(n,\,l,\,g,\,h)$
$\land\quad (k \in \text{nset}\,(n))$
$\land\quad \text{mrhoi}\,(n,\,k,\,l,\,g,\,h,\,lp,\,gp,\,hp)$
$\land\quad \text{lg}\,(n,\,l,\,g)$
$\land\quad \text{at}\,(l,\,k,\,2)$
$\land\quad \text{at}\,(lp,\,k,\,3)$
$\land\quad \text{at}\,(g,\,\text{nth}\,(h,\,i),\,4))$
$\to\quad \text{exist-hint-8-12-3-4}\,(n,\,lp,\,gp,\,hp,\,k)$

THEOREM: i-not-in-g34

$((\neg\ \text{union-at-n}\,(g,\,i,\,\texttt{'(3 4)}))$
$\land\quad \text{molws}\,(n,\,l,\,g,\,h)$
$\land\quad (i \in \text{nset}\,(n))$
$\land\quad (k \in \text{nset}\,(n))$
$\land\quad \text{mrhoi}\,(n,\,k,\,l,\,g,\,h,\,lp,\,gp,\,hp)$
$\land\quad \text{lg}\,(n,\,l,\,g)$
$\land\quad \text{at}\,(l,\,k,\,2)$
$\land\quad \text{at}\,(lp,\,k,\,3)$
$\land\quad \text{b1c}\,(n,\,l,\,g,\,h,\,i)$
$\land\quad \text{union-at-n}\,(l,\,i,\,\texttt{'(8 9 10 11 12)}))$
$\to\quad \text{exist-hint-8-12-3-4}\,(n,\,lp,\,gp,\,hp,\,k)$

THEOREM: i-in-int-8-12-3-4

(union-at-n (*g*, *i*, `'(3 4)`)
∧ molws (*n*, *l*, *g*, *h*)
∧ ($i \in$ nset (*n*))
∧ ($k \in$ nset (*n*))
∧ mrhoi (*n*, *k*, *l*, *g*, *h*, *lp*, *gp*, *hp*)
∧ ($i \neq k$)
∧ union-at-n (*l*, *i*, `'(8 9 10 11 12)`))
→ intersect-8-12-3-4-at-n (*i*, *lp*, *gp*)

THEOREM: i-in-g34
(union-at-n (*g*, *i*, `'(3 4)`)
∧ molws (*n*, *l*, *g*, *h*)
∧ ($i \in$ nset (*n*))
∧ ($k \in$ nset (*n*))
∧ mrhoi (*n*, *k*, *l*, *g*, *h*, *lp*, *gp*, *hp*)
∧ ($i \neq k$)
∧ at (*l*, *k*, `2`)
∧ at (*lp*, *k*, `3`)
∧ union-at-n (*l*, *i*, `'(8 9 10 11 12)`))
→ exist-hint-8-12-3-4 (*n*, *lp*, *gp*, *hp*, *k*)

THEOREM: k-in-l2
(molws (*n*, *l*, *g*, *h*)
∧ ($i \in$ nset (*n*))
∧ ($k \in$ nset (*n*))
∧ mrhoi (*n*, *k*, *l*, *g*, *h*, *lp*, *gp*, *hp*)
∧ b1d (*n*, *l*, *h*, *k*)
∧ ($i \neq k$)
∧ lg (*n*, *l*, *g*)
∧ at (*l*, *k*, `2`)
∧ at (*lp*, *k*, `3`)
∧ b1c (*n*, *l*, *g*, *h*, *i*)
∧ union-at-n (*l*, *i*, `'(8 9 10 11 12)`))
→ exist-hint-8-12-3-4 (*n*, *lp*, *gp*, *hp*, *k*)

THEOREM: lp3-then-l3-or-l2
(molws (*n*, *l*, *g*, *h*)
∧ ($k \in$ nset (*n*))
∧ mrhoi (*n*, *k*, *l*, *g*, *h*, *lp*, *gp*, *hp*)
∧ (¬ at (*l*, *k*, `3`))
∧ at (*lp*, *k*, `3`))
→ at (*l*, *k*, `2`)

THEOREM: lm-k-not-in-l3
(molws (*n*, *l*, *g*, *h*)

$\wedge$ $(i \in \text{nset}(n))$
$\wedge$ $(k \in \text{nset}(n))$
$\wedge$ $\text{mrhoi}(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ $\text{b1d}(n,\ l,\ h,\ k)$
$\wedge$ $(i \neq k)$
$\wedge$ $\text{lg}(n,\ l,\ g)$
$\wedge$ $(\neg\ \text{at}(l,\ k,\ \texttt{3}))$
$\wedge$ $\text{at}(lp,\ k,\ \texttt{3})$
$\wedge$ $\text{b1c}(n,\ l,\ g,\ h,\ i)$
$\wedge$ $\text{union-at-n}(l,\ i,\ \texttt{'(8 9 10 11 12)}))$
$\rightarrow$ $\text{exist-hint-8-12-3-4}(n,\ lp,\ gp,\ hp,\ k)$

THEOREM: k-not-in-l3
$(\text{molws}(n,\ l,\ g,\ h)$
$\wedge$ $(i \in \text{nset}(n))$
$\wedge$ $(k \in \text{nset}(n))$
$\wedge$ $\text{mrhoi}(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ $\text{b1d}(n,\ l,\ h,\ k)$
$\wedge$ $(i \neq k)$
$\wedge$ $\text{lg}(n,\ l,\ g)$
$\wedge$ $(\neg\ \text{at}(l,\ k,\ \texttt{3}))$
$\wedge$ $\text{b1b}(n,\ l,\ g,\ h,\ i,\ k)$
$\wedge$ $\text{b1c}(n,\ l,\ g,\ h,\ i))$
$\rightarrow$ $\text{b1b}(n,\ lp,\ gp,\ hp,\ i,\ k)$

THEOREM: b1b-i-neq-k-j-eq-k
$(\text{molws}(n,\ l,\ g,\ h)$
$\wedge$ $(i \in \text{nset}(n))$
$\wedge$ $(k \in \text{nset}(n))$
$\wedge$ $\text{mrhoi}(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ $\text{b1d}(n,\ l,\ h,\ k)$
$\wedge$ $(i \neq k)$
$\wedge$ $\text{lg}(n,\ l,\ g)$
$\wedge$ $\text{b1b}(n,\ l,\ g,\ h,\ i,\ k)$
$\wedge$ $\text{b1c}(n,\ l,\ g,\ h,\ i))$
$\rightarrow$ $\text{b1b}(n,\ lp,\ gp,\ hp,\ i,\ k)$

THEOREM: lm-i-neq-k-in-int-8-12-3-4
$(\text{molws}(n,\ l,\ g,\ h)$
$\wedge$ $(i \in \text{nset}(n))$
$\wedge$ $(k \in \text{nset}(n))$
$\wedge$ $\text{mrhoi}(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ $(i \neq k)$
$\wedge$ $\text{lg}(n,\ l,\ g)$
$\wedge$ $\text{at}(g,\ i,\ \texttt{4}))$

$\rightarrow$ intersect-8-12-3-4-at-n $(i,\ lp,\ gp)$

THEOREM: i-neq-k-in-int-8-12-3-4
(molws $(n,\ l,\ g,\ h)$
 $\land$ $(i \in \text{nset}\,(n))$
 $\land$ $(k \in \text{nset}\,(n))$
 $\land$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 $\land$ $(i \neq k)$
 $\land$ union-at-n $(l,\ i,$ `'(8 9 10 11 12)`$)$
 $\land$ lg $(n,\ l,\ g)$
 $\land$ at $(l,$ exist-hint-8-12-3-4 $(n,\ l,\ g,\ h,\ j),$ `12`$)$
 $\land$ b3a $(l,\ g,$ exist-hint-8-12-3-4 $(n,\ l,\ g,\ h,\ j),\ i))$
$\rightarrow$ intersect-8-12-3-4-at-n $(i,\ lp,\ gp)$

THEOREM: h-j-leq-i
(union-at-n $(l,\ i,$ `'(8 9 10 11 12)`$)$
 $\land$ exist-hint-8-12-3-4 $(n,\ l,\ g,\ h,\ j)$
 $\land$ at $(l,$ exist-hint-8-12-3-4 $(n,\ l,\ g,\ h,\ j),$ `12`$)$
 $\land$ b2a $(l,$ exist-hint-8-12-3-4 $(n,\ l,\ g,\ h,\ j),\ i))$
$\rightarrow$ $(i \not< \text{nth}\,(h,\ j))$

THEOREM: i-neq-k-ex-hint-in-l12
(molws $(n,\ l,\ g,\ h)$
 $\land$ $(i \in \text{nset}\,(n))$
 $\land$ $(k \in \text{nset}\,(n))$
 $\land$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 $\land$ $(i \neq k)$
 $\land$ lg $(n,\ l,\ g)$
 $\land$ union-at-n $(l,\ i,$ `'(8 9 10 11 12)`$)$
 $\land$ exist-hint-8-12-3-4 $(n,\ l,\ g,\ h,\ j)$
 $\land$ at $(l,$ exist-hint-8-12-3-4 $(n,\ l,\ g,\ h,\ j),$ `12`$)$
 $\land$ b2a $(l,$ exist-hint-8-12-3-4 $(n,\ l,\ g,\ h,\ j),\ i)$
 $\land$ b3a $(l,\ g,$ exist-hint-8-12-3-4 $(n,\ l,\ g,\ h,\ j),\ i))$
$\rightarrow$ exist-hint-8-12-3-4 $(n,\ lp,\ gp,\ h,\ j)$

THEOREM: i-neq-k-ex-hint-not-in-l12
(molws $(n,\ l,\ g,\ h)$
 $\land$ $(k \in \text{nset}\,(n))$
 $\land$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 $\land$ lg $(n,\ l,\ g)$
 $\land$ exist-hint-8-12-3-4 $(n,\ l,\ g,\ h,\ j)$
 $\land$ $(\neg$ at $(l,$ exist-hint-8-12-3-4 $(n,\ l,\ g,\ h,\ j),$ `12`$)))$
$\rightarrow$ exist-hint-8-12-3-4 $(n,\ lp,\ gp,\ h,\ j)$

