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|#

EVENT: Start with the initial **nqthm** theory.

;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;; mutex-molecular.ev ;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;

;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;; com.ev ;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;*sequence and finite set utilities

;;;The ith entry in l.

DEFINITION:

nth(*l*, *i*)
= if listp(*l*)
 then if *i* = 1 then car(*l*)
 else nth(cdr(*l*), *i* - 1) endif

```

elseif  $l \in \mathbf{N}$ 
then if  $i = 1$  then  $l$ 
      else f endif
else f endif

```

EVENT: Disable nth.

```
;;;update ith entry of l to be k
```

```

DEFINITION:
move( $l, i, k$ )
= if  $i = 0$  then  $l$ 
  elseif  $l \simeq \text{nil}$ 
  then if  $i = 1$  then  $k$ 
        else  $l$  endif
  elseif  $i = 1$  then cons( $k, \text{cdr}(l)$ )
  else cons(car( $l$ ), move(cdr( $l$ ),  $i - 1, k$ )) endif

```

EVENT: Disable move.

```
DEFINITION: at( $l, i, k$ ) = (nth( $l, i$ ) =  $k$ )
```

EVENT: Disable at.

```

DEFINITION:
length( $l$ )
= if listp( $l$ ) then 1 + length(cdr( $l$ ))
  else ZERO endif

```

EVENT: Disable length.

```
;;;The nth entry in l is in the list i.
```

```
DEFINITION: union-at-n( $l, n, i$ ) = (nth( $l, n$ )  $\in i$ )
```

EVENT: Disable union-at-n.

```
;;;Any entry in l is in the list i.
```

```

DEFINITION:
all-union( $l, n, i$ )
= if  $n \simeq 0$  then t
  else union-at-n( $l, n, i$ )  $\wedge$  all-union( $l, n - 1, i$ ) endif

```

EVENT: Disable all-union.

;;;There exists an entry in l which belongs to
;;;the list i, moreover when exists, some such
;;;j is returned.

DEFINITION:

exist-union(l, n, i)
= **if** $n \simeq 0$ **then f**
 elseif union-at-n(l, n, i) **then** n
 else exist-union($l, n - 1, i$) **endif**

EVENT: Disable exist-union.

;;;n is in the intersection of l8-12 and g34.

DEFINITION:

intersect-8-12-3-4-at-n(n, l, g)
= (union-at-n($l, n, '(8\ 9\ 10\ 11\ 12)$) \wedge union-at-n($g, n, '(3\ 4)$))

EVENT: Disable intersect-8-12-3-4-at-n.

;;;There exists n in the intersection of l8-12 and g34.

DEFINITION:

exist-intersect-8-12-3-4(n, l, g)
= **if** $n \simeq 0$ **then f**
 elseif intersect-8-12-3-4-at-n(n, l, g) **then** n
 else exist-intersect-8-12-3-4($n - 1, l, g$) **endif**

EVENT: Disable exist-intersect-8-12-3-4.

;*Flag invariant.

DEFINITION:

lg-1-at-n(n, l, g)
= ((at($l, n, 0$) \wedge at($g, n, 0$))
 \vee (at($l, n, 1$) \wedge at($g, n, 0$))
 \vee (at($l, n, 2$) \wedge at($g, n, 0$))
 \vee (at($l, n, 3$) \wedge at($g, n, 1$))
 \vee (at($l, n, 4$) \wedge at($g, n, 1$)))

EVENT: Disable lg-1-at-n.

DEFINITION:

$$\begin{aligned} \text{lg-2-at-n}(n, l, g) &= ((\text{at}(l, n, 5) \wedge \text{at}(g, n, 3)) \\ &\quad \vee (\text{at}(l, n, 6) \wedge \text{at}(g, n, 3)) \\ &\quad \vee (\text{at}(l, n, 7) \wedge \text{at}(g, n, 2)) \\ &\quad \vee (\text{at}(l, n, 8) \wedge \text{at}(g, n, 3)) \\ &\quad \vee (\text{at}(l, n, 8) \wedge \text{at}(g, n, 2))) \end{aligned}$$

EVENT: Disable lg-2-at-n.

DEFINITION:

$$\begin{aligned} \text{lg-3-at-n}(n, l, g) &= ((\text{at}(l, n, 9) \wedge \text{at}(g, n, 4)) \\ &\quad \vee (\text{at}(l, n, 10) \wedge \text{at}(g, n, 4)) \\ &\quad \vee (\text{at}(l, n, 11) \wedge \text{at}(g, n, 4)) \\ &\quad \vee (\text{at}(l, n, 12) \wedge \text{at}(g, n, 4))) \end{aligned}$$

EVENT: Disable lg-3-at-n.

DEFINITION:

$$\begin{aligned} \text{lg-at-n}(n, l, g) &= (\text{lg-1-at-n}(n, l, g) \wedge \text{lg-2-at-n}(n, l, g) \wedge \text{lg-3-at-n}(n, l, g)) \end{aligned}$$

EVENT: Disable lg-at-n.

DEFINITION:

$$\begin{aligned} \text{lg}(n, l, g) &= \text{if } n \simeq 0 \text{ then } \mathbf{t} \\ &\quad \text{else } \text{lg-at-n}(n, l, g) \wedge \text{lg}(n - 1, l, g) \text{ endif} \end{aligned}$$

EVENT: Disable lg.

;*The set {1..n}.

DEFINITION:

$$\begin{aligned} \text{nset}(n) &= \text{if } n \simeq 0 \text{ then } \mathbf{nil} \\ &\quad \text{else } \text{cons}(n, \text{nset}(n - 1)) \text{ endif} \end{aligned}$$

EVENT: Disable nset.

;;n belongs to nset.

THEOREM: n-in-nset

$$(n \neq 0) \rightarrow (n \in \text{nset}(n))$$

;;;Any element in nset is a number.

THEOREM: nset-number

$$(k \in \text{nset}(n)) \rightarrow (k \in \mathbf{N})$$

;;;If a nonzero number plus one belongs to nset,
;;;then so does the nonzero number itself.

THEOREM: add1-nset

$$((k \neq 0) \wedge ((1 + k) \in \text{nset}(n))) \rightarrow (k \in \text{nset}(n))$$

;;;Any list has its length at least nonzero.

THEOREM: list-ln

$$\text{listp}(l) \rightarrow (\text{length}(l) \neq 0)$$

;;;(move l k i) is again a list if l is a list.

THEOREM: move-is-list

$$\text{listp}(l) \rightarrow \text{listp}(\text{move}(l, k, i))$$

EVENT: Enable length.

;;;(move l k i) has i as its kth entry.
;;;(enable length) is critical to prove this lemma.

THEOREM: move-nth

$$(\text{listp}(l) \wedge (k \in \text{nset}(\text{length}(l)))) \rightarrow (\text{nth}(\text{move}(l, k, i), k) = i)$$

THEOREM: zero-not-member-nset

$$0 \notin \text{nset}(n)$$

;;;Lists l and (move l k i) have the same length.

THEOREM: move-unchange-length

$$\begin{aligned} &(\text{listp}(l) \wedge (k \in \text{nset}(\text{length}(l)))) \\ &\rightarrow (\text{length}(\text{move}(l, k, i)) = \text{length}(l)) \end{aligned}$$

;;;Lists l and (move l k i) have the same entries
;;;except kth one.

THEOREM: move-unchange-other-than-nth

$$\begin{aligned} &(\text{listp}(l) \wedge (k \in \text{nset}(\text{length}(l))) \wedge (j \neq k)) \\ &\rightarrow (\text{nth}(\text{move}(l, k, i), j) = \text{nth}(l, j)) \end{aligned}$$

THEOREM: member-ex-union
 $\text{exist-union}(l, n, i) \rightarrow (\text{exist-union}(l, n, i) \in \text{nset}(n))$
 ;;;(exist-union l n i) is a number.

THEOREM: number-ex-union
 $\text{exist-union}(l, n, i) \rightarrow (\text{exist-union}(l, n, i) \in \mathbf{N})$
 ;;;(exist-intersect-8-12-3-4 n l g) belongs to nset.

THEOREM: member-intersect
 $\text{exist-intersect-8-12-3-4}(n, l, g)$
 $\rightarrow (\text{exist-intersect-8-12-3-4}(n, l, g) \in \text{nset}(n))$
 ;;;(exist-intersect-8-12-3-4 n l g) is a number.

THEOREM: number-intersect
 $\text{exist-intersect-8-12-3-4}(n, l, g) \rightarrow (\text{exist-intersect-8-12-3-4}(n, l, g) \in \mathbf{N})$
 ;;;any member of nset is nonzero.

THEOREM: k-not-0
 $(k \in \text{nset}(n)) \rightarrow (k \neq 0)$
 ;*lemmas for a0

;;;If j's entry in l is between 8..12 then
 ;;;(exist-union l n '(8 9 10 11 12)) holds.

THEOREM: j-ex-l8-12
 $((j \in \text{nset}(n)) \wedge \text{union-at-n}(l, j, '(8 9 10 11 12)))$
 $\rightarrow \text{exist-union}(l, n, '(8 9 10 11 12))$

;;;Witness of (exist-union lp n '(8 9 10 11 12))
 ;;;has in lp its entry between 8...12.

THEOREM: ex-lp8-12-in-lp8-12
 $\text{exist-union}(lp, n, '(8 9 10 11 12))$
 $\rightarrow \text{union-at-n}(lp,$
 $\quad \text{exist-union}(lp, n, '(8 9 10 11 12)),$
 $\quad '(8 9 10 11 12))$

;;;If (not (exist-union l n '(8 9 10 11 12)))
 ;;;holds, then (not (exist-union g n '(4))) by lg.

THEOREM: ex-if4
 $((\neg \text{exist-union}(l, n, '(8 9 10 11 12))) \wedge \text{lg}(n, l, g))$
 $\rightarrow (\neg \text{exist-union}(g, n, '(4)))$

;;;If (not (exist-union g n '(1))) holds,
;;; then there is no entry either 3 or 4.

THEOREM: l34-empty
 $((j \in \text{nset}(n)) \wedge \text{lg}(n, l, g) \wedge (\neg \text{exist-union}(g, n, '(1))))$
 $\rightarrow (\neg \text{union-at-n}(l, j, '(3\ 4)))$

;;;If j's entry in lp is 4, then (certainly)
;;;it is either 3 or 4.