THEOREM: lm1-b1b-i-j-neq-k

$(\text{molws}\,(n,\ l,\ g,\ h)$
$\land\quad (i \in \text{nset}\,(n))$
$\land\quad (j \in \text{nset}\,(n))$
$\land\quad (k \in \text{nset}\,(n))$
$\land\quad \text{mrhoi}\,(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\land\quad (i \neq k)$
$\land\quad \text{lg}\,(n,\ l,\ g)$
$\land\quad \text{union-at-n}\,(l,\ i,\ $ `'(8 9 10 11 12)` $)$
$\land\quad \text{exist-hint-8-12-3-4}\,(n,\ l,\ g,\ h,\ j)$
$\land\quad \text{b2a}\,(l,\ \text{exist-hint-8-12-3-4}\,(n,\ l,\ g,\ h,\ j),\ i)$
$\land\quad \text{b3a}\,(l,\ g,\ \text{exist-hint-8-12-3-4}\,(n,\ l,\ g,\ h,\ j),\ i))$
$\rightarrow\quad \text{exist-hint-8-12-3-4}\,(n,\ lp,\ gp,\ h,\ j)$

THEOREM: lm-b1b-i-j-neq-k
$(\text{molws}\,(n,\ l,\ g,\ h)$
$\land\quad (i \in \text{nset}\,(n))$
$\land\quad (j \in \text{nset}\,(n))$
$\land\quad (k \in \text{nset}\,(n))$
$\land\quad \text{mrhoi}\,(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\land\quad (i \neq k)$
$\land\quad (j \neq k)$
$\land\quad \text{lg}\,(n,\ l,\ g)$
$\land\quad \text{union-at-n}\,(l,\ i,\ $ `'(8 9 10 11 12)` $)$
$\land\quad \text{b2a}\,(l,\ \text{exist-hint-8-12-3-4}\,(n,\ l,\ g,\ h,\ j),\ i)$
$\land\quad \text{b3a}\,(l,\ g,\ \text{exist-hint-8-12-3-4}\,(n,\ l,\ g,\ h,\ j),\ i)$
$\land\quad \text{exist-hint-8-12-3-4}\,(n,\ l,\ g,\ h,\ j))$
$\rightarrow\quad \text{exist-hint-8-12-3-4}\,(n,\ lp,\ gp,\ hp,\ j)$

THEOREM: b1b-i-j-neq-k
$(\text{molws}\,(n,\ l,\ g,\ h)$
$\land\quad (i \in \text{nset}\,(n))$
$\land\quad (j \in \text{nset}\,(n))$
$\land\quad (k \in \text{nset}\,(n))$
$\land\quad \text{mrhoi}\,(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\land\quad (i \neq k)$
$\land\quad (j \neq k)$
$\land\quad \text{lg}\,(n,\ l,\ g)$
$\land\quad \text{b1b}\,(n,\ l,\ g,\ h,\ i,\ j)$
$\land\quad \text{b2a}\,(l,\ \text{exist-hint-8-12-3-4}\,(n,\ l,\ g,\ h,\ j),\ i)$
$\land\quad \text{b3a}\,(l,\ g,\ \text{exist-hint-8-12-3-4}\,(n,\ l,\ g,\ h,\ j),\ i))$
$\rightarrow\quad \text{b1b}\,(n,\ lp,\ gp,\ hp,\ i,\ j)$

THEOREM: b1b-i-neq-k
$(\text{molws}\,(n,\ l,\ g,\ h)$
$\land\quad (i \in \text{nset}\,(n))$

$\wedge \quad (j \in \text{nset}(n))$
$\wedge \quad (k \in \text{nset}(n))$
$\wedge \quad \text{mrhoi}(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge \quad \text{b1d}(n,\ l,\ h,\ k)$
$\wedge \quad (i \neq k)$
$\wedge \quad \text{lg}(n,\ l,\ g)$
$\wedge \quad \text{b0b}(n,\ l,\ h,\ i,\ j)$
$\wedge \quad \text{b1b}(n,\ l,\ g,\ h,\ i,\ j)$
$\wedge \quad \text{b1c}(n,\ l,\ g,\ h,\ i)$
$\wedge \quad \text{b2a}(l,\ \text{exist-hint-8-12-3-4}(n,\ l,\ g,\ h,\ j),\ i)$
$\wedge \quad \text{b3a}(l,\ g,\ \text{exist-hint-8-12-3-4}(n,\ l,\ g,\ h,\ j),\ i))$
$\rightarrow \quad \text{b1b}(n,\ lp,\ gp,\ hp,\ i,\ j)$

```
;;;I wonder if (b1d n l h i) and
;;;(b1d n l h j) are better than
;;;(b1d n l h k).
;;;(b1b n l g h (nth h i) j) and (b1b n l g h (nth h k) j).
;;;What about (b2a l (exist-hint-8-12-3-4 n l g h j) i)
;;;           (b3a l g (exist-hint-8-12-3-4 n l g h j) i) ?
```

THEOREM: mrho-preserves-b1b
$(\text{molws}(n,\ l,\ g,\ h)$
$\wedge \quad (i \in \text{nset}(n))$
$\wedge \quad (j \in \text{nset}(n))$
$\wedge \quad (k \in \text{nset}(n))$
$\wedge \quad \text{mrhoi}(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge \quad \text{b1d}(n,\ l,\ h,\ k)$
$\wedge \quad \text{lg}(n,\ l,\ g)$
$\wedge \quad \text{b0b}(n,\ l,\ h,\ i,\ j)$
$\wedge \quad \text{b1b}(n,\ l,\ g,\ h,\ \text{nth}(h,\ k),\ j)$
$\wedge \quad \text{b1b}(n,\ l,\ g,\ h,\ i,\ j)$
$\wedge \quad \text{b1c}(n,\ l,\ g,\ h,\ i)$
$\wedge \quad \text{b2a}(l,\ \text{exist-hint-8-12-3-4}(n,\ l,\ g,\ h,\ j),\ i)$
$\wedge \quad \text{b3a}(l,\ g,\ \text{exist-hint-8-12-3-4}(n,\ l,\ g,\ h,\ j),\ i))$
$\rightarrow \quad \text{b1b}(n,\ lp,\ gp,\ hp,\ i,\ j)$

```
;;;;;;;;;;;;;   b1c   ;;;;;;;;;;;;;;;;

;;;;;;;;;;;;;common in mole and atom.
```

THEOREM: not-g34-then-not-g4
$(\neg\ \text{union-at-n}(g,\ i,\ \texttt{'(3 4)})) \rightarrow (\neg\ \text{at}(g,\ i,\ \texttt{4}))$

THEOREM: contra-if4

$((j \in \mathrm{nset}(n)) \wedge \mathrm{lg}(n, l, g) \wedge \mathrm{at}(g, j, 4))$
$\rightarrow$ union-at-n $(l, j,$ `'(9 10 11 12)`$)$

```
;;;;;;;;;;;;;;common in mole and atom end.
```

THEOREM: lp8-not-l5-then-l7
$(\mathrm{molws}(n, l, g, h)$
$\wedge$ $(k \in \mathrm{nset}(n))$
$\wedge$ $\mathrm{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\wedge$ $(\neg\, \mathrm{at}(l, k, 5))$
$\wedge$ $\mathrm{at}(lp, k, 8))$
$\rightarrow$ $\mathrm{at}(l, k, 7)$

THEOREM: lp8-not-g34-then-k-in-l7
$(\mathrm{molws}(n, l, g, h)$
$\wedge$ $(k \in \mathrm{nset}(n))$
$\wedge$ $\mathrm{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\wedge$ $\mathrm{lg}(n, l, g)$
$\wedge$ $\mathrm{at}(lp, k, 8)$
$\wedge$ union-at-n $(lp, k,$ `'(8 9 10 11 12)`$)$
$\wedge$ $(\neg$ union-at-n $(gp, k,$ `'(3 4)`$)))$
$\rightarrow$ $\mathrm{at}(l, k, 7)$

THEOREM: lm-k-in-l7
$(\mathrm{molws}(n, l, g, h)$
$\wedge$ $(k \in \mathrm{nset}(n))$
$\wedge$ $\mathrm{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\wedge$ $\mathrm{lg}(n, l, g)$
$\wedge$ $\mathrm{lg}(n, lp, gp)$
$\wedge$ union-at-n $(lp, k,$ `'(8 9 10 11 12)`$)$
$\wedge$ $(\neg$ union-at-n $(gp, k,$ `'(3 4)`$)))$
$\rightarrow$ $\mathrm{at}(l, k, 7)$

THEOREM: k-in-l7
$(\mathrm{molws}(n, l, g, h)$
$\wedge$ $(k \in \mathrm{nset}(n))$
$\wedge$ $\mathrm{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\wedge$ $\mathrm{lg}(n, l, g)$
$\wedge$ union-at-n $(lp, k,$ `'(8 9 10 11 12)`$)$
$\wedge$ $(\neg$ union-at-n $(gp, k,$ `'(3 4)`$)))$
$\rightarrow$ $\mathrm{at}(l, k, 7)$

THEOREM: h-k-cond-l7
$(\mathrm{molws}(n, l, g, h)$

$\wedge \quad (k \in \text{nset}\,(n))$
$\wedge \quad \text{mrhoi}\,(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge \quad \text{at}\,(l,\ k,\ \texttt{7})$
$\wedge \quad \text{union-at-n}\,(lp,\ k,\ \texttt{'(8 9 10 11 12)}))$
$\rightarrow \quad (\text{nth}\,(hp,\ k) = \text{nth}\,(h,\ k))$