THEOREM: lp4-then-un34
 $\text{at}(lp, j, 4) \rightarrow \text{union-at-n}(lp, j, '(3\ 4))$

;;;If (exist-intersect-8-12-3-4 n l g) holds,
;;;then so does (exist-union g n '(3 4)).

THEOREM: int-8-12-3-4-then-un34
 $\text{exist-intersect-8-12-3-4}(n, l, g) \rightarrow \text{exist-union}(g, n, '(3\ 4))$

;*lemmas for a1

;;;i is the witness of
;;;(exist-intersect-8-12-3-4 n lp gp).

THEOREM: int-wtn
 $((j \in \text{nset}(n)) \wedge \text{intersect-8-12-3-4-at-n}(j, lp, gp))$
 $\rightarrow \text{exist-intersect-8-12-3-4}(n, lp, gp)$

;;;If there exists j such that j's entry in lp
;;;is between 8..12 and entry in gp is either 3 or 4
;;;then (intersect-8-12-3-4-at-n j lp gp) holds.

THEOREM: un8-12-and-un34-then-int
 $(\text{union-at-n}(lp, j, '(8\ 9\ 10\ 11\ 12)) \wedge \text{union-at-n}(gp, j, '(3\ 4)))$
 $\rightarrow \text{intersect-8-12-3-4-at-n}(j, lp, gp)$

;;;By the two lemmas above,
;;;(exist-intersect-8-12-3-4 n lp gp) holds provided
;;;that there exists j such that j's entry in lp is
;;;between 8..12 and entry in gp is either 3 or 4.

;* ep-18-12

;;;If the k's entry in l is 5, then the k's entry
;;;in g is 3 by lg.

THEOREM: lg-15-g3
 $((k \in \text{nset}(n)) \wedge \text{lg}(n, l, g) \wedge \text{at}(l, k, 5)) \rightarrow \text{at}(g, k, 3)$

;;;If the k's entry in gp is 3 then certainly
;;;it is either 3 or 4.

THEOREM: gp3-then-un34
 $\text{at}(gp, k, 3) \rightarrow \text{union-at-n}(gp, k, '(3\ 4))$

;;;nep-18-12

;;;If the k's entry in l is between 8..12 then
;;;it is either between 8..11 or equal to 12.

THEOREM: case-k
 $(\text{union-at-n}(l, k, '(8\ 9\ 10\ 11\ 12))$
 $\wedge (\neg \text{union-at-n}(l, k, '(8\ 9\ 10\ 11))))$
 $\rightarrow \text{at}(l, k, 12)$

;;;;k-not-18-12

;;;If (exist-intersect-8-12-3-4 n l g) holds
;;;then the witness has its entry in g either equal
;;;to 3 or 4.

THEOREM: intersect-8-12-3-4-then-3-4
 $\text{exist-intersect-8-12-3-4}(n, l, g)$
 $\rightarrow \text{union-at-n}(g, \text{exist-intersect-8-12-3-4}(n, l, g), '(3\ 4))$

;;;If (exist-intersect-8-12-3-4 n l g) holds,
;;; then the witness has its entry in g between 8 and 12.

THEOREM: intersect-8-12-3-4-then-8-12
 $\text{exist-intersect-8-12-3-4}(n, l, g)$
 $\rightarrow \text{union-at-n}(l, \text{exist-intersect-8-12-3-4}(n, l, g), '(8\ 9\ 10\ 11\ 12))$

;;;k-in-18-11

;;;If k's entry in lp is between 9 and 12,
;;;then it is certainly between 8 and 12.

THEOREM: un9-12-then-un8-12
 $\text{union-at-n}(lp, k, '(9\ 10\ 11\ 12))$
 $\rightarrow \text{union-at-n}(lp, k, '(8\ 9\ 10\ 11\ 12))$

;;;If the i's entry in l is between 9 and 12,
;;;then the k's entry in g is 4.

THEOREM: if4
 $((j \in \text{nset}(n)) \wedge \text{lg}(n, l, g) \wedge \text{union-at-n}(l, j, '(9\ 10\ 11\ 12)))$
 $\rightarrow \text{at}(g, j, 4)$

;;;k-in-l12

;;;If (exist-union lp n '(8 9 10 11 12)) holds then
 ;;;its witness does not have its entry in lp equal to 1.

THEOREM: ex-lp8-12-not-in-lp0
 $\text{exist-union}(lp, n, '(8\ 9\ 10\ 11\ 12))$
 $\rightarrow (\neg \text{at}(lp, \text{exist-union}(lp, n, '(8\ 9\ 10\ 11\ 12)), 0))$

;;;If k's entry in lp is between 8 and 12,
 ;;; then it is either between 8 and 11 or 12.

THEOREM: k-in-lp9-12-or-lp8
 $(\text{union-at-n}(lp, k, '(8\ 9\ 10\ 11\ 12))$
 $\wedge (\neg \text{union-at-n}(lp, k, '(9\ 10\ 11\ 12))))$
 $\rightarrow \text{at}(lp, k, 8)$

;;;If the k's entry is either 5 or 7,
 ;;;then it is between 5 and 7.

THEOREM: un57-then-un5-12
 $\text{union-at-n}(l, k, '(5\ 7)) \rightarrow \text{union-at-n}(l, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12))$

;;;If the k's entry in l is between 8 and 11,
 ;;;then it is between 5 and 12.

THEOREM: un8-11-then-un5-12
 $\text{union-at-n}(l, k, '(8\ 9\ 10\ 11))$
 $\rightarrow \text{union-at-n}(l, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12))$

;;;If the k's entry in l is between 8 and 12,
 ;;;then it is between 5 and 12.

THEOREM: un8-12-then-un5-12
 $\text{union-at-n}(l, k, '(8\ 9\ 10\ 11\ 12))$
 $\rightarrow \text{union-at-n}(l, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12))$

;*lemmas for a2

;;;i-eq-k-j-neq-k

;;;If the k's entry in l is either 10 or 11,
 ;;;then the k's entry in l is between 10 and 12.

THEOREM: un10-11-then-un10-12
union-at-n($l, k, '(10\ 11)$) \rightarrow union-at-n($l, k, '(10\ 11\ 12)$)

;;;If the j's entry in g is either 0 or 1 then
;;;the j's entry in l is not between 5 and 12.

THEOREM: if1
(($j \in \text{nset}(n)$) \wedge lg(n, l, g) \wedge union-at-n($g, j, '(0\ 1)$))
 \rightarrow (\neg union-at-n($l, j, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)$))

;;;j-eq-k-i-neq-k

;;;If the k's entry in l is between 5 and 7,
;;;then it is certainly between 5 and 12.

THEOREM: un5-7-then-un5-11
union-at-n($l, k, '(5\ 6\ 7)$) \rightarrow union-at-n($l, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11)$)

;;;If the k's entry in lp is between 5 and 7 then
;;;it is certain between 5 and 11.

THEOREM: un57-then-un5-11
union-at-n($l, k, '(5\ 7)$) \rightarrow union-at-n($l, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11)$)

;;;If the k's entry in l is between 8 and 11,
;;;then it is certainly between 5 and 11.

THEOREM: un8-11-then-un5-11
union-at-n($l, k, '(8\ 9\ 10\ 11)$)
 \rightarrow union-at-n($l, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11)$)

;;;If the k's entry in lp is between 5 and 12 and
;;;the k's entry in lp is between 5 and 7, then
;;;the k's entry in lp in fact is between 9 and 12.

THEOREM: k-in-lp5-7-or-lp8-or-lp9-12
(union-at-n($lp, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)$)
 \wedge (\neg union-at-n($lp, k, '(5\ 6\ 7)$))
 \wedge (\neg at($lp, k, 8$)))
 \rightarrow union-at-n($lp, k, '(9\ 10\ 11\ 12)$)

;;;If the k's entry in l is between 5 and 11,
;;; then it is certainly between 5 and 12.

THEOREM: un5-11-then-un5-12
union-at-n($l, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11)$)
 \rightarrow union-at-n($l, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)$)

;;;If the k's entry in l is between 10 and 12,
 ;;; then it is certainly between 8 and 12.

THEOREM: un10-12-then-un8-12
 $\text{union-at-n}(l, i, '(10\ 11\ 12)) \rightarrow \text{union-at-n}(l, i, '(8\ 9\ 10\ 11\ 12))$

;;;j-eq-k-i-neq-k

;;;If (exist-union l n '(8 9 10 11 12)) does not hold,
 ;;;then the i's entry in l is not between 10 and 12.

THEOREM: i-not-l10-12
 $((i \in \text{nset}(n)) \wedge (\neg \text{exist-union}(l, n, '(8\ 9\ 10\ 11\ 12))))$
 $\rightarrow (\neg \text{union-at-n}(l, i, '(10\ 11\ 12)))$

;*lemmas for a3

;;;j-eq-k-i-neq-k

;;;If the k's entry in l is between 5 and 11,
 ;;;then the k's entry in l is between 9 and 11.

THEOREM: un5-11-eq-un58-or-un8-11
 $(\text{union-at-n}(l, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11))$
 $\wedge (\neg \text{union-at-n}(l, k, '(5\ 6\ 7\ 8))))$
 $\rightarrow \text{union-at-n}(l, k, '(9\ 10\ 11))$

;;;If the k's entry in g is 4,
 ;;;then the k's entry in l is between 5 and 8.

THEOREM: a3-if4
 $((k \in \text{nset}(n)) \wedge \text{lg}(n, l, g) \wedge \text{at}(g, k, 4))$
 $\rightarrow (\neg \text{union-at-n}(l, k, '(5\ 6\ 7\ 8)))$

;;;If the k's entry in l is between 5 and 11,
 ;;;and the k's entry in l is between 5 and 12,
 ;;;then the k's entry in l is 9 and 11.

THEOREM: k-in-l5-11-g4-then-l9-11
 $((k \in \text{nset}(n))$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{union-at-n}(l, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11))$
 $\wedge \text{at}(g, k, 4))$
 $\rightarrow \text{union-at-n}(l, k, '(9\ 10\ 11))$

;;;If the i's entry in l is 12,
 ;;;then the i's entry in l is between 8 and 12.

THEOREM: l12-then-un8-12
 $at(l, i, 12) \rightarrow union-at-n(l, i, '(8\ 9\ 10\ 11\ 12))$

;;;If (exist-union l n '(8 9 10 11 12)) does not hold,
 ;;;then the i's entry in l is 12.