THEOREM: lm-h-k-g4
$(\text{molws}\,(n,\ l,\ g,\ h)$
$\wedge \quad (k \in \text{nset}\,(n))$
$\wedge \quad \text{mrhoi}\,(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge \quad \text{b1d}\,(n,\ l,\ h,\ k)$
$\wedge \quad \text{lg}\,(n,\ l,\ g)$
$\wedge \quad \text{at}\,(l,\ k,\ \texttt{7})$
$\wedge \quad \text{union-at-n}\,(lp,\ k,\ \texttt{'(8 9 10 11 12)}))$
$\rightarrow \quad ((\text{nth}\,(hp,\ k) \in \text{nset}\,(n)) \wedge \text{at}\,(g,\ \text{nth}\,(hp,\ k),\ \texttt{4}))$

THEOREM: h-k-g4
$(\text{molws}\,(n,\ l,\ g,\ h)$
$\wedge \quad (k \in \text{nset}\,(n))$
$\wedge \quad \text{mrhoi}\,(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge \quad \text{b1d}\,(n,\ l,\ h,\ k)$
$\wedge \quad \text{lg}\,(n,\ l,\ g)$
$\wedge \quad \text{union-at-n}\,(lp,\ k,\ \texttt{'(8 9 10 11 12)})$
$\wedge \quad (\neg\ \text{union-at-n}\,(gp,\ k,\ \texttt{'(3 4)})))$
$\rightarrow \quad ((\text{nth}\,(hp,\ k) \in \text{nset}\,(n)) \wedge \text{at}\,(g,\ \text{nth}\,(hp,\ k),\ \texttt{4}))$

THEOREM: lm1-i-eq-k-then-h-k-neq-k
$(\text{molws}\,(n,\ l,\ g,\ h)$
$\wedge \quad (k \in \text{nset}\,(n))$
$\wedge \quad \text{mrhoi}\,(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge \quad \text{b1d}\,(n,\ l,\ h,\ k)$
$\wedge \quad \text{lg}\,(n,\ l,\ g)$
$\wedge \quad \text{at}\,(l,\ k,\ \texttt{7})$
$\wedge \quad \text{at}\,(g,\ \text{nth}\,(hp,\ k),\ \texttt{4}))$
$\rightarrow \quad (\neg\ \text{at}\,(hp,\ k,\ k))$

```
;;;Need k-in-l7.
```

THEOREM: lm-i-eq-k-then-h-k-neq-k
$(\text{molws}\,(n,\ l,\ g,\ h)$
$\wedge \quad (k \in \text{nset}\,(n))$
$\wedge \quad \text{mrhoi}\,(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge \quad \text{b1d}\,(n,\ l,\ h,\ k)$
$\wedge \quad \text{lg}\,(n,\ l,\ g)$
$\wedge \quad \text{at}\,(l,\ k,\ \texttt{7})$

$\wedge$ union-at-n (lp, k, '(8 9 10 11 12))
$\wedge$ ($\neg$ union-at-n (gp, k, '(3 4))))
$\rightarrow$ ($\neg$ at (hp, k, k))

THEOREM: i-eq-k-then-h-k-neq-k
(molws (n, l, g, h)
$\wedge$ ($k \in$ nset (n))
$\wedge$ mrhoi (n, k, l, g, h, lp, gp, hp)
$\wedge$ b1d (n, l, h, k)
$\wedge$ lg (n, l, g)
$\wedge$ union-at-n (lp, k, '(8 9 10 11 12))
$\wedge$ ($\neg$ union-at-n (gp, k, '(3 4))))
$\rightarrow$ ($\neg$ at (hp, k, k))

THEOREM: b1c-i-eq-k-hp-k-neq-k
(molws (n, l, g, h)
$\wedge$ ($k \in$ nset (n))
$\wedge$ mrhoi (n, k, l, g, h, lp, gp, hp)
$\wedge$ b1d (n, l, h, k)
$\wedge$ ($\neg$ at (hp, k, k))
$\wedge$ lg (n, l, g)
$\wedge$ union-at-n (lp, k, '(8 9 10 11 12))
$\wedge$ ($\neg$ union-at-n (gp, k, '(3 4))))
$\rightarrow$ ((nth (hp, k) $\in$ nset (n)) $\wedge$ at (gp, nth (hp, k), 4))

THEOREM: b1c-i-eq-k
(molws (n, l, g, h)
$\wedge$ ($k \in$ nset (n))
$\wedge$ mrhoi (n, k, l, g, h, lp, gp, hp)
$\wedge$ b1d (n, l, h, k)
$\wedge$ lg (n, l, g))
$\rightarrow$ b1c (n, lp, gp, hp, k)

THEOREM: l9-11-then-in-lp9-12
(molws (n, l, g, h)
$\wedge$ ($k \in$ nset (n))
$\wedge$ mrhoi (n, k, l, g, h, lp, gp, hp)
$\wedge$ ($\neg$ at (l, k, 12))
$\wedge$ union-at-n (l, k, '(9 10 11 12)))
$\rightarrow$ union-at-n (lp, k, '(9 10 11 12))

THEOREM: k-in-lp9-12
(molws (n, l, g, h)
$\wedge$ ($k \in$ nset (n))
$\wedge$ mrhoi (n, k, l, g, h, lp, gp, hp)

$\land$ lg(n, l, g)
$\land$ at$(g, k, \texttt{4})$
$\land$ $(\neg$ at$(l, k, \texttt{12})))$
$\rightarrow$ union-at-n$(lp, k, \texttt{'(9 10 11 12)})$

THEOREM: lm-k-not-in-l12-imp
(molws(n, l, g, h)
$\land$ $(k \in$ nset$(n))$
$\land$ mrhoi$(n, k, l, g, h, lp, gp, hp)$
$\land$ lg(n, l, g)
$\land$ lg(n, lp, gp)
$\land$ at$(g, k, \texttt{4})$
$\land$ $(\neg$ at$(l, k, \texttt{12})))$
$\rightarrow$ at$(gp, k, \texttt{4})$

THEOREM: k-not-in-l12-imp
(molws(n, l, g, h)
$\land$ $(k \in$ nset$(n))$
$\land$ mrhoi$(n, k, l, g, h, lp, gp, hp)$
$\land$ lg(n, l, g)
$\land$ at$(g, k, \texttt{4})$
$\land$ $(\neg$ at$(l, k, \texttt{12})))$
$\rightarrow$ at$(gp, k, \texttt{4})$

THEOREM: k-not-in-l12
(molws(n, l, g, h)
$\land$ $(i \in$ nset$(n))$
$\land$ $(k \in$ nset$(n))$
$\land$ mrhoi$(n, k, l, g, h, lp, gp, hp)$
$\land$ b3a(l, g, k, i)
$\land$ union-at-n$(l, i, \texttt{'(8 9 10 11 12)})$
$\land$ $(\neg$ union-at-n$(g, i, \texttt{'(3 4)})))$
$\rightarrow$ $(\neg$ at$(l, k, \texttt{12}))$

THEOREM: lm1-b1c-i-neq-k-h-i-eq-k
(molws(n, l, g, h)
$\land$ $(i \in$ nset$(n))$
$\land$ $(k \in$ nset$(n))$
$\land$ mrhoi$(n, k, l, g, h, lp, gp, hp)$
$\land$ (nth$(h, i) = k)$
$\land$ lg(n, l, g)
$\land$ b1c(n, l, g, h, i)
$\land$ b3a$(l, g,$ nth$(h, i), i)$
$\land$ union-at-n$(l, i, \texttt{'(8 9 10 11 12)})$
$\land$ $(\neg$ union-at-n$(g, i, \texttt{'(3 4)})))$
$\rightarrow$ at$(gp, k, \texttt{4})$

THEOREM: lm-b1c-i-neq-k-h-i-eq-k
(molws $(n,\ l,\ g,\ h)$
 $\land$ $(i \in \text{nset}\,(n))$
 $\land$ $(k \in \text{nset}\,(n))$
 $\land$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 $\land$ at $(h,\ i,\ k)$
 $\land$ lg $(n,\ l,\ g)$
 $\land$ b1c $(n,\ l,\ g,\ h,\ i)$
 $\land$ b3a $(l,\ g,\ \text{nth}\,(h,\ i),\ i)$
 $\land$ union-at-n $(l,\ i,$ '(8 9 10 11 12))
 $\land$ $(\neg$ union-at-n $(g,\ i,$ '(3 4))))
$\rightarrow$ $((\text{nth}\,(h,\ i) \in \text{nset}\,(n)) \land \text{at}\,(gp,\ \text{nth}\,(h,\ i),\ 4))$

THEOREM: b3a-h-rholemma
(molws $(n,\ l,\ g,\ h)$
 $\land$ $(i \in \text{nset}\,(n))$
 $\land$ $(k \in \text{nset}\,(n))$
 $\land$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 $\land$ $(i \neq k)$
 $\land$ b3a $(l,\ g,\ \text{nth}\,(h,\ i),\ i))$
$\rightarrow$ b3a $(l,\ g,\ \text{nth}\,(hp,\ i),\ i)$

```
;;;m-l-same-lp and m-gp-same-g are used.
```

THEOREM: b1c-i-neq-k-h-i-eq-k
(molws $(n,\ l,\ g,\ h)$
 $\land$ $(i \in \text{nset}\,(n))$
 $\land$ $(k \in \text{nset}\,(n))$
 $\land$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 $\land$ $(i \neq k)$
 $\land$ at $(h,\ i,\ k)$
 $\land$ lg $(n,\ l,\ g)$
 $\land$ b1c $(n,\ l,\ g,\ h,\ i)$
 $\land$ b3a $(l,\ g,\ \text{nth}\,(h,\ i),\ i))$
$\rightarrow$ b1c $(n,\ lp,\ gp,\ hp,\ i)$