THEOREM: i-not-in-l12
 $((i \in nset(n)) \wedge (\neg exist-union(l, n, '(8\ 9\ 10\ 11\ 12))))$
 $\rightarrow (\neg at(l, i, 12))$

;;;j-neq-k-i-eq-k

;;;If the k's entry in l is 11,
 ;;; then the k's entry in l is between 10 and 12.

THEOREM: l11-then-un10-12
 $at(l, k, 11) \rightarrow union-at-n(l, k, '(10\ 11\ 12))$

;;;If the j's entry in g is either 2 or 3,
 ;;;then the j's entry in l is between 5 and 8 by lg.

THEOREM: if3
 $((j \in nset(n)) \wedge lg(n, l, g) \wedge (\neg union-at-n(g, j, '(2\ 3))))$
 $\rightarrow (\neg union-at-n(l, j, '(5\ 6\ 7\ 8)))$

;;;If the j's entry in l is between 5 and 12 and
 ;;;the j's entry in l is between 5 and 8, then
 ;;;the j's entry in l is 9 and 12.

THEOREM: l5-12-eq-l5-8-or-l9-12
 $(union-at-n(l, j, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12))$
 $\wedge (\neg union-at-n(l, j, '(5\ 6\ 7\ 8))))$
 $\rightarrow union-at-n(l, j, '(9\ 10\ 11\ 12))$

;;;i-j-eq-k

;;;If the k's entry in lp is 12,
 ;;;then it is certainly between 5 and 12.

THEOREM: l12-then-un9-12
 $at(lp, k, 12) \rightarrow union-at-n(lp, k, '(9\ 10\ 11\ 12))$

*lemmas for b1a

;;;If the u's entry in g is 4,
 ;;;then the u's entry in l is between 8 and 12 by lg.

THEOREM: bla-if4

$((u \in \text{nset}(n)) \wedge \text{lg}(n, l, g) \wedge \text{at}(g, u, 4))$
 $\rightarrow \text{union-at-n}(l, u, '(8\ 9\ 10\ 11\ 12))$

;*lemmas for b1b

;;;If the k's entry in lp is between 9 and 12,
;;;then the k's entry in gp is iether 3 or 4 by lg.

THEOREM: lp9-12-then-k-in-g34

$((k \in \text{nset}(n)) \wedge \text{union-at-n}(lp, k, '(9\ 10\ 11\ 12)) \wedge \text{lg}(n, lp, gp))$
 $\rightarrow \text{union-at-n}(gp, k, '(3\ 4))$

;;;If the k's entry in lp is between 8 and 12, and
;;;it is not 8, then it is certainly between 9 and 12.

THEOREM: un8-12-then-l8-or-l9-12

$(\text{union-at-n}(lp, k, '(8\ 9\ 10\ 11\ 12)) \wedge (\neg \text{at}(lp, k, 8)))$
 $\rightarrow \text{union-at-n}(lp, k, '(9\ 10\ 11\ 12))$

;;;;;;;;;;;;; moldefn.ev ;;;;;;;;;;;;;;
;* Well-formed states

DEFINITION:

molws(n, l, g, h)

= $((n \in \mathbf{N})$
 $\wedge \text{listp}(l)$
 $\wedge \text{listp}(g)$
 $\wedge \text{listp}(h)$
 $\wedge (\text{length}(l) = n)$
 $\wedge (\text{length}(g) = n)$
 $\wedge (\text{length}(h) = n)$
 $\wedge \text{all-union}(l, n, '(0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12))$
 $\wedge \text{all-union}(g, n, '(0\ 1\ 2\ 3\ 4))$
 $\wedge \text{all-union}(h, n, \text{nset}(1 + n))$

EVENT: Disable molws.

;* Transitions

DEFINITION:

mrhoi0($n, i, l, g, h, lp, gp, hp$)

= $(\text{at}(l, i, 0) \wedge (gp = g) \wedge (lp = \text{move}(l, i, 1)) \wedge (hp = h))$

DEFINITION:

$$\begin{aligned} & \text{mrhoi1a}(n, i, l, g, h, lp, gp, hp) \\ &= (\text{at}(l, i, 1) \wedge (gp = g) \wedge (lp = \text{move}(l, i, 2)) \wedge (hp = h)) \end{aligned}$$

DEFINITION:

$$\begin{aligned} & \text{mrhoi1b}(n, i, l, g, h, lp, gp, hp) \\ &= (\text{at}(l, i, 1) \wedge (gp = g) \wedge (lp = l) \wedge (hp = h)) \end{aligned}$$

DEFINITION:

$$\begin{aligned} & \text{mrhoi2}(n, i, l, g, h, lp, gp, hp) \\ &= (\text{at}(l, i, 2) \\ & \quad \wedge (lp = \text{move}(l, i, 3)) \\ & \quad \wedge (gp = \text{move}(g, i, 1)) \\ & \quad \wedge (hp = \text{move}(h, i, 1))) \end{aligned}$$

DEFINITION:

$$\begin{aligned} & \text{mrhoi3a}(n, i, l, g, h, lp, gp, hp) \\ &= (\text{at}(l, i, 3) \\ & \quad \wedge (gp = g) \\ & \quad \wedge (hp = h) \\ & \quad \wedge \text{at}(h, i, 1 + n) \\ & \quad \wedge (lp = \text{move}(l, i, 4))) \end{aligned}$$

DEFINITION:

$$\begin{aligned} & \text{mrhoi3b}(n, i, l, g, h, lp, gp, hp) \\ &= (\text{at}(l, i, 3) \\ & \quad \wedge (gp = g) \\ & \quad \wedge (lp = l) \\ & \quad \wedge (\text{nth}(h, i) < (1 + n)) \\ & \quad \wedge (hp = \text{move}(h, i, 1 + \text{nth}(h, i))) \\ & \quad \wedge \text{union-at-n}(g, \text{nth}(h, i), '(0 1 2))) \end{aligned}$$

DEFINITION:

$$\begin{aligned} & \text{mrhoi4}(n, i, l, g, h, lp, gp, hp) \\ &= (\text{at}(l, i, 4) \\ & \quad \wedge (gp = \text{move}(g, i, 3)) \\ & \quad \wedge (lp = \text{move}(l, i, 5)) \\ & \quad \wedge (hp = \text{move}(h, i, 1))) \end{aligned}$$

DEFINITION:

$$\begin{aligned} & \text{mrhoi5a}(n, i, l, g, h, lp, gp, hp) \\ &= (\text{at}(l, i, 5) \\ & \quad \wedge (gp = g) \\ & \quad \wedge (hp = h) \\ & \quad \wedge \text{at}(h, i, 1 + n) \\ & \quad \wedge (lp = \text{move}(l, i, 8))) \end{aligned}$$

DEFINITION:
 $\text{mrhoi5b}(n, i, l, g, h, lp, gp, hp)$
 $=$ $(\text{at}(l, i, 5)$
 \wedge $(gp = g)$
 \wedge $(hp = h)$
 \wedge $(\text{nth}(h, i) < (1 + n))$
 \wedge $\text{at}(g, \text{nth}(h, i), 1)$
 \wedge $(lp = \text{move}(l, i, 6)))$

DEFINITION:
 $\text{mrhoi5c}(n, i, l, g, h, lp, gp, hp)$
 $=$ $(\text{at}(l, i, 5)$
 \wedge $(gp = g)$
 \wedge $(lp = l)$
 \wedge $(\text{nth}(h, i) < (1 + n))$
 \wedge $(\neg \text{at}(g, \text{nth}(h, i), 1))$
 \wedge $(hp = \text{move}(h, i, 1 + \text{nth}(h, i))))$

DEFINITION:
 $\text{mrhoi6}(n, i, l, g, h, lp, gp, hp)$
 $=$ $(\text{at}(l, i, 6)$
 \wedge $(gp = \text{move}(g, i, 2))$
 \wedge $(lp = \text{move}(l, i, 7))$
 \wedge $(hp = \text{move}(h, i, 1)))$

DEFINITION:
 $\text{mrhoi7a}(n, i, l, g, h, lp, gp, hp)$
 $=$ $(\text{at}(l, i, 7)$
 \wedge $(lp = \text{move}(l, i, 8))$
 \wedge $\text{at}(g, \text{nth}(h, i), 4)$
 \wedge $(gp = g)$
 \wedge $(hp = h))$

DEFINITION:
 $\text{mrhoi7b}(n, i, l, g, h, lp, gp, hp)$
 $=$ $(\text{at}(l, i, 7)$
 \wedge $(\neg \text{at}(g, \text{nth}(h, i), 4))$
 \wedge $(lp = l)$
 \wedge $(gp = g)$
 \wedge $(hp = \text{move}(h, i, 1 + ((\text{nth}(h, i) - 1) \bmod n))))$

DEFINITION:
 $\text{mrhoi8}(n, i, l, g, h, lp, gp, hp)$
 $=$ $(\text{at}(l, i, 8)$
 \wedge $(gp = \text{move}(g, i, 4))$

$$\begin{aligned} & \wedge (lp = \text{move}(l, i, 9)) \\ & \wedge (hp = \text{move}(h, i, 1)) \end{aligned}$$

DEFINITION:

$$\begin{aligned} & \text{mrhoi9a}(n, i, l, g, h, lp, gp, hp) \\ = & (\text{at}(l, i, 9) \\ & \wedge \text{at}(h, i, i) \\ & \wedge (lp = \text{move}(l, i, 10)) \\ & \wedge (gp = g) \\ & \wedge (hp = h)) \end{aligned}$$

DEFINITION:

$$\begin{aligned} & \text{mrhoi9b}(n, i, l, g, h, lp, gp, hp) \\ = & (\text{at}(l, i, 9) \\ & \wedge (\text{nth}(h, i) < i) \\ & \wedge \text{union-at-n}(g, \text{nth}(h, i), '(0 1)) \\ & \wedge (hp = \text{move}(h, i, 1 + \text{nth}(h, i))) \\ & \wedge (gp = g) \\ & \wedge (lp = l)) \end{aligned}$$