THEOREM: lm-b1c-i-h-i-neq-k
(molws $(n,\ l,\ g,\ h)$
 $\land$ $(i \in \text{nset}\,(n))$
 $\land$ $(k \in \text{nset}\,(n))$
 $\land$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 $\land$ $(\neg\ \text{at}\,(h,\ i,\ k))$
 $\land$ b1c $(n,\ l,\ g,\ h,\ i)$
 $\land$ union-at-n $(l,\ i,$ '(8 9 10 11 12))
 $\land$ $(\neg$ union-at-n $(g,\ i,$ '(3 4))))
$\rightarrow$ $((\text{nth}\,(h,\ i) \in \text{nset}\,(n)) \land \text{at}\,(gp,\ \text{nth}\,(h,\ i),\ 4))$

THEOREM: b1c-i-h-i-neq-k

$$(\text{molws}(n, l, g, h) \land (i \in \text{nset}(n)) \land (k \in \text{nset}(n)) \land \text{mrhoi}(n, k, l, g, h, lp, gp, hp) \land (i \neq k) \land (\neg \text{at}(h, i, k)) \land \text{b1c}(n, l, g, h, i)) \rightarrow \text{b1c}(n, lp, gp, hp, i)$$

THEOREM: b1c-i-neq-k

$$(\text{molws}(n, l, g, h) \land (i \in \text{nset}(n)) \land (k \in \text{nset}(n)) \land \text{mrhoi}(n, k, l, g, h, lp, gp, hp) \land (i \neq k) \land \text{lg}(n, l, g) \land \text{b1c}(n, l, g, h, i) \land \text{b3a}(l, g, \text{nth}(h, i), i)) \rightarrow \text{b1c}(n, lp, gp, hp, i)$$

THEOREM: mrho-preserves-b1c

$$(\text{molws}(n, l, g, h) \land (i \in \text{nset}(n)) \land (k \in \text{nset}(n)) \land \text{mrhoi}(n, k, l, g, h, lp, gp, hp) \land \text{b1d}(n, l, h, k) \land \text{lg}(n, l, g) \land \text{b1c}(n, l, g, h, i) \land \text{b3a}(l, g, \text{nth}(h, i), i)) \rightarrow \text{b1c}(n, lp, gp, hp, i)$$

```
;;;;;;;;;;;;;   b1d   ;;;;;;;;;;;;;;;;
```

THEOREM: remainder-quotient

$$(x \textbf{ mod } (1 + x)) = \text{fix}(x)$$

THEOREM: lm1-member-remainder

$$(x \not< n) \rightarrow ((1 + x) \notin \text{nset}(n - 1))$$

THEOREM: lm-member-remainder

$$((1 + x) \in \text{nset}(n - 1)) \rightarrow ((1 + (x \textbf{ mod } n)) \in \text{nset}(n - 1))$$

THEOREM: member-remainder

$$(j \in \text{nset}(n)) \rightarrow ((1 + ((j - 1) \textbf{ mod } n)) \in \text{nset}(n))$$

THEOREM: one-nset
$(n \not\simeq 0) \rightarrow (1 \in \text{nset}(n))$

THEOREM: lm-b1d-i-eq-k
$(\text{listp}(l)$
$\land\ \text{listp}(h)$
$\land\ (n \in \mathbf{N})$
$\land\ (\text{nth}(h, k) \in \mathbf{N})$
$\land\ (\text{length}(l) = n)$
$\land\ (\text{length}(h) = n)$
$\land\ (k \in \text{nset}(n))$
$\land\ \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\land\ \text{b1d}(n, l, h, k))$
$\rightarrow\ \text{b1d}(n, lp, hp, k)$

THEOREM: b1d-i-eq-k
$(\text{molws}(n, l, g, h)$
$\land\ (k \in \text{nset}(n))$
$\land\ \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\land\ \text{b1d}(n, l, h, k))$
$\rightarrow\ \text{b1d}(n, lp, hp, k)$

THEOREM: b1d-neq-k
$(\text{molws}(n, l, g, h)$
$\land\ (i \in \text{nset}(n))$
$\land\ (k \in \text{nset}(n))$
$\land\ \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\land\ (i \neq k)$
$\land\ \text{b1d}(n, l, h, i))$
$\rightarrow\ \text{b1d}(n, lp, hp, i)$

THEOREM: mrhoi-preserves-b1d
$(\text{molws}(n, l, g, h)$
$\land\ (i \in \text{nset}(n))$
$\land\ (k \in \text{nset}(n))$
$\land\ \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\land\ \text{b1d}(n, l, h, i))$
$\rightarrow\ \text{b1d}(n, lp, hp, i)$

```
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;   b2.ev   ;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;;;;;;;;;;;   b2a   ;;;;;;;;;;;;;;;;

;* i-eq-k-j-neq-k
```

Theorem: j-lt-h-k
(molws (n, l, g, h)
$\wedge$ $(j \in \text{nset}(n))$
$\wedge$ $(k \in \text{nset}(n))$
$\wedge$ mrhoi $(n, k, l, g, h, lp, gp, hp)$
$\wedge$ $(j < k)$
$\wedge$ at $(l, k, \texttt{9})$
$\wedge$ union-at-n $(lp, k,$ `'(10 11 12)`$))$
$\rightarrow$ $(j < \text{nth}(h, k))$

Theorem: lm-case-k-in-l9
(molws (n, l, g, h)
$\wedge$ $(j \in \text{nset}(n))$
$\wedge$ $(k \in \text{nset}(n))$
$\wedge$ mrhoi $(n, k, l, g, h, lp, gp, hp)$
$\wedge$ $(j \neq k)$
$\wedge$ $(j < k)$
$\wedge$ b2b (l, h, k, j)
$\wedge$ at $(l, k, \texttt{9})$
$\wedge$ $(j < \text{nth}(h, k))$
$\wedge$ union-at-n $(lp, k,$ `'(10 11 12)`$))$
$\rightarrow$ $(\neg$ union-at-n $(l, j,$ `'(5 6 7 8 9 10 11 12)`$)))$

```
;;;In the rewrite rule a set of hypotheses is replaced
;;;by anther set of formulas. Thus if in a proof
;;;intended beforehand there is a formula belonging to
;;;more than one set of hypotheses which are expected to
;;;be replaced, Bmp is very likely to be unsuccessful.
;;;If j is not equal to k and the k's entry in l is
;;;between 10 and 12, then the j's entry in lp is not
;;;between 5 and 12.
```

Theorem: case-k-in-l9
(molws (n, l, g, h)
$\wedge$ $(j \in \text{nset}(n))$
$\wedge$ $(k \in \text{nset}(n))$
$\wedge$ mrhoi $(n, k, l, g, h, lp, gp, hp)$
$\wedge$ $(j \neq k)$
$\wedge$ $(j < k)$
$\wedge$ b2b (l, h, k, j)
$\wedge$ at $(l, k, \texttt{9})$
$\wedge$ union-at-n $(lp, k,$ `'(10 11 12)`$))$
$\rightarrow$ $(\neg$ union-at-n $(lp, j,$ `'(5 6 7 8 9 10 11 12)`$)))$

```
;;;need un10-11-then-un10-12.
```

Theorem: case-k-in-l10-11
(molws $(n,\ l,\ g,\ h)$
 $\wedge$ $(j \in \text{nset}\,(n))$
 $\wedge$ $(k \in \text{nset}\,(n))$
 $\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 $\wedge$ $(j \neq k)$
 $\wedge$ $(j < k)$
 $\wedge$ b2a $(l,\ k,\ j)$
 $\wedge$ union-at-n $(l,\ k,$ '(10 11)))
$\rightarrow$ ($\neg$ union-at-n $(lp,\ j,$ '(5 6 7 8 9 10 11 12)))

Theorem: k-in-l10-11-or-l9
(molws $(n,\ l,\ g,\ h)$
 $\wedge$ $(k \in \text{nset}\,(n))$
 $\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 $\wedge$ union-at-n $(lp,\ k,$ '(10 11 12))
 $\wedge$ ($\neg$ union-at-n $(l,\ k,$ '(10 11))))
$\rightarrow$ at $(l,\ k,$ 9)

Theorem: lm-b2a-i-eq-k-j-neq-k
(molws $(n,\ l,\ g,\ h)$
 $\wedge$ $(j \in \text{nset}\,(n))$
 $\wedge$ $(k \in \text{nset}\,(n))$
 $\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 $\wedge$ $(j \neq k)$
 $\wedge$ $(j < k)$
 $\wedge$ lg $(n,\ l,\ g)$
 $\wedge$ b2a $(l,\ k,\ j)$
 $\wedge$ b2b $(l,\ h,\ k,\ j)$
 $\wedge$ union-at-n $(lp,\ k,$ '(10 11 12)))
$\rightarrow$ ($\neg$ union-at-n $(lp,\ j,$ '(5 6 7 8 9 10 11 12)))

```
;;;I proved
;;;(prove-lemma lm-b2a-i-eq-k-j-neq-k (rewrite)
;;;   (implies (and (molws n l g h)
;;;                 (member j (nset n))
;;;                 (member k (nset n))
;;;                 (mrhoi n k l g h lp gp hp)
;;;                 (not (equal j k))
;;;                 (lessp j k)
;;;                 (lg n l g)
;;;                 (b2a l k j)
;;;                 (b2b l h k j)
;;;                 (union-at-n lp k '(10 11 12)))
;;;             (not (union-at-n l j '(5 6 7 8 9 10 11 12)))))
```

```
;;;and tried to prove the following lemma counting on
;;;m-lp-same-l, but it was unsuccessful.
```

THEOREM: b2a-i-eq-k-j-neq-k
(molws (n, l, g, h)
 ∧ ($j \in$ nset (n))
 ∧ ($k \in$ nset (n))
 ∧ mrhoi (n, k, l, g, h, lp, gp, hp)
 ∧ ($j \neq k$)
 ∧ ($j < k$)
 ∧ lg (n, l, g)
 ∧ b2b (l, h, k, j)
 ∧ b2a (l, k, j))
→ b2a (lp, k, j)

```
;* i-neq-k-j-eq-k

;;;If the k's entry in l is not 4 and
;;;the k's entry in lp is between 5 and 7, then
;;;the k's entry in l is between 5 and 7.
```

THEOREM: m-k-in-lp5-7-not-l4-then-l5-7
(molws (n, l, g, h)
 ∧ ($k \in$ nset (n))
 ∧ mrhoi (n, k, l, g, h, lp, gp, hp)
 ∧ (¬ at (l, k, 4))
 ∧ union-at-n (lp, k, `'(5 6 7)`))
→ union-at-n (l, k, `'(5 6 7)`)

```
;;;If the k's entry in lp is between 5 and 7 then
;;;the k's entry in l is  certainly between 5 and 12.
```

THEOREM: m-k-in-lp5-7-then-l5-11
(molws (n, l, g, h)
 ∧ ($k \in$ nset (n))
 ∧ mrhoi (n, k, l, g, h, lp, gp, hp)
 ∧ (¬ at (l, k, 4))
 ∧ union-at-n (lp, k, `'(5 6 7)`))
→ union-at-n (l, k, `'(5 6 7 8 9 10 11)`)

```
;;;If the k's entry in lp is 8 then k's entry
;;;in l is either 5 or 7.
```

THEOREM: m-lp8-k-in-l57

(molws (n, l, g, h)
 ∧ ($k \in$ nset (n))
 ∧ mrhoi (n, k, l, g, h, lp, gp, hp)
 ∧ at (lp, k, 8))
→ union-at-n (l, k, '(5 7))

```
;;;If the k's entry in lp is 8,
;;;then the k's entry in l is between 5 and 11.
```

Theorem: m-k-in-lp8-then-l5-11
(molws (n, l, g, h)
 ∧ ($k \in$ nset (n))
 ∧ mrhoi (n, k, l, g, h, lp, gp, hp)
 ∧ at (lp, k, 8))
→ union-at-n (l, k, '(5 6 7 8 9 10 11))

```
;;;If the k's entry in lp is between 9 and 12,
;;;then the k's entry in l is between 5 and 12.
```

Theorem: m-k-in-lp9-12-then-l5-11
(molws (n, l, g, h)
 ∧ ($k \in$ nset (n))
 ∧ mrhoi (n, k, l, g, h, lp, gp, hp)
 ∧ union-at-n (lp, k, '(9 10 11 12)))
→ union-at-n (l, k, '(5 6 7 8 9 10 11))

```
;;;If the k's entry in l is 4 an the k's entry in lp is
;;;between 5 and 12, then the k's entry in l is
;;;between 5 and 11.
```

Theorem: m-k-in-l5-11
(molws (n, l, g, h)
 ∧ ($k \in$ nset (n))
 ∧ mrhoi (n, k, l, g, h, lp, gp, hp)
 ∧ (¬ at (l, k, 4))
 ∧ union-at-n (lp, k, '(5 6 7 8 9 10 11 12)))
→ union-at-n (l, k, '(5 6 7 8 9 10 11))

Theorem: m-k-not-in-l4
(molws (n, l, g, h)
 ∧ ($k \in$ nset (n))
 ∧ mrhoi (n, k, l, g, h, lp, gp, hp)
 ∧ (¬ at (l, k, 4))
 ∧ (¬ union-at-n (l, k, '(5 6 7 8 9 10 11 12))))
→ (¬ union-at-n (lp, k, '(5 6 7 8 9 10 11 12)))

THEOREM: m-k-not-in-lp5-12
(molws $(n,\ l,\ g,\ h)$
 $\land$ $(i \in \text{nset}\,(n))$
 $\land$ $(k \in \text{nset}\,(n))$
 $\land$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 $\land$ b1a $(l,\ i,\ k)$
 $\land$ union-at-n $(l,\ i,$ `'(10 11 12)`$)$
 $\land$ $(\neg$ union-at-n $(l,\ k,$ `'(5 6 7 8 9 10 11 12)`$)))$
$\rightarrow$ $(\neg$ union-at-n $(lp,\ k,$ `'(5 6 7 8 9 10 11 12)`$))$

THEOREM: lm-b2a-i-neq-k-j-eq-k
(molws $(n,\ l,\ g,\ h)$
 $\land$ $(i \in \text{nset}\,(n))$
 $\land$ $(k \in \text{nset}\,(n))$
 $\land$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 $\land$ $(i \neq k)$
 $\land$ $(k < i)$
 $\land$ b1a $(l,\ i,\ k)$
 $\land$ b2a $(l,\ i,\ k)$
 $\land$ union-at-n $(lp,\ i,$ `'(10 11 12)`$))$
$\rightarrow$ $(\neg$ union-at-n $(lp,\ k,$ `'(5 6 7 8 9 10 11 12)`$))$

THEOREM: b2a-i-neq-k-j-eq-k
(molws $(n,\ l,\ g,\ h)$
 $\land$ $(i \in \text{nset}\,(n))$
 $\land$ $(k \in \text{nset}\,(n))$
 $\land$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 $\land$ $(i \neq k)$
 $\land$ $(k < i)$
 $\land$ b1a $(l,\ i,\ k)$
 $\land$ b2a $(l,\ i,\ k))$
$\rightarrow$ b2a $(lp,\ i,\ k)$

`;* i-j-neq-k-neq-k`

THEOREM: b2a-i-j-neq-k
(molws $(n,\ l,\ g,\ h)$
 $\land$ $(i \in \text{nset}\,(n))$
 $\land$ $(j \in \text{nset}\,(n))$
 $\land$ $(k \in \text{nset}\,(n))$
 $\land$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 $\land$ $(i \neq k)$
 $\land$ $(j \neq k)$
 $\land$ $(j < i)$

$\wedge$ b2a $(l, i, j))$
$\rightarrow$ b2a (lp, i, j)

THEOREM: b2a-i-neq-k
(molws (n, l, g, h)
$\wedge$ $(i \in$ nset $(n))$
$\wedge$ $(j \in$ nset $(n))$
$\wedge$ $(k \in$ nset $(n))$
$\wedge$ mrhoi $(n, k, l, g, h, lp, gp, hp)$
$\wedge$ $(i \neq k)$
$\wedge$ $(j < i)$
$\wedge$ b1a (l, i, j)
$\wedge$ b2a (l, i, j)
$\wedge$ b2b $(l, h, i, j))$
$\rightarrow$ b2a (lp, i, j)

THEOREM: b2a-i-eq-k
(molws (n, l, g, h)
$\wedge$ $(j \in$ nset $(n))$
$\wedge$ $(k \in$ nset $(n))$
$\wedge$ mrhoi $(n, k, l, g, h, lp, gp, hp)$
$\wedge$ $(j < k)$
$\wedge$ lg (n, l, g)
$\wedge$ b2a (l, k, j)
$\wedge$ b2b $(l, h, k, j))$
$\rightarrow$ b2a (lp, k, j)

THEOREM: mrho-preserves-b2a
(molws (n, l, g, h)
$\wedge$ $(i \in$ nset $(n))$
$\wedge$ $(j \in$ nset $(n))$
$\wedge$ $(k \in$ nset $(n))$
$\wedge$ mrhoi $(n, k, l, g, h, lp, gp, hp)$
$\wedge$ $(j < i)$
$\wedge$ lg (n, l, g)
$\wedge$ b1a (l, i, j)
$\wedge$ b2a (l, i, j)
$\wedge$ b2b $(l, h, i, j))$
$\rightarrow$ b2a (lp, i, j)

```
;;;;;;;;;;;;;   b2b   ;;;;;;;;;;;;;;;;
```

```
;;;;;;;;;;;;;Common in atom and mole.
```

THEOREM: l9-then-un8-12
at $(l, i,$ `9`$)$ $\rightarrow$ union-at-n $(l, i,$ `'(8 9 10 11 12)`$)$

`;;;;;;;;;;;;;Common in atom and mole end.`

THEOREM: lg-nth-h-k
(molws $(n,\ l,\ g,\ h)$
 ∧ $(j \in \text{nset}\,(n))$
 ∧ $(k \in \text{nset}\,(n))$
 ∧ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 ∧ lg $(n,\ l,\ g)$
 ∧ at $(h,\ k,\ j)$
 ∧ union-at-n $(g,\ \text{nth}\,(h,\ k),$ `'(0 1)`$))$
 → $(\neg$ union-at-n $(l,\ j,$ `'(5 6 7 8 9 10 11 12)`$))$

THEOREM: l9-g01
(molws $(n,\ l,\ g,\ h)$
 ∧ $(j \in \text{nset}\,(n))$
 ∧ $(k \in \text{nset}\,(n))$
 ∧ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 ∧ at $(h,\ k,\ j)$
 ∧ at $(l,\ k,\ 9)$
 ∧ at $(lp,\ k,\ 9))$
 → union-at-n $(g,\ \text{nth}\,(h,\ k),$ `'(0 1)`$)$

THEOREM: l9-nth-h-k-eq-j
(at $(h,\ k,\ j)$
 ∧ molws $(n,\ l,\ g,\ h)$
 ∧ $(j \in \text{nset}\,(n))$
 ∧ $(k \in \text{nset}\,(n))$
 ∧ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 ∧ lg $(n,\ l,\ g)$
 ∧ at $(l,\ k,\ 9)$
 ∧ at $(lp,\ k,\ 9))$
 → $(\neg$ union-at-n $(l,\ j,$ `'(5 6 7 8 9 10 11 12)`$))$

THEOREM: lm-j-not-in-l5-12
(molws $(n,\ l,\ g,\ h)$
 ∧ $(j \in \text{nset}\,(n))$
 ∧ $(k \in \text{nset}\,(n))$
 ∧ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 ∧ $(j < k)$
 ∧ lg $(n,\ l,\ g)$
 ∧ b2b $(l,\ h,\ k,\ j)$
 ∧ at $(l,\ k,\ 9)$
 ∧ at $(lp,\ k,\ 9)$
 ∧ $((j - 1) < \text{nth}\,(h,\ k)))$
 → $(\neg$ union-at-n $(l,\ j,$ `'(5 6 7 8 9 10 11 12)`$))$