DEFINITION:

$$\begin{aligned} & \text{mrhoi10}(n, i, l, g, h, lp, gp, hp) \\ = & (\text{at}(l, i, 10) \\ & \wedge (lp = \text{move}(l, i, 11)) \\ & \wedge (gp = g) \\ & \wedge (hp = \text{move}(h, i, 1 + i))) \end{aligned}$$

DEFINITION:

$$\begin{aligned} & \text{mrhoi11a}(n, i, l, g, h, lp, gp, hp) \\ = & (\text{at}(l, i, 11) \\ & \wedge \text{at}(h, i, 1 + n) \\ & \wedge (lp = \text{move}(l, i, 12)) \\ & \wedge (gp = g) \\ & \wedge (hp = h)) \end{aligned}$$

DEFINITION:

$$\begin{aligned} & \text{mrhoi11b}(n, i, l, g, h, lp, gp, hp) \\ = & (\text{at}(l, i, 11) \\ & \wedge (\text{nth}(h, i) < (1 + n)) \\ & \wedge (\neg \text{union-at-n}(g, \text{nth}(h, i), '(2 3))) \\ & \wedge (hp = \text{move}(h, i, 1 + \text{nth}(h, i))) \\ & \wedge (gp = g) \\ & \wedge (lp = l)) \end{aligned}$$

DEFINITION:


```

mrhoi12(n, i, l, g, h, lp, gp, hp)
= (at(l, i, 12)
  ∧ (hp = h)
  ∧ (gp = move(g, i, 0))
  ∧ (lp = move(l, i, 0)))

```

DEFINITION:

```

mrhoi(n, i, l, g, h, lp, gp, hp)
= (mrhoi0(n, i, l, g, h, lp, gp, hp)
  ∨ mrhoi1a(n, i, l, g, h, lp, gp, hp)
  ∨ mrhoi1b(n, i, l, g, h, lp, gp, hp)
  ∨ mrhoi2(n, i, l, g, h, lp, gp, hp)
  ∨ mrhoi3a(n, i, l, g, h, lp, gp, hp)
  ∨ mrhoi3b(n, i, l, g, h, lp, gp, hp)
  ∨ mrhoi4(n, i, l, g, h, lp, gp, hp)
  ∨ mrhoi5a(n, i, l, g, h, lp, gp, hp)
  ∨ mrhoi5b(n, i, l, g, h, lp, gp, hp)
  ∨ mrhoi5c(n, i, l, g, h, lp, gp, hp)
  ∨ mrhoi6(n, i, l, g, h, lp, gp, hp)
  ∨ mrhoi7a(n, i, l, g, h, lp, gp, hp)
  ∨ mrhoi7b(n, i, l, g, h, lp, gp, hp)
  ∨ mrhoi8(n, i, l, g, h, lp, gp, hp)
  ∨ mrhoi9a(n, i, l, g, h, lp, gp, hp)
  ∨ mrhoi9b(n, i, l, g, h, lp, gp, hp)
  ∨ mrhoi10(n, i, l, g, h, lp, gp, hp)
  ∨ mrhoi11a(n, i, l, g, h, lp, gp, hp)
  ∨ mrhoi11b(n, i, l, g, h, lp, gp, hp)
  ∨ mrhoi12(n, i, l, g, h, lp, gp, hp))

```

EVENT: Disable mrhoi.

```

;* Invariants

```

```

;;;b0

```

DEFINITION:

```

b0a(n, l, h, i, j) = ((at(l, i, 5) ∧ (j < nth(h, i))) → (¬ at(l, j, 4)))

```

EVENT: Disable b0a.

DEFINITION:

```

b0b(n, l, h, i, j)
= ((at(l, i, 5) ∧ (j < nth(h, i)) ∧ at(l, j, 3)) → (i ≠ nth(h, j)))

```

EVENT: Disable b0b.

;;b1

DEFINITION:

$$\text{b1a}(l, i, j) = (\text{union-at-n}(l, i, '(8\ 9\ 10\ 11\ 12)) \rightarrow (\neg \text{at}(l, j, 4)))$$

EVENT: Disable b1a.

DEFINITION:

$$\begin{aligned} &\text{hint-8-12-3-4-at-n}(n, l, g, h, j) \\ &= (\text{intersect-8-12-3-4-at-n}(n, l, g) \wedge (n \not\prec \text{nth}(h, j))) \end{aligned}$$

EVENT: Disable hint-8-12-3-4-at-n.

DEFINITION:

$$\begin{aligned} &\text{exist-hint-8-12-3-4}(n, l, g, h, j) \\ &= \text{if } n \simeq 0 \text{ then f} \\ &\quad \text{elseif hint-8-12-3-4-at-n}(n, l, g, h, j) \text{ then } n \\ &\quad \text{else exist-hint-8-12-3-4}(n - 1, l, g, h, j) \text{ endif} \end{aligned}$$

EVENT: Disable exist-hint-8-12-3-4.

DEFINITION:

$$\begin{aligned} &\text{b1b}(n, l, g, h, i, j) \\ &= ((\text{union-at-n}(l, i, '(8\ 9\ 10\ 11\ 12)) \wedge \text{at}(l, j, 3)) \\ &\quad \rightarrow \text{exist-hint-8-12-3-4}(n, l, g, h, j)) \end{aligned}$$

EVENT: Disable b1b.

DEFINITION:

$$\begin{aligned} &\text{b1c}(n, l, g, h, i) \\ &= ((\text{union-at-n}(l, i, '(8\ 9\ 10\ 11\ 12)) \\ &\quad \wedge (\neg \text{union-at-n}(g, i, '(3\ 4)))) \\ &\quad \rightarrow ((\text{nth}(h, i) \in \text{nset}(n)) \wedge \text{at}(g, \text{nth}(h, i), 4))) \end{aligned}$$

EVENT: Disable b1c.

DEFINITION:

$$\text{b1d}(n, l, h, i) = (\text{at}(l, i, 7) \rightarrow (\text{nth}(h, i) \in \text{nset}(n)))$$

EVENT: Disable b1d.

;;; b2

DEFINITION:

$$\begin{aligned} & \text{b2a}(l, i, j) \\ = & ((j < i) \wedge \text{union-at-n}(l, i, '(10\ 11\ 12))) \\ & \rightarrow (\neg \text{union-at-n}(l, j, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)))) \end{aligned}$$

EVENT: Disable b2a.

DEFINITION:

$$\begin{aligned} & \text{b2b}(l, h, i, j) \\ = & ((j < i) \wedge \text{at}(l, i, 9) \wedge (j < \text{nth}(h, i))) \\ & \rightarrow (\neg \text{union-at-n}(l, j, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)))) \end{aligned}$$

EVENT: Disable b2b.

;;;b3

DEFINITION:

$$\begin{aligned} & \text{b3a}(l, g, i, j) \\ = & ((\text{at}(l, i, 12) \wedge \text{union-at-n}(l, j, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12))) \\ & \rightarrow \text{at}(g, j, 4)) \end{aligned}$$

EVENT: Disable b3a.

DEFINITION:

$$\begin{aligned} & \text{b3b}(l, g, h, i, j) \\ = & ((\text{at}(l, i, 11) \\ & \wedge (j < \text{nth}(h, i)) \\ & \wedge \text{union-at-n}(l, j, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12))) \\ & \rightarrow \text{at}(g, j, 4)) \end{aligned}$$

EVENT: Disable b3b.

;; molbasic.ev ;;

THEOREM: hint-member

$$\begin{aligned} & \text{exist-hint-8-12-3-4}(n, l, g, h, j) \\ \rightarrow & (\text{exist-hint-8-12-3-4}(n, l, g, h, j) \in \text{nset}(n)) \end{aligned}$$

THEOREM: n-not-less-j

$$(n < j) \rightarrow (j \notin \text{nset}(n))$$

;;;molws implies that n is a number.

THEOREM: molws-num-n
 $\text{molws}(n, l, g, h) \rightarrow (n \in \mathbf{N})$

;;;molws implies that l is a list.

THEOREM: molws-list-l
 $\text{molws}(n, l, g, h) \rightarrow \text{listp}(l)$

;;;molws implies that g is a list.

THEOREM: molws-list-g
 $\text{molws}(n, l, g, h) \rightarrow \text{listp}(g)$

;;;molws implies that h is a list.

THEOREM: molws-list-h
 $\text{molws}(n, l, g, h) \rightarrow \text{listp}(h)$

;;;molws implies that length of l is n.

THEOREM: molws-ln-l
 $\text{molws}(n, l, g, h) \rightarrow (\text{length}(l) = n)$

;;;molws implies that length of g is n.

THEOREM: molws-ln-g
 $\text{molws}(n, l, g, h) \rightarrow (\text{length}(g) = n)$

;;;molws implies that length of h is n.

THEOREM: molws-ln-h
 $\text{molws}(n, l, g, h) \rightarrow (\text{length}(h) = n)$

;;;molws and mrho imply that lp is a list.

THEOREM: molws-ln-lp
 $(\text{molws}(n, l, g, h) \wedge (k \in \text{nset}(n)) \wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp))$
 $\rightarrow \text{listp}(lp)$

;;;molws and mrho imply that gp is a list.

THEOREM: molws-ln-gp
 $(\text{molws}(n, l, g, h) \wedge (k \in \text{nset}(n)) \wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp))$
 $\rightarrow \text{listp}(gp)$

;;;molws and mrho imply that hp is a list.

THEOREM: molws-ln-hp
 $(\text{molws}(n, l, g, h) \wedge (k \in \text{nset}(n)) \wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp))$
 $\rightarrow \text{listp}(hp)$

;;;Another version of nset-number.
 ;;;This is available in the theorem
 ;;;where molws is disabled.

THEOREM: molws-num-k
 $(\text{molws}(n, l, g, h) \wedge (k \in \text{nset}(n))) \rightarrow (k \in \mathbf{N})$

THEOREM: molws-union-h
 $\text{molws}(n, l, g, h) \rightarrow \text{all-union}(h, n, \text{nset}(1 + n))$

THEOREM: lm-nth-numberp
 $((i \in \mathbf{N}) \wedge \text{all-union}(h, n, \text{nset}(i)) \wedge (k \in \text{nset}(n))) \rightarrow (\text{nth}(h, k) \in \mathbf{N})$

THEOREM: nth-numberp
 $(\text{molws}(n, l, g, h) \wedge (k \in \text{nset}(n))) \rightarrow (\text{nth}(h, k) \in \mathbf{N})$

;;;molws implies that n is nonzero.