THEOREM: cond-l9
$(\text{molws}(n, l, g, h)$
$\land \quad (j \in \text{nset}(n))$
$\land \quad (k \in \text{nset}(n))$
$\land \quad \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\land \quad \text{at}(l, k, \texttt{9})$
$\land \quad (j < \text{nth}(hp, k)))$
$\rightarrow \quad ((j - 1) < \text{nth}(h, k))$

THEOREM: j-not-in-l5-12
$(\text{molws}(n, l, g, h)$
$\land \quad (j \in \text{nset}(n))$
$\land \quad (k \in \text{nset}(n))$
$\land \quad \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\land \quad (j < k)$
$\land \quad \text{lg}(n, l, g)$
$\land \quad \text{b2b}(l, h, k, j)$
$\land \quad \text{at}(l, k, \texttt{9})$
$\land \quad \text{at}(lp, k, \texttt{9})$
$\land \quad (j < \text{nth}(hp, k)))$
$\rightarrow \quad (\neg\ \text{union-at-n}(l, j, \texttt{'(5 6 7 8 9 10 11 12)}))$

THEOREM: k-in-l9
$(\text{molws}(n, l, g, h)$
$\land \quad (j \in \text{nset}(n))$
$\land \quad (k \in \text{nset}(n))$
$\land \quad \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\land \quad \text{at}(lp, k, \texttt{9})$
$\land \quad (j < \text{nth}(hp, k)))$
$\rightarrow \quad \text{at}(l, k, \texttt{9})$

```
;;;The order of the hints if crucial.
```

THEOREM: lm-b2b-i-eq-k-j-neq-k
$(\text{molws}(n, l, g, h)$
$\land \quad (j \in \text{nset}(n))$
$\land \quad (k \in \text{nset}(n))$
$\land \quad \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\land \quad (j \neq k)$
$\land \quad (j < k)$
$\land \quad \text{lg}(n, l, g)$
$\land \quad \text{b2b}(l, h, k, j)$
$\land \quad \text{at}(lp, k, \texttt{9})$
$\land \quad (j < \text{nth}(hp, k)))$
$\rightarrow \quad (\neg\ \text{union-at-n}(lp, j, \texttt{'(5 6 7 8 9 10 11 12)}))$

THEOREM: b2b-i-eq-k-j-neq-k
$(\text{molws}(n, l, g, h)$
$\land \quad (j \in \text{nset}(n))$
$\land \quad (k \in \text{nset}(n))$
$\land \quad \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\land \quad (j \neq k)$
$\land \quad (j < k)$
$\land \quad \text{lg}(n, l, g)$
$\land \quad \text{b2b}(l, h, k, j))$
$\rightarrow \quad \text{b2b}(lp, hp, k, j)$

THEOREM: b2b-i-eq-k
$(\text{molws}(n, l, g, h)$
$\land \quad (j \in \text{nset}(n))$
$\land \quad (k \in \text{nset}(n))$
$\land \quad \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\land \quad (j < k)$
$\land \quad \text{lg}(n, l, g)$
$\land \quad \text{b2b}(l, h, k, j))$
$\rightarrow \quad \text{b2b}(lp, hp, k, j)$

THEOREM: not-k-in-l5-12-imp
$(\text{molws}(n, l, g, h)$
$\land \quad (i \in \text{nset}(n))$
$\land \quad (k \in \text{nset}(n))$
$\land \quad \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\land \quad \text{b1a}(l, i, k)$
$\land \quad \text{at}(l, i, \texttt{9})$
$\land \quad (\neg\, \text{union-at-n}(l, k, \texttt{'(5 6 7 8 9 10 11 12)})))$
$\rightarrow \quad (\neg\, \text{union-at-n}(lp, k, \texttt{'(5 6 7 8 9 10 11 12)}))$

```
;;;The order of hyptheses is crucial.
```

THEOREM: lm-b2b-i-neq-k-j-eq-k
$(\text{molws}(n, l, g, h)$
$\land \quad (i \in \text{nset}(n))$
$\land \quad (k \in \text{nset}(n))$
$\land \quad \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\land \quad (i \neq k)$
$\land \quad (k < i)$
$\land \quad \text{b1a}(l, i, k)$
$\land \quad \text{b2b}(l, h, i, k)$
$\land \quad \text{at}(l, i, \texttt{9})$
$\land \quad (k < \text{nth}(h, i)))$
$\rightarrow \quad (\neg\, \text{union-at-n}(lp, k, \texttt{'(5 6 7 8 9 10 11 12)}))$

THEOREM: b2b-i-neq-k-j-eq-k
$(\mathrm{molws}(n, l, g, h)$
$\land \quad (i \in \mathrm{nset}(n))$
$\land \quad (k \in \mathrm{nset}(n))$
$\land \quad \mathrm{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\land \quad (i \neq k)$
$\land \quad (k < i)$
$\land \quad \mathrm{b1a}(l, i, k)$
$\land \quad \mathrm{b2b}(l, h, i, k))$
$\to \quad \mathrm{b2b}(lp, hp, i, k)$

```
;;;The position of (member k (nset n)) is
;;;crucial to trigger rholemmas.
```

THEOREM: b2b-i-j-neq-k
$(\mathrm{molws}(n, l, g, h)$
$\land \quad (i \in \mathrm{nset}(n))$
$\land \quad (j \in \mathrm{nset}(n))$
$\land \quad (k \in \mathrm{nset}(n))$
$\land \quad \mathrm{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\land \quad (i \neq k)$
$\land \quad (j \neq k)$
$\land \quad (j < i)$
$\land \quad \mathrm{b2b}(l, h, i, j))$
$\to \quad \mathrm{b2b}(lp, hp, i, j)$

THEOREM: b2b-i-neq-k
$(\mathrm{molws}(n, l, g, h)$
$\land \quad (i \in \mathrm{nset}(n))$
$\land \quad (j \in \mathrm{nset}(n))$
$\land \quad (k \in \mathrm{nset}(n))$
$\land \quad \mathrm{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\land \quad (i \neq k)$
$\land \quad (j < i)$
$\land \quad \mathrm{b1a}(l, i, j)$
$\land \quad \mathrm{b2b}(l, h, i, j))$
$\to \quad \mathrm{b2b}(lp, hp, i, j)$

THEOREM: mrho-preserves-b2b
$(\mathrm{molws}(n, l, g, h)$
$\land \quad (i \in \mathrm{nset}(n))$
$\land \quad (j \in \mathrm{nset}(n))$
$\land \quad (k \in \mathrm{nset}(n))$
$\land \quad \mathrm{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\land \quad (j < i)$

$\wedge$ lg $(n,\ l,\ g)$
$\wedge$ b1a $(l,\ i,\ j)$
$\wedge$ b2b $(l,\ h,\ i,\ j))$
$\rightarrow$ b2b $(lp,\ hp,\ i,\ j)$

```
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;   b3.ev   ;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;;;;;;;;;;;   b3a   ;;;;;;;;;;;;;;;;

;* i-neq-k-j-eq-k
```

THEOREM: lm-b3a-k-in-l9-11
(molws $(n,\ l,\ g,\ h)$
$\wedge$ $(i \in$ nset $(n))$
$\wedge$ $(k \in$ nset $(n))$
$\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ lg $(n,\ l,\ g)$
$\wedge$ b3a $(l,\ g,\ i,\ k)$
$\wedge$ at $(l,\ i,$ `12`$)$
$\wedge$ union-at-n $(l,\ k,$ `'(5 6 7 8 9 10 11)`$))$
$\rightarrow$ union-at-n $(l,\ k,$ `'(9 10 11)`$)$

THEOREM: b3a-k-in-l9-11
(molws $(n,\ l,\ g,\ h)$
$\wedge$ $(i \in$ nset $(n))$
$\wedge$ $(k \in$ nset $(n))$
$\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ lg $(n,\ l,\ g)$
$\wedge$ b1a $(l,\ i,\ k)$
$\wedge$ b3a $(l,\ g,\ i,\ k)$
$\wedge$ at $(l,\ i,$ `12`$)$
$\wedge$ union-at-n $(lp,\ k,$ `'(5 6 7 8 9 10 11 12)`$))$
$\rightarrow$ union-at-n $(l,\ k,$ `'(9 10 11)`$)$

THEOREM: m-k-in-lp9-12
(molws $(n,\ l,\ g,\ h)$
$\wedge$ $(k \in$ nset $(n))$
$\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ union-at-n $(l,\ k,$ `'(9 10 11)`$))$
$\rightarrow$ union-at-n $(lp,\ k,$ `'(9 10 11 12)`$)$

THEOREM: lm-b3a-i-neq-k-j-eq-k
(molws $(n,\ l,\ g,\ h)$
$\wedge$ $(i \in$ nset $(n))$
$\wedge$ $(k \in$ nset $(n))$

$\wedge$ mrhoi $(n, k, l, g, h, lp, gp, hp)$
$\wedge$ lg (n, l, g)
$\wedge$ lg (n, lp, gp)
$\wedge$ b1a (l, i, k)
$\wedge$ b3a (l, g, i, k)
$\wedge$ at $(l, i, \texttt{12})$
$\wedge$ union-at-n $(lp, k,$ `'(5 6 7 8 9 10 11 12)`$))$
$\rightarrow$ at $(gp, k, \texttt{4})$

THEOREM: b3a-i-neq-k-j-eq-k
(molws (n, l, g, h)
$\wedge$ $(i \in$ nset $(n))$
$\wedge$ $(k \in$ nset $(n))$
$\wedge$ mrhoi $(n, k, l, g, h, lp, gp, hp)$
$\wedge$ $(i \neq k)$
$\wedge$ lg (n, l, g)
$\wedge$ b1a (l, i, k)
$\wedge$ b3a $(l, g, i, k))$
$\rightarrow$ b3a (lp, gp, i, k)

```
;* i-eq-k-j-neq-k
```

THEOREM: cond-lp12
(molws (n, l, g, h)
$\wedge$ $(k \in$ nset $(n))$
$\wedge$ $(j \in$ nset $(n))$
$\wedge$ mrhoi $(n, k, l, g, h, lp, gp, hp)$
$\wedge$ at $(lp, k, \texttt{12})$
$\wedge$ at $(l, k, \texttt{11}))$
$\rightarrow$ $(j <$ nth $(h, k))$

THEOREM: b3a-j-in-l5-12
(molws (n, l, g, h)
$\wedge$ $(j \in$ nset $(n))$
$\wedge$ $(k \in$ nset $(n))$
$\wedge$ mrhoi $(n, k, l, g, h, lp, gp, hp)$
$\wedge$ lg (n, l, g)
$\wedge$ b3b (l, g, h, k, j)
$\wedge$ at $(l, k, \texttt{11})$
$\wedge$ at $(lp, k, \texttt{12})$
$\wedge$ union-at-n $(l, j,$ `'(5 6 7 8 9 10 11 12)`$))$
$\rightarrow$ at $(g, j, \texttt{4})$

THEOREM: m-k-in-l11
(molws (n, l, g, h)

$\wedge$ $(k \in \mathrm{nset}(n))$
$\wedge$ $\mathrm{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\wedge$ $\mathrm{at}(lp, k, \texttt{12}))$
$\rightarrow$ $\mathrm{at}(l, k, \texttt{11})$

THEOREM: lm-b3a-i-eq-k-j-neq-k
$(\mathrm{molws}(n, l, g, h)$
$\wedge$ $(j \in \mathrm{nset}(n))$
$\wedge$ $(k \in \mathrm{nset}(n))$
$\wedge$ $\mathrm{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\wedge$ $(j \neq k)$
$\wedge$ $\mathrm{lg}(n, l, g)$
$\wedge$ $\mathrm{b3b}(l, g, h, k, j)$
$\wedge$ $\mathrm{at}(lp, k, \texttt{12})$
$\wedge$ $\mathrm{union\text{-}at\text{-}n}(lp, j, \texttt{'(5 6 7 8 9 10 11 12)}))$
$\rightarrow$ $\mathrm{at}(g, j, \texttt{4})$

THEOREM: b3a-i-eq-k-j-neq-k
$(\mathrm{molws}(n, l, g, h)$
$\wedge$ $(j \in \mathrm{nset}(n))$
$\wedge$ $(k \in \mathrm{nset}(n))$
$\wedge$ $\mathrm{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\wedge$ $(j \neq k)$
$\wedge$ $\mathrm{lg}(n, l, g)$
$\wedge$ $\mathrm{b3b}(l, g, h, k, j))$
$\rightarrow$ $\mathrm{b3a}(lp, gp, k, j)$

```
;* i-j-neq-k
```

THEOREM: b3a-i-j-neq-k
$(\mathrm{molws}(n, l, g, h)$
$\wedge$ $(i \in \mathrm{nset}(n))$
$\wedge$ $(j \in \mathrm{nset}(n))$
$\wedge$ $(k \in \mathrm{nset}(n))$
$\wedge$ $\mathrm{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\wedge$ $(i \neq k)$
$\wedge$ $(j \neq k)$
$\wedge$ $\mathrm{b3a}(l, g, i, j))$
$\rightarrow$ $\mathrm{b3a}(lp, gp, i, j)$

THEOREM: b3a-i-neq-k
$(\mathrm{molws}(n, l, g, h)$
$\wedge$ $(i \in \mathrm{nset}(n))$
$\wedge$ $(j \in \mathrm{nset}(n))$

$\wedge \quad (k \in \text{nset}(n))$
$\wedge \quad \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\wedge \quad (i \neq k)$
$\wedge \quad \text{lg}(n, l, g)$
$\wedge \quad \text{b1a}(l, i, j)$
$\wedge \quad \text{b3a}(l, g, i, j))$
$\rightarrow \quad \text{b3a}(lp, gp, i, j)$

```
;* i-j-eq-k
```

THEOREM: b3a-i-j-eq-k
$(\text{molws}(n, l, g, h)$
$\wedge \quad (k \in \text{nset}(n))$
$\wedge \quad \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\wedge \quad \text{lg}(n, l, g)$
$\wedge \quad \text{b3a}(l, g, k, k)$
$\wedge \quad \text{b3b}(l, g, h, k, k))$
$\rightarrow \quad \text{b3a}(lp, gp, k, k)$

THEOREM: b3a-i-eq-k
$(\text{molws}(n, l, g, h)$
$\wedge \quad (j \in \text{nset}(n))$
$\wedge \quad (k \in \text{nset}(n))$
$\wedge \quad \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\wedge \quad \text{lg}(n, l, g)$
$\wedge \quad \text{b3a}(l, g, k, j)$
$\wedge \quad \text{b3b}(l, g, h, k, j))$
$\rightarrow \quad \text{b3a}(lp, gp, k, j)$

THEOREM: mrho-preserves-b3a
$(\text{molws}(n, l, g, h)$
$\wedge \quad (i \in \text{nset}(n))$
$\wedge \quad (j \in \text{nset}(n))$
$\wedge \quad (k \in \text{nset}(n))$
$\wedge \quad \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\wedge \quad \text{lg}(n, l, g)$
$\wedge \quad \text{b1a}(l, i, j)$
$\wedge \quad \text{b3a}(l, g, i, j)$
$\wedge \quad \text{b3b}(l, g, h, i, j))$
$\rightarrow \quad \text{b3a}(lp, gp, i, j)$

```
;;;;;;;;;;;;;   b3b   ;;;;;;;;;;;;;;;;

;;;;;;;;;;;;;;common in atom and mole.
```

THEOREM: l10-then-un10-12
at $(l,\ k,\ \texttt{10}) \rightarrow$ union-at-n $(l,\ k,\ \texttt{'(10 11 12)})$

THEOREM: l11-then-un9-12
at $(lp,\ k,\ \texttt{11}) \rightarrow$ union-at-n $(lp,\ k,\ \texttt{'(9 10 11 12)})$

THEOREM: l11-then-un8-12
at $(l,\ i,\ \texttt{11}) \rightarrow$ union-at-n $(l,\ i,\ \texttt{'(8 9 10 11 12)})$

```
;;;;;;;;;;;;;common in atom and mole end.

;* i-neq-k-j-eq-k
```

THEOREM: lm-b3b-k-in-l9-11
(molws $(n,\ l,\ g,\ h)$
 ∧ $(i \in \text{nset}(n))$
 ∧ $(k \in \text{nset}(n))$
 ∧ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 ∧ lg $(n,\ l,\ g)$
 ∧ b3b $(l,\ g,\ h,\ i,\ k)$
 ∧ at $(l,\ i,\ \texttt{11})$
 ∧ $(k < \text{nth}(h,\ i))$
 ∧ union-at-n $(l,\ k,\ \texttt{'(5 6 7 8 9 10 11)}))$
→ union-at-n $(l,\ k,\ \texttt{'(9 10 11)})$

THEOREM: b3b-k-in-l9-11
(molws $(n,\ l,\ g,\ h)$
 ∧ $(i \in \text{nset}(n))$
 ∧ $(k \in \text{nset}(n))$
 ∧ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 ∧ b1a $(l,\ i,\ k)$
 ∧ b3b $(l,\ g,\ h,\ i,\ k)$
 ∧ lg $(n,\ l,\ g)$
 ∧ at $(l,\ i,\ \texttt{11})$
 ∧ $(k < \text{nth}(h,\ i))$
 ∧ union-at-n $(lp,\ k,\ \texttt{'(5 6 7 8 9 10 11 12)}))$
→ union-at-n $(l,\ k,\ \texttt{'(9 10 11)})$

THEOREM: lm-b3b-i-neq-k-j-eq-k
(molws $(n,\ l,\ g,\ h)$
 ∧ $(i \in \text{nset}(n))$
 ∧ $(k \in \text{nset}(n))$
 ∧ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 ∧ lg $(n,\ l,\ g)$

$\wedge$ lg (n, lp, gp)
$\wedge$ b1a (l, i, k)
$\wedge$ b3b (l, g, h, i, k)
$\wedge$ at (l, i, `11`)
$\wedge$ ($k <$ nth (h, i))
$\wedge$ union-at-n (lp, k, `'(5 6 7 8 9 10 11 12)`))
$\rightarrow$ at (gp, k, `4`)

THEOREM: b3b-i-neq-k-j-eq-k
(molws (n, l, g, h)
$\wedge$ ($i \in$ nset (n))
$\wedge$ ($k \in$ nset (n))
$\wedge$ mrhoi (n, k, l, g, h, lp, gp, hp)
$\wedge$ ($i \neq k$)
$\wedge$ lg (n, l, g)
$\wedge$ b1a (l, i, k)
$\wedge$ b3b (l, g, h, i, k))
$\rightarrow$ b3b (lp, gp, hp, i, k)

```
;* i-eq-k-j-neq-k
```

THEOREM: j-in-g4
(molws (n, l, g, h)
$\wedge$ ($j \in$ nset (n))
$\wedge$ ($k \in$ nset (n))
$\wedge$ mrhoi (n, k, l, g, h, lp, gp, hp)
$\wedge$ lg (n, l, g)
$\wedge$ at (h, k, j)
$\wedge$ ($\neg$ union-at-n (g, nth (h, k), `'(2 3)`))
$\wedge$ union-at-n (l, j, `'(5 6 7 8 9 10 11 12)`))
$\rightarrow$ at (g, j, `4`)

THEOREM: l11-g14
(molws (n, l, g, h)
$\wedge$ ($j \in$ nset (n))
$\wedge$ ($k \in$ nset (n))
$\wedge$ mrhoi (n, k, l, g, h, lp, gp, hp)
$\wedge$ at (h, k, j)
$\wedge$ at (l, k, `11`)
$\wedge$ at (lp, k, `11`))
$\rightarrow$ ($\neg$ union-at-n (g, nth (h, k), `'(2 3)`))