THEOREM: molws-n-not-0
 $\text{molws}(n, l, g, h) \rightarrow (n \neq 0)$

;;;Auxiliary lemma.

THEOREM: lm-l-mrholemma
 $(\text{listp}(l)$
 $\wedge (j \in \text{nset}(\text{length}(l)))$
 $\wedge (k \in \text{nset}(\text{length}(l)))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (k \neq j))$
 $\rightarrow (\text{nth}(l, j) = \text{nth}(lp, j))$

EVENT: Disable lm-l-mrholemma.

;;;Mrholemma for list l.

THEOREM: l-mrholemma
 $(\text{molws}(n, l, g, h)$
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (k \neq j))$
 $\rightarrow (\text{nth}(l, j) = \text{nth}(lp, j))$

;;;Auxiliary lemma.

THEOREM: lm-g-mrholemma

(listp g)
 \wedge ($j \in \text{nset}(\text{length}(g))$)
 \wedge ($k \in \text{nset}(\text{length}(g))$)
 \wedge $\text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 \wedge ($k \neq j$)
 \rightarrow ($\text{nth}(g, j) = \text{nth}(gp, j)$)

EVENT: Disable lm-g-mrholemma.

;;;Mrholemma for list g.

THEOREM: g-mrholemma

(molws n, l, g, h)
 \wedge ($j \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge $\text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 \wedge ($k \neq j$)
 \rightarrow ($\text{nth}(g, j) = \text{nth}(gp, j)$)

;;;Auxiliary lemma.

THEOREM: lm-h-mrholemma

(listp h)
 \wedge ($j \in \text{nset}(\text{length}(h))$)
 \wedge ($k \in \text{nset}(\text{length}(h))$)
 \wedge $\text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 \wedge ($k \neq j$)
 \rightarrow ($\text{nth}(h, j) = \text{nth}(hp, j)$)

EVENT: Disable lm-h-mrholemma.

;;;Mrholemma for list g.

THEOREM: h-mrholemma

(molws n, l, g, h)
 \wedge ($j \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge $\text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 \wedge ($k \neq j$)
 \rightarrow ($\text{nth}(h, j) = \text{nth}(hp, j)$)

;;; lp-gp-same-l-g

;;;Another version of Rholemma for l.
;;;It applies to (union-at-n l j m) in stead of
;;;(nth l j).

THEOREM: m-lp-same-l

(molws (n, l, g, h)
 \wedge listp (m)
 \wedge ($j \in \text{nset } (n)$)
 \wedge ($k \in \text{nset } (n)$)
 \wedge mrhoi ($n, k, l, g, h, lp, gp, hp$)
 \wedge ($j \neq k$)
 \wedge union-at-n (l, j, m)
 \rightarrow union-at-n (lp, j, m))

;;;Contrast to the one above,
;;;the order of l and lp is reversed.

THEOREM: m-l-same-lp

(molws (n, l, g, h)
 \wedge listp (m)
 \wedge ($j \in \text{nset } (n)$)
 \wedge ($k \in \text{nset } (n)$)
 \wedge mrhoi ($n, k, l, g, h, lp, gp, hp$)
 \wedge ($j \neq k$)
 \wedge union-at-n (lp, j, m)
 \rightarrow union-at-n (l, j, m))

THEOREM: m-lp-same-l-not

(molws (n, l, g, h)
 \wedge listp (m)
 \wedge ($j \in \text{nset } (n)$)
 \wedge ($k \in \text{nset } (n)$)
 \wedge mrhoi ($n, k, l, g, h, lp, gp, hp$)
 \wedge ($j \neq k$)
 \wedge (\neg union-at-n (lp, j, m)))
 \rightarrow (\neg union-at-n (l, j, m))

;;;Another version of Rholemma for g.

THEOREM: m-gp-same-g

(molws (n, l, g, h)
 \wedge listp (m)
 \wedge ($j \in \text{nset } (n)$)

$\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (j \neq k)$
 $\wedge \text{union-at-n}(g, j, m)$
 $\rightarrow \text{union-at-n}(gp, j, m)$

;;;Contrast to the one above,
 ;;;the order of g and gp is reversed.

THEOREM: m-g-same-gp
 $(\text{molws}(n, l, g, h)$
 $\wedge \text{listp}(m)$
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (j \neq k)$
 $\wedge \text{union-at-n}(gp, j, m))$
 $\rightarrow \text{union-at-n}(g, j, m)$

THEOREM: m-gp-same-g-not
 $(\text{molws}(n, l, g, h)$
 $\wedge \text{listp}(m)$
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (j \neq k)$
 $\wedge (\neg \text{union-at-n}(gp, j, m))$
 $\rightarrow (\neg \text{union-at-n}(g, j, m))$

;;;Another version of Rholemma for h.

THEOREM: m-hp-same-h
 $(\text{molws}(n, l, g, h)$
 $\wedge \text{listp}(m)$
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (j \neq k)$
 $\wedge \text{union-at-n}(h, j, m)$
 $\rightarrow \text{union-at-n}(hp, j, m)$

;;;Contrast to the one above,
 ;;;the order of g and gp is reversed.

THEOREM: m-h-same-hp


```

(molws (n, l, g, h)
  ^ listp (m)
  ^ (j ∈ nset (n))
  ^ (k ∈ nset (n))
  ^ mrhoi (n, k, l, g, h, lp, gp, hp)
  ^ (j ≠ k)
  ^ union-at-n (hp, j, m))
→ union-at-n (h, j, m)

```

```

;;;It applies to (at l j m) in stead of
;;;(nth l j).

```

THEOREM: m-l-same-lp-at

```

(molws (n, l, g, h)
  ^ (j ∈ nset (n))
  ^ (k ∈ nset (n))
  ^ (m ∈ N)
  ^ mrhoi (n, k, l, g, h, lp, gp, hp)
  ^ (k ≠ j)
  ^ at (lp, j, m))
→ at (l, j, m)

```

THEOREM: m-gp-same-g-at

```

(molws (n, l, g, h)
  ^ (j ∈ nset (n))
  ^ (k ∈ nset (n))
  ^ (m ∈ N)
  ^ mrhoi (n, k, l, g, h, lp, gp, hp)
  ^ (k ≠ j)
  ^ at (g, j, m))
→ at (gp, j, m)

```

THEOREM: m-l-same-lp-at-not

```

(molws (n, l, g, h)
  ^ (m ∈ N)
  ^ (j ∈ nset (n))
  ^ (k ∈ nset (n))
  ^ mrhoi (n, k, l, g, h, lp, gp, hp)
  ^ (j ≠ k)
  ^ (¬ at (l, j, m)))
→ (¬ at (lp, j, m))

```

```

;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;; mllg.ev ;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;mrhoi0

```

THEOREM: n-neq-k-mrhoi0

(listp (l)
 \wedge listp (g)
 \wedge ($n \in \mathbf{N}$)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge ($k \neq n$)
 \wedge at ($l, k, 0$)
 \wedge lg-at-n (n, l, g)
 \rightarrow lg-at-n ($n, \text{move}(l, k, 1), g$)

EVENT: Disable n-neq-k-mrhoi0.

THEOREM: n-eq-k-mrhoi0

(listp (l)
 \wedge listp (g)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge at ($l, k, 0$)
 \wedge lg-at-n (k, l, g)
 \rightarrow lg-at-n ($k, \text{move}(l, k, 1), g$)

EVENT: Disable n-eq-k-mrhoi0.

THEOREM: lg-at-mrhoi0

(listp (l)
 \wedge listp (g)
 \wedge ($n \in \mathbf{N}$)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge at ($l, k, 0$)
 \wedge lg-at-n (n, l, g)
 \rightarrow lg-at-n ($n, \text{move}(l, k, 1), g$)

EVENT: Disable lg-at-mrhoi0.

THEOREM: lg-mrhoi0

(listp (l)
 \wedge listp (g)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge ($n \in \mathbf{N}$)
 \wedge at ($l, k, 0$)
 \wedge lg (n, l, g)
 \rightarrow lg ($n, \text{move}(l, k, 1), g$)

EVENT: Disable lg-mrhoi0.

THEOREM: mrhoi0-preserves-lg
 $(\text{molws}(n, l, g, h)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi0}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{lg}(n, l, g)$
 $\rightarrow \text{lg}(n, lp, gp)$

;;;mrhoi1a

THEOREM: n-neq-k-mrhoi1a
 $(\text{listp}(l)$
 $\wedge \text{listp}(g)$
 $\wedge (n \in \mathbf{N})$
 $\wedge (k \in \text{nset}(\text{length}(l)))$
 $\wedge (k \neq n)$
 $\wedge \text{at}(l, k, 1)$
 $\wedge \text{lg-at-n}(n, l, g)$
 $\rightarrow \text{lg-at-n}(n, \text{move}(l, k, 2), g)$

EVENT: Disable n-neq-k-mrhoi1a.

THEOREM: n-eq-k-mrhoi1a
 $(\text{listp}(l)$
 $\wedge \text{listp}(g)$
 $\wedge (k \in \text{nset}(\text{length}(l)))$
 $\wedge \text{at}(l, k, 1)$
 $\wedge \text{lg-at-n}(k, l, g)$
 $\rightarrow \text{lg-at-n}(n, \text{move}(l, k, 2), g)$

EVENT: Disable n-eq-k-mrhoi1a.

THEOREM: lg-at-mrhoi1a
 $(\text{listp}(l)$
 $\wedge \text{listp}(g)$
 $\wedge (n \in \mathbf{N})$
 $\wedge (k \in \text{nset}(\text{length}(l)))$
 $\wedge \text{at}(l, k, 1)$
 $\wedge \text{lg-at-n}(n, l, g)$
 $\rightarrow \text{lg-at-n}(n, \text{move}(l, k, 2), g)$

EVENT: Disable lg-at-mrhoi1a.

THEOREM: lg-mrhoi1a

$(\text{listp}(l)$
 $\wedge \text{listp}(g)$
 $\wedge (k \in \text{nset}(\text{length}(l)))$
 $\wedge (n \in \mathbf{N})$
 $\wedge \text{at}(l, k, 1)$
 $\wedge \text{lg}(n, l, g)$
 $\rightarrow \text{lg}(n, \text{move}(l, k, 2), g)$

EVENT: Disable lg-mrhoi1a.

THEOREM: mrhoi1a-preserves-lg
 $(\text{molws}(n, l, g, h)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi1a}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{lg}(n, l, g)$
 $\rightarrow \text{lg}(n, lp, gp)$

;;;mrhoi1b

THEOREM: mrhoi1b-preserves-lg
 $(\text{molws}(n, l, g, h)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi1b}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{lg}(n, l, g)$
 $\rightarrow \text{lg}(n, lp, gp)$

;;;mrhoi2

THEOREM: n-neq-k-mrhoi2
 $(\text{listp}(l)$
 $\wedge \text{listp}(g)$
 $\wedge (n \in \mathbf{N})$
 $\wedge (k \in \text{nset}(\text{length}(l)))$
 $\wedge (k \neq n)$
 $\wedge \text{at}(l, k, 2)$
 $\wedge \text{lg-at-n}(n, l, g)$
 $\rightarrow \text{lg-at-n}(n, \text{move}(l, k, 3), \text{move}(g, k, 1))$

EVENT: Disable n-neq-k-mrhoi2.

THEOREM: n-eq-k-mrhoi2
 $(\text{listp}(l)$
 $\wedge \text{listp}(g)$
 $\wedge (k \in \text{nset}(\text{length}(l)))$

$$\begin{aligned}
& \wedge \text{at}(l, k, 2) \\
& \wedge \text{lg-at-n}(k, l, g) \\
& \rightarrow \text{lg-at-n}(n, \text{move}(l, k, 3), \text{move}(g, k, 1))
\end{aligned}$$

EVENT: Disable n-eq-k-mrhoi2.

THEOREM: lg-at-mrhoi2

$$\begin{aligned}
& (\text{listp}(l) \\
& \wedge \text{listp}(g) \\
& \wedge (n \in \mathbf{N}) \\
& \wedge (k \in \text{nset}(\text{length}(l))) \\
& \wedge \text{at}(l, k, 2) \\
& \wedge \text{lg-at-n}(n, l, g) \\
& \rightarrow \text{lg-at-n}(n, \text{move}(l, k, 3), \text{move}(g, k, 1))
\end{aligned}$$

EVENT: Disable lg-at-mrhoi2.

THEOREM: lg-mrhoi2

$$\begin{aligned}
& (\text{listp}(l) \\
& \wedge \text{listp}(g) \\
& \wedge (k \in \text{nset}(\text{length}(l))) \\
& \wedge (n \in \mathbf{N}) \\
& \wedge \text{at}(l, k, 2) \\
& \wedge \text{lg}(n, l, g) \\
& \rightarrow \text{lg}(n, \text{move}(l, k, 3), \text{move}(g, k, 1))
\end{aligned}$$

EVENT: Disable lg-mrhoi2.

THEOREM: mrhoi2-preserves-lg

$$\begin{aligned}
& (\text{molws}(n, l, g, h) \\
& \wedge (k \in \text{nset}(n)) \\
& \wedge \text{mrhoi2}(n, k, l, g, h, lp, gp, hp) \\
& \wedge \text{lg}(n, l, g) \\
& \rightarrow \text{lg}(n, lp, gp)
\end{aligned}$$

;;;mrhoi3a

THEOREM: n-neq-k-mrhoi3a

$$\begin{aligned}
& (\text{listp}(l) \\
& \wedge \text{listp}(g) \\
& \wedge (n \in \mathbf{N}) \\
& \wedge (k \in \text{nset}(\text{length}(l))) \\
& \wedge (k \neq n)
\end{aligned}$$

\wedge at($l, k, 3$)
 \wedge lg-at-n(n, l, g)
 \rightarrow lg-at-n($n, \text{move}(l, k, 4), g$)

EVENT: Disable n-neq-k-mrhoi3a.

THEOREM: n-eq-k-mrhoi3a

(listp(l)
 \wedge listp(g)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge at($l, k, 3$)
 \wedge lg-at-n(k, l, g)
 \rightarrow lg-at-n($k, \text{move}(l, k, 4), g$)

EVENT: Disable n-eq-k-mrhoi3a.

THEOREM: lg-at-mrhoi3a

(listp(l)
 \wedge listp(g)
 \wedge ($n \in \mathbf{N}$)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge at($l, k, 3$)
 \wedge lg-at-n(n, l, g)
 \rightarrow lg-at-n($n, \text{move}(l, k, 4), g$)

EVENT: Disable lg-at-mrhoi3a.

THEOREM: lg-mrhoi3a

(listp(l)
 \wedge listp(g)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge ($n \in \mathbf{N}$)
 \wedge at($l, k, 3$)
 \wedge lg(n, l, g)
 \rightarrow lg($n, \text{move}(l, k, 4), g$)

EVENT: Disable lg-mrhoi3a.

THEOREM: mrhoi3a-preserves-lg

(molws(n, l, g, h)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi3a($n, k, l, g, h, lp, gp, hp$)
 \wedge lg(n, l, g)
 \rightarrow lg(n, lp, gp)

;;;mrhoi3b

THEOREM: mrhoi3b-preserves-lg

(molws (n, l, g, h)
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi3b}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{lg}(n, l, g)$
 $\rightarrow \text{lg}(n, lp, gp)$

;;;mrhoi4

THEOREM: n-neq-k-mrhoi4

(listp (l)
 $\wedge \text{listp}(g)$
 $\wedge (n \in \mathbf{N})$
 $\wedge (k \in \text{nset}(\text{length}(l)))$
 $\wedge (k \neq n)$
 $\wedge \text{at}(l, k, 4)$
 $\wedge \text{lg-at-n}(n, l, g)$
 $\rightarrow \text{lg-at-n}(n, \text{move}(l, k, 5), \text{move}(g, k, 3))$

EVENT: Disable n-neq-k-mrhoi4.

THEOREM: n-eq-k-mrhoi4

(listp (l)
 $\wedge \text{listp}(g)$
 $\wedge (k \in \text{nset}(\text{length}(l)))$
 $\wedge \text{at}(l, k, 4)$
 $\wedge \text{lg-at-n}(k, l, g)$
 $\rightarrow \text{lg-at-n}(n, \text{move}(l, k, 5), \text{move}(g, k, 3))$

EVENT: Disable n-eq-k-mrhoi4.

THEOREM: lg-at-mrhoi4

(listp (l)
 $\wedge \text{listp}(g)$
 $\wedge (n \in \mathbf{N})$
 $\wedge (k \in \text{nset}(\text{length}(l)))$
 $\wedge \text{at}(l, k, 4)$
 $\wedge \text{lg-at-n}(n, l, g)$
 $\rightarrow \text{lg-at-n}(n, \text{move}(l, k, 5), \text{move}(g, k, 3))$

EVENT: Disable lg-at-mrhoi4.

THEOREM: lg-mrnoi4

(listp (l)
 \wedge listp (g)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge ($n \in \mathbf{N}$)
 \wedge at ($l, k, 4$)
 \wedge lg (n, l, g)
 \rightarrow lg ($n, \text{move}(l, k, 5), \text{move}(g, k, 3)$)

EVENT: Disable lg-mrnoi4.

THEOREM: mrnoi4-preserves-lg

(molws (n, l, g, h)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrnoi4 ($n, k, l, g, h, lp, gp, hp$)
 \wedge lg (n, l, g)
 \rightarrow lg (n, lp, gp)

;;mrnoi5a

THEOREM: n-neq-k-mrnoi5a

(listp (l)
 \wedge listp (g)
 \wedge ($n \in \mathbf{N}$)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge ($k \neq n$)
 \wedge at ($l, k, 5$)
 \wedge lg-at-n (n, l, g)
 \rightarrow lg-at-n ($n, \text{move}(l, k, 8), g$)

EVENT: Disable n-neq-k-mrnoi5a.

THEOREM: n-eq-k-mrnoi5a

(listp (l)
 \wedge listp (g)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge at ($l, k, 5$)
 \wedge lg-at-n (k, l, g)
 \rightarrow lg-at-n ($k, \text{move}(l, k, 8), g$)

EVENT: Disable n-eq-k-mrnoi5a.

THEOREM: lg-at-mrnoi5a

$(\text{listp}(l)$
 $\wedge \text{listp}(g)$
 $\wedge (n \in \mathbf{N})$
 $\wedge (k \in \text{nset}(\text{length}(l)))$
 $\wedge \text{at}(l, k, 5)$
 $\wedge \text{lg-at-n}(n, l, g)$
 $\rightarrow \text{lg-at-n}(n, \text{move}(l, k, 8), g)$

EVENT: Disable lg-at-mrhei5a.

THEOREM: lg-mrhei5a

$(\text{listp}(l)$
 $\wedge \text{listp}(g)$
 $\wedge (k \in \text{nset}(\text{length}(l)))$
 $\wedge (n \in \mathbf{N})$
 $\wedge \text{at}(l, k, 5)$
 $\wedge \text{lg}(n, l, g)$
 $\rightarrow \text{lg}(n, \text{move}(l, k, 8), g)$

EVENT: Disable lg-mrhei5a.

THEOREM: mrhei5a-preserves-lg

$(\text{molws}(n, l, g, h)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhei5a}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{lg}(n, l, g)$
 $\rightarrow \text{lg}(n, lp, gp)$

;;;mrhei5b

THEOREM: n-neq-k-mrhei5b

$(\text{listp}(l)$
 $\wedge \text{listp}(g)$
 $\wedge (n \in \mathbf{N})$
 $\wedge (k \in \text{nset}(\text{length}(l)))$
 $\wedge (k \neq n)$
 $\wedge \text{at}(l, k, 5)$
 $\wedge \text{lg-at-n}(n, l, g)$
 $\rightarrow \text{lg-at-n}(n, \text{move}(l, k, 6), g)$

EVENT: Disable n-neq-k-mrhei5b.

THEOREM: n-eq-k-mrhei5b

$(\text{listp}(l)$
 $\wedge \text{listp}(g)$
 $\wedge (k \in \text{nset}(\text{length}(l)))$
 $\wedge \text{at}(l, k, 5)$
 $\wedge \text{lg-at-n}(k, l, g)$
 $\rightarrow \text{lg-at-n}(k, \text{move}(l, k, 6), g)$

EVENT: Disable n-eq-k-mrhoi5b.

THEOREM: lg-at-mrhoi5b
 $(\text{listp}(l)$
 $\wedge \text{listp}(g)$
 $\wedge (n \in \mathbf{N})$
 $\wedge (k \in \text{nset}(\text{length}(l)))$
 $\wedge \text{at}(l, k, 5)$
 $\wedge \text{lg-at-n}(n, l, g)$
 $\rightarrow \text{lg-at-n}(n, \text{move}(l, k, 6), g)$

EVENT: Disable lg-at-mrhoi5b.

THEOREM: lg-mrhoi5b
 $(\text{listp}(l)$
 $\wedge \text{listp}(g)$
 $\wedge (k \in \text{nset}(\text{length}(l)))$
 $\wedge (n \in \mathbf{N})$
 $\wedge \text{at}(l, k, 5)$
 $\wedge \text{lg}(n, l, g)$
 $\rightarrow \text{lg}(n, \text{move}(l, k, 6), g)$

EVENT: Disable lg-mrhoi5b.

THEOREM: mrhoi5b-preserves-lg
 $(\text{molws}(n, l, g, h)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi5b}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{lg}(n, l, g)$
 $\rightarrow \text{lg}(n, lp, gp)$

;;;mrhoi5c

THEOREM: mrhoi5c-preserves-lg
 $(\text{molws}(n, l, g, h)$
 $\wedge (k \in \text{nset}(n))$

$$\begin{aligned}
& \wedge \text{mrhoi5c}(n, k, l, g, h, lp, gp, hp) \\
& \wedge \text{lg}(n, l, g) \\
& \rightarrow \text{lg}(n, lp, gp)
\end{aligned}$$

;;mrhoi6

THEOREM: n-neq-k-mrhoi6

$$\begin{aligned}
& (\text{listp}(l) \\
& \wedge \text{listp}(g) \\
& \wedge (n \in \mathbf{N}) \\
& \wedge (k \in \text{nset}(\text{length}(l))) \\
& \wedge (k \neq n) \\
& \wedge \text{at}(l, k, 6) \\
& \wedge \text{lg-at-n}(n, l, g)) \\
& \rightarrow \text{lg-at-n}(n, \text{move}(l, k, 7), \text{move}(g, k, 2))
\end{aligned}$$

EVENT: Disable n-neq-k-mrhoi6.

THEOREM: n-eq-k-mrhoi6

$$\begin{aligned}
& (\text{listp}(l) \\
& \wedge \text{listp}(g) \\
& \wedge (k \in \text{nset}(\text{length}(l))) \\
& \wedge \text{at}(l, k, 6) \\
& \wedge \text{lg-at-n}(k, l, g)) \\
& \rightarrow \text{lg-at-n}(n, \text{move}(l, k, 7), \text{move}(g, k, 2))
\end{aligned}$$

EVENT: Disable n-eq-k-mrhoi6.

THEOREM: lg-at-mrhoi6

$$\begin{aligned}
& (\text{listp}(l) \\
& \wedge \text{listp}(g) \\
& \wedge (n \in \mathbf{N}) \\
& \wedge (k \in \text{nset}(\text{length}(l))) \\
& \wedge \text{at}(l, k, 6) \\
& \wedge \text{lg-at-n}(n, l, g)) \\
& \rightarrow \text{lg-at-n}(n, \text{move}(l, k, 7), \text{move}(g, k, 2))
\end{aligned}$$

EVENT: Disable lg-at-mrhoi6.

THEOREM: lg-mrhoi6

$$\begin{aligned}
& (\text{listp}(l) \\
& \wedge \text{listp}(g) \\
& \wedge (k \in \text{nset}(\text{length}(l)))
\end{aligned}$$

$\wedge (n \in \mathbf{N})$
 $\wedge \text{at}(l, k, 6)$
 $\wedge \text{lg}(n, l, g)$
 $\rightarrow \text{lg}(n, \text{move}(l, k, 7), \text{move}(g, k, 2))$

EVENT: Disable lg-mrhoi6.

THEOREM: mrhoi6-preserves-lg
 $(\text{molws}(n, l, g, h)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi6}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{lg}(n, l, g)$
 $\rightarrow \text{lg}(n, lp, gp)$

;;;mrhoi7a

THEOREM: n-neq-k-mrhoi7a
 $(\text{listp}(l)$
 $\wedge \text{listp}(g)$
 $\wedge (n \in \mathbf{N})$
 $\wedge (k \in \text{nset}(\text{length}(l)))$
 $\wedge (k \neq n)$
 $\wedge \text{at}(l, k, 7)$
 $\wedge \text{lg-at-n}(n, l, g)$
 $\rightarrow \text{lg-at-n}(n, \text{move}(l, k, 8), g)$

EVENT: Disable n-neq-k-mrhoi7a.

THEOREM: n-eq-k-mrhoi7a
 $(\text{listp}(l)$
 $\wedge \text{listp}(g)$
 $\wedge (k \in \text{nset}(\text{length}(l)))$
 $\wedge \text{at}(l, k, 7)$
 $\wedge \text{lg-at-n}(k, l, g)$
 $\rightarrow \text{lg-at-n}(k, \text{move}(l, k, 8), g)$

EVENT: Disable n-eq-k-mrhoi7a.

THEOREM: lg-at-mrhoi7a
 $(\text{listp}(l)$
 $\wedge \text{listp}(g)$
 $\wedge (n \in \mathbf{N})$
 $\wedge (k \in \text{nset}(\text{length}(l)))$

$$\begin{aligned}
& \wedge \text{at}(l, k, 7) \\
& \wedge \text{lg-at-n}(n, l, g) \\
& \rightarrow \text{lg-at-n}(n, \text{move}(l, k, 8), g)
\end{aligned}$$

EVENT: Disable lg-at-mrnoi7a.

THEOREM: lg-mrnoi7a

$$\begin{aligned}
& (\text{listp}(l) \\
& \wedge \text{listp}(g) \\
& \wedge (k \in \text{nset}(\text{length}(l))) \\
& \wedge (n \in \mathbf{N}) \\
& \wedge \text{at}(l, k, 7) \\
& \wedge \text{lg}(n, l, g) \\
& \rightarrow \text{lg}(n, \text{move}(l, k, 8), g)
\end{aligned}$$

EVENT: Disable lg-mrnoi7a.

THEOREM: mrnoi7a-preserves-lg

$$\begin{aligned}
& (\text{molws}(n, l, g, h) \\
& \wedge (k \in \text{nset}(n)) \\
& \wedge \text{mrnoi7a}(n, k, l, g, h, lp, gp, hp) \\
& \wedge \text{lg}(n, l, g) \\
& \rightarrow \text{lg}(n, lp, gp)
\end{aligned}$$

;;;mrnoi7b

THEOREM: mrnoi7b-preserves-lg

$$\begin{aligned}
& (\text{molws}(n, l, g, h) \\
& \wedge (k \in \text{nset}(n)) \\
& \wedge \text{mrnoi7b}(n, k, l, g, h, lp, gp, hp) \\
& \wedge \text{lg}(n, l, g) \\
& \rightarrow \text{lg}(n, lp, gp)
\end{aligned}$$

;;;mrnoi8

THEOREM: n-neq-k-mrnoi8

$$\begin{aligned}
& (\text{listp}(l) \\
& \wedge \text{listp}(g) \\
& \wedge (n \in \mathbf{N}) \\
& \wedge (k \in \text{nset}(\text{length}(l))) \\
& \wedge (k \neq n) \\
& \wedge \text{at}(l, k, 8) \\
& \wedge \text{lg-at-n}(n, l, g) \\
& \rightarrow \text{lg-at-n}(n, \text{move}(l, k, 9), \text{move}(g, k, 4))
\end{aligned}$$

EVENT: Disable n-neq-k-mrhoi8.

THEOREM: n-eq-k-mrhoi8
(listp (l)
 \wedge listp (g)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge at ($l, k, 8$)
 \wedge lg-at-n (k, l, g)
 \rightarrow lg-at-n ($n, \text{move}(l, k, 9), \text{move}(g, k, 4)$))

EVENT: Disable n-eq-k-mrhoi8.

THEOREM: lg-at-mrhoi8
(listp (l)
 \wedge listp (g)
 \wedge ($n \in \mathbf{N}$)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge at ($l, k, 8$)
 \wedge lg-at-n (n, l, g)
 \rightarrow lg-at-n ($n, \text{move}(l, k, 9), \text{move}(g, k, 4)$))

EVENT: Disable lg-at-mrhoi8.

THEOREM: lg-mrhoi8
(listp (l)
 \wedge listp (g)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge ($n \in \mathbf{N}$)
 \wedge at ($l, k, 8$)
 \wedge lg (n, l, g)
 \rightarrow lg ($n, \text{move}(l, k, 9), \text{move}(g, k, 4)$))

EVENT: Disable lg-mrhoi8.

THEOREM: mrhoi8-preserves-lg
(molws (n, l, g, h)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi8 ($n, k, l, g, h, lp, gp, hp$)
 \wedge lg (n, l, g)
 \rightarrow lg (n, lp, gp))

;;mrhoi9a

THEOREM: n-neq-k-mrhoi9a

(listp (l)
 \wedge listp (g)
 \wedge ($n \in \mathbf{N}$)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge ($k \neq n$)
 \wedge at ($l, k, \mathbf{9}$)
 \wedge lg-at-n (n, l, g)
 \rightarrow lg-at-n ($n, \text{move}(l, k, \mathbf{10}), g$)

EVENT: Disable n-neq-k-mrhoi9a.

THEOREM: n-eq-k-mrhoi9a

(listp (l)
 \wedge listp (g)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge at ($l, k, \mathbf{9}$)
 \wedge lg-at-n (k, l, g)
 \rightarrow lg-at-n ($k, \text{move}(l, k, \mathbf{10}), g$)

EVENT: Disable n-eq-k-mrhoi9a.

THEOREM: lg-at-mrhoi9a

(listp (l)
 \wedge listp (g)
 \wedge ($n \in \mathbf{N}$)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge at ($l, k, \mathbf{9}$)
 \wedge lg-at-n (n, l, g)
 \rightarrow lg-at-n ($n, \text{move}(l, k, \mathbf{10}), g$)

EVENT: Disable lg-at-mrhoi9a.

THEOREM: lg-mrhoi9a

(listp (l)
 \wedge listp (g)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge ($n \in \mathbf{N}$)
 \wedge at ($l, k, \mathbf{9}$)
 \wedge lg (n, l, g)
 \rightarrow lg ($n, \text{move}(l, k, \mathbf{10}), g$)

EVENT: Disable lg-mrhoi9a.

THEOREM: mrhoi9a-preserves-lg
 (molws (n, l, g, h)
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi9a}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{lg}(n, l, g)$
 $\rightarrow \text{lg}(n, lp, gp)$

;;;mrhoi9b

THEOREM: mrhoi9b-preserves-lg
 (molws (n, l, g, h)
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi9b}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{lg}(n, l, g)$
 $\rightarrow \text{lg}(n, lp, gp)$

;;;mrhoi10

THEOREM: n-neq-k-mrhoi10
 (listp (l)
 $\wedge \text{listp}(g)$
 $\wedge (n \in \mathbf{N})$
 $\wedge (k \in \text{nset}(\text{length}(l)))$
 $\wedge (k \neq n)$
 $\wedge \text{at}(l, k, 10)$
 $\wedge \text{lg-at-n}(n, l, g)$
 $\rightarrow \text{lg-at-n}(n, \text{move}(l, k, 11), g)$

EVENT: Disable n-neq-k-mrhoi10.

THEOREM: n-eq-k-mrhoi10
 (listp (l)
 $\wedge \text{listp}(g)$
 $\wedge (k \in \text{nset}(\text{length}(l)))$
 $\wedge \text{at}(l, k, 10)$
 $\wedge \text{lg-at-n}(k, l, g)$
 $\rightarrow \text{lg-at-n}(k, \text{move}(l, k, 11), g)$

EVENT: Disable n-eq-k-mrhoi10.

THEOREM: lg-at-mrhoi10
 (listp (l)
 $\wedge \text{listp}(g)$
 $\wedge (n \in \mathbf{N})$

$\wedge (k \in \text{nset}(\text{length}(l)))$
 $\wedge \text{at}(l, k, 10)$
 $\wedge \text{lg-at-n}(n, l, g)$
 $\rightarrow \text{lg-at-n}(n, \text{move}(l, k, 11), g)$

EVENT: Disable lg-at-mrhoi10.

THEOREM: lg-mrhoi10
 $(\text{listp}(l)$
 $\wedge \text{listp}(g)$
 $\wedge (k \in \text{nset}(\text{length}(l)))$
 $\wedge (n \in \mathbf{N})$
 $\wedge \text{at}(l, k, 10)$
 $\wedge \text{lg}(n, l, g)$
 $\rightarrow \text{lg}(n, \text{move}(l, k, 11), g)$

EVENT: Disable lg-mrhoi10.

THEOREM: mrhoi10-preserves-lg
 $(\text{molws}(n, l, g, h)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi10}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{lg}(n, l, g)$
 $\rightarrow \text{lg}(n, lp, gp)$

;;;mrhoi11a

THEOREM: n-neq-k-mrhoi11a
 $(\text{listp}(l)$
 $\wedge \text{listp}(g)$
 $\wedge (n \in \mathbf{N})$
 $\wedge (k \in \text{nset}(\text{length}(l)))$
 $\wedge (k \neq n)$
 $\wedge \text{at}(l, k, 11)$
 $\wedge \text{lg-at-n}(n, l, g)$
 $\rightarrow \text{lg-at-n}(n, \text{move}(l, k, 12), g)$

EVENT: Disable n-neq-k-mrhoi11a.

THEOREM: n-eq-k-mrhoi11a
 $(\text{listp}(l)$
 $\wedge \text{listp}(g)$
 $\wedge (k \in \text{nset}(\text{length}(l)))$

\wedge at($l, k, 11$)
 \wedge lg-at-n(k, l, g)
 \rightarrow lg-at-n($k, \text{move}(l, k, 12), g$)

EVENT: Disable n-eq-k-mrhoi11a.

THEOREM: lg-at-mrhoi11a
 (listp(l)
 \wedge listp(g)
 \wedge ($n \in \mathbf{N}$)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge at($l, k, 11$)
 \wedge lg-at-n(n, l, g)
 \rightarrow lg-at-n($n, \text{move}(l, k, 12), g$)

EVENT: Disable lg-at-mrhoi11a.

THEOREM: lg-mrhoi11a
 (listp(l)
 \wedge listp(g)
 \wedge ($k \in \text{nset}(\text{length}(l))$)
 \wedge ($n \in \mathbf{N}$)
 \wedge at($l, k, 11$)
 \wedge lg(n, l, g)
 \rightarrow lg($n, \text{move}(l, k, 12), g$)

EVENT: Disable lg-mrhoi11a.

THEOREM: mrhoi11a-preserves-lg
 (molws(n, l, g, h)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi11a($n, k, l, g, h, lp, gp, hp$)
 \wedge lg(n, l, g)
 \rightarrow lg(n, lp, gp)

;;;mrhoi11b

THEOREM: mrhoi11b-preserves-lg
 (molws(n, l, g, h)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi11b($n, k, l, g, h, lp, gp, hp$)
 \wedge lg(n, l, g)
 \rightarrow lg(n, lp, gp)

;;mrhoi12

THEOREM: n-neq-k-mrhoi12

(listp l)
^ listp g
^ ($n \in \mathbf{N}$)
^ ($k \in \text{nset}(\text{length}(l))$)
^ ($k \neq n$)
^ at $(l, k, 12)$
^ lg-at-n (n, l, g)
→ lg-at-n $(n, \text{move}(l, k, 0), \text{move}(g, k, 0))$

EVENT: Disable n-neq-k-mrhoi12.

THEOREM: n-eq-k-mrhoi12

(listp l)
^ listp g
^ ($k \in \text{nset}(\text{length}(l))$)
^ at $(l, k, 12)$
^ lg-at-n (k, l, g)
→ lg-at-n $(n, \text{move}(l, k, 0), \text{move}(g, k, 0))$

EVENT: Disable n-eq-k-mrhoi12.

THEOREM: lg-at-mrhoi12

(listp l)
^ listp g
^ ($n \in \mathbf{N}$)
^ ($k \in \text{nset}(\text{length}(l))$)
^ at $(l, k, 12)$
^ lg-at-n (n, l, g)
→ lg-at-n $(n, \text{move}(l, k, 0), \text{move}(g, k, 0))$

EVENT: Disable lg-at-mrhoi12.

THEOREM: lg-mrhoi12

(listp l)
^ listp g
^ ($k \in \text{nset}(\text{length}(l))$)
^ ($n \in \mathbf{N}$)
^ at $(l, k, 12)$
^ lg (n, l, g)
→ lg $(n, \text{move}(l, k, 0), \text{move}(g, k, 0))$

EVENT: Disable lg-mrhoi12.

THEOREM: mrhoi12-preserves-lg
(molws (n, l, g, h)
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi12}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{lg}(n, l, g)$
 $\rightarrow \text{lg}(n, lp, gp)$

THEOREM: mrho-preserves-lg
(molws (n, l, g, h)
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{lg}(n, l, g)$
 $\rightarrow \text{lg}(n, lp, gp)$

EVENT: Disable mrhoi0-preserves-lg.

EVENT: Disable mrhoi1a-preserves-lg.

EVENT: Disable mrhoi1b-preserves-lg.

EVENT: Disable mrhoi2-preserves-lg.

EVENT: Disable mrhoi3a-preserves-lg.

EVENT: Disable mrhoi3b-preserves-lg.

EVENT: Disable mrhoi4-preserves-lg.

EVENT: Disable mrhoi5a-preserves-lg.

EVENT: Disable mrhoi5b-preserves-lg.

EVENT: Disable mrhoi5c-preserves-lg.

EVENT: Disable mrhoi6-preserves-lg.

EVENT: Disable mrhoi7a-preserves-lg.

EVENT: Disable mrhoi7b-preserves-lg.

EVENT: Disable mrhoi8-preserves-lg.

EVENT: Disable mrhoi9a-preserves-lg.

EVENT: Disable mrhoi9b-preserves-lg.

EVENT: Disable mrhoi10-preserves-lg.

EVENT: Disable mrhoi11a-preserves-lg.

EVENT: Disable mrhoi11b-preserves-lg.

EVENT: Disable mrhoi12-preserves-lg.

;;; b0.ev ;;
;;;;;;;;;;;;;;;;;;;;;;;;; b0a ;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;

;;;;;;;;;;;;;;;;;;;;;;;;;Common in mole and atom.

THEOREM: b0a-if1
 $((j \in \text{nset}(n)) \wedge \text{lg}(n, l, g) \wedge (\neg \text{at}(g, j, 1))) \rightarrow (\neg \text{at}(l, j, 4))$

;;;;;;;;;;;;;;;;;;;;;;;;;comm end.

THEOREM: if1-nth-h-k
 $(\text{molws}(n, l, g, h)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge (j \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{at}(h, k, j)$
 $\wedge (\neg \text{at}(g, \text{nth}(h, k), 1)))$
 $\rightarrow (\neg \text{at}(l, j, 4))$

THEOREM: l5-not-g1
 $(\text{molws}(n, l, g, h)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge (j \in \text{nset}(n))$

$$\begin{aligned}
& \wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp) \\
& \wedge \text{at}(h, k, j) \\
& \wedge \text{at}(l, k, 5) \\
& \wedge \text{at}(lp, k, 5) \\
& \rightarrow (\neg \text{at}(g, \text{nth}(h, k), 1))
\end{aligned}$$

THEOREM: l5-nth-h-k-eq-j

$$\begin{aligned}
& (\text{at}(h, k, j) \\
& \wedge \text{molws}(n, l, g, h) \\
& \wedge (k \in \text{nset}(n)) \\
& \wedge (j \in \text{nset}(n)) \\
& \wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp) \\
& \wedge \text{lg}(n, l, g) \\
& \wedge \text{at}(l, k, 5) \\
& \wedge \text{at}(lp, k, 5) \\
& \rightarrow (\neg \text{at}(l, j, 4))
\end{aligned}$$

THEOREM: l5-j-lt-nth-k

$$\begin{aligned}
& ((j < \text{nth}(h, k)) \\
& \wedge \text{molws}(n, l, g, h) \\
& \wedge (k \in \text{nset}(n)) \\
& \wedge (j \in \text{nset}(n)) \\
& \wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp) \\
& \wedge \text{b0a}(n, l, h, k, j