THEOREM: l11-nth-h-k-eq-j
(at (h, k, j)

$\wedge$ molws $(n,\ l,\ g,\ h)$
$\wedge$ $(j \in \text{nset}\,(n))$
$\wedge$ $(k \in \text{nset}\,(n))$
$\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ lg $(n,\ l,\ g)$
$\wedge$ at $(l,\ k,\ 11)$
$\wedge$ at $(lp,\ k,\ 11)$
$\wedge$ union-at-n $(l,\ j,$ `'(5 6 7 8 9 10 11 12)`$))$
$\rightarrow$ at $(g,\ j,\ 4)$

THEOREM: lm-j-in-l5-12
(molws $(n,\ l,\ g,\ h)$
$\wedge$ $(j \in \text{nset}\,(n))$
$\wedge$ $(k \in \text{nset}\,(n))$
$\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ lg $(n,\ l,\ g)$
$\wedge$ b3b $(l,\ g,\ h,\ k,\ j)$
$\wedge$ at $(l,\ k,\ 11)$
$\wedge$ at $(lp,\ k,\ 11)$
$\wedge$ $((j - 1) < \text{nth}\,(h,\ k))$
$\wedge$ union-at-n $(l,\ j,$ `'(5 6 7 8 9 10 11 12)`$))$
$\rightarrow$ at $(g,\ j,\ 4)$

THEOREM: cond-l11
(molws $(n,\ l,\ g,\ h)$
$\wedge$ $(j \in \text{nset}\,(n))$
$\wedge$ $(k \in \text{nset}\,(n))$
$\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ at $(l,\ k,\ 11)$
$\wedge$ $(j < \text{nth}\,(hp,\ k)))$
$\rightarrow$ $((j - 1) < \text{nth}\,(h,\ k))$

THEOREM: j-in-l5-12
(molws $(n,\ l,\ g,\ h)$
$\wedge$ $(j \in \text{nset}\,(n))$
$\wedge$ $(k \in \text{nset}\,(n))$
$\wedge$ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
$\wedge$ $(j \neq k)$
$\wedge$ lg $(n,\ l,\ g)$
$\wedge$ b3b $(l,\ g,\ h,\ k,\ j)$
$\wedge$ at $(l,\ k,\ 11)$
$\wedge$ at $(lp,\ k,\ 11)$
$\wedge$ $(j < \text{nth}\,(hp,\ k))$
$\wedge$ union-at-n $(l,\ j,$ `'(5 6 7 8 9 10 11 12)`$))$
$\rightarrow$ at $(g,\ j,\ 4)$

THEOREM: j-leq-add1k-then-k-not-in-l10
(molws $(n,\ l,\ g,\ h)$
 ∧ $(k \in \text{nset}\,(n))$
 ∧ $(j \in \text{nset}\,(n))$
 ∧ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 ∧ b2a $(l,\ k,\ j)$
 ∧ union-at-n $(l,\ j,$ `'(5 6 7 8 9 10 11 12)`$)$
 ∧ $(j \neq k)$
 ∧ $(j < (1 + k)))$
 → $(\neg$ at $(l,\ k,$ `10`$))$

THEOREM: not-j-leq-add1k-then-k-not-in-l10
(molws $(n,\ l,\ g,\ h)$
 ∧ $(k \in \text{nset}\,(n))$
 ∧ $(j \in \text{nset}\,(n))$
 ∧ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 ∧ at $(lp,\ k,$ `11`$)$
 ∧ $(j < \text{nth}\,(hp,\ k))$
 ∧ $(j \not< (1 + k)))$
 → $(\neg$ at $(l,\ k,$ `10`$))$

```
;;;The order of (member k (nset n)) and
;;;(member j (nset n)) are switched deliberately.
```

THEOREM: k-not-in-l10
(molws $(n,\ l,\ g,\ h)$
 ∧ $(k \in \text{nset}\,(n))$
 ∧ $(j \in \text{nset}\,(n))$
 ∧ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 ∧ $(j \neq k)$
 ∧ b2a $(l,\ k,\ j)$
 ∧ union-at-n $(l,\ j,$ `'(5 6 7 8 9 10 11 12)`$)$
 ∧ at $(lp,\ k,$ `11`$)$
 ∧ $(j < \text{nth}\,(hp,\ k)))$
 → $(\neg$ at $(l,\ k,$ `10`$))$

THEOREM: lp11-then-l11-or-l10
(molws $(n,\ l,\ g,\ h)$
 ∧ $(k \in \text{nset}\,(n))$
 ∧ mrhoi $(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp)$
 ∧ at $(lp,\ k,$ `11`$)$
 ∧ $(\neg$ at $(l,\ k,$ `10`$)))$
 → at $(l,\ k,$ `11`$)$

```
;;;When the order of (member j (nset n)) and
```

```
;;;(member k (nset n)) is switched, the order of
;;;hints must be switched, in order to make the proof
;;;successful.
```

THEOREM: b3b-k-in-l11
(molws (n, l, g, h)
 ∧ $(j \in \text{nset}(n))$
 ∧ $(k \in \text{nset}(n))$
 ∧ mrhoi $(n, k, l, g, h, lp, gp, hp)$
 ∧ $(j \neq k)$
 ∧ b2a (l, k, j)
 ∧ union-at-n $(l, j,$ `'(5 6 7 8 9 10 11 12)`)
 ∧ at $(lp, k,$ `11`)
 ∧ $(j < \text{nth}(hp, k)))$
→ at $(l, k,$ `11`)

```
;;;When the order of (member j (nset n)) and
;;;(member k (nset n)) is switched then the order of
;;;hints must be switched in order to make the proof
;;;successful.
```

THEOREM: lm-b3b-i-eq-k-j-neq-k
(molws (n, l, g, h)
 ∧ $(j \in \text{nset}(n))$
 ∧ $(k \in \text{nset}(n))$
 ∧ mrhoi $(n, k, l, g, h, lp, gp, hp)$
 ∧ $(j \neq k)$
 ∧ lg (n, l, g)
 ∧ b2a (l, k, j)
 ∧ b3b (l, g, h, k, j)
 ∧ at $(lp, k,$ `11`)
 ∧ $(j < \text{nth}(hp, k))$
 ∧ union-at-n $(l, j,$ `'(5 6 7 8 9 10 11 12)`))
→ at $(g, j,$ `4`)

THEOREM: b3b-i-eq-k-j-neq-k
(molws (n, l, g, h)
 ∧ $(j \in \text{nset}(n))$
 ∧ $(k \in \text{nset}(n))$
 ∧ mrhoi $(n, k, l, g, h, lp, gp, hp)$
 ∧ $(j \neq k)$
 ∧ lg (n, l, g)
 ∧ b3b (l, g, h, k, j)
 ∧ b2a $(l, k, j))$
→ b3b (lp, gp, hp, k, j)

THEOREM: b3b-i-j-neq-k

$(\text{molws}(n, l, g, h)$
$\wedge \quad (i \in \text{nset}(n))$
$\wedge \quad (j \in \text{nset}(n))$
$\wedge \quad (k \in \text{nset}(n))$
$\wedge \quad \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\wedge \quad (i \neq k)$
$\wedge \quad (j \neq k)$
$\wedge \quad \text{b3b}(l, g, h, i, j))$
$\rightarrow \quad \text{b3b}(lp, gp, hp, i, j)$

THEOREM: b3b-i-neq-k

$(\text{molws}(n, l, g, h)$
$\wedge \quad (i \in \text{nset}(n))$
$\wedge \quad (j \in \text{nset}(n))$
$\wedge \quad (k \in \text{nset}(n))$
$\wedge \quad \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\wedge \quad (i \neq k)$
$\wedge \quad \text{lg}(n, l, g)$
$\wedge \quad \text{b1a}(l, i, j)$
$\wedge \quad \text{b3b}(l, g, h, i, j))$
$\rightarrow \quad \text{b3b}(lp, gp, hp, i, j)$

THEOREM: b3b-i-j-eq-k

$(\text{molws}(n, l, g, h)$
$\wedge \quad (k \in \text{nset}(n))$
$\wedge \quad \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\wedge \quad \text{lg}(n, l, g)$
$\wedge \quad \text{b3b}(l, g, h, k, k))$
$\rightarrow \quad \text{b3b}(lp, gp, hp, k, k)$

THEOREM: b3b-i-eq-k

$(\text{molws}(n, l, g, h)$
$\wedge \quad (j \in \text{nset}(n))$
$\wedge \quad (k \in \text{nset}(n))$
$\wedge \quad \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
$\wedge \quad \text{lg}(n, l, g)$
$\wedge \quad \text{b1a}(l, k, j)$
$\wedge \quad \text{b3b}(l, g, h, k, j)$
$\wedge \quad \text{b2a}(l, k, j))$
$\rightarrow \quad \text{b3b}(lp, gp, hp, k, j)$

THEOREM: mrho-preserves-b3b

$(\text{molws}(n, l, g, h)$
$\wedge \quad (i \in \text{nset}(n))$

$$
\begin{array}{ll}
\wedge & (j \in \mathrm{nset}\,(n)) \\
\wedge & (k \in \mathrm{nset}\,(n)) \\
\wedge & \mathrm{mrhoi}\,(n,\ k,\ l,\ g,\ h,\ lp,\ gp,\ hp) \\
\wedge & \mathrm{lg}\,(n,\ l,\ g) \\
\wedge & \mathrm{b1a}\,(l,\ i,\ j) \\
\wedge & \mathrm{b3b}\,(l,\ g,\ h,\ i,\ j) \\
\wedge & \mathrm{b2a}\,(l,\ i,\ j)) \\
\rightarrow & \mathrm{b3b}\,(lp,\ gp,\ hp,\ i,\ j)
\end{array}
$$

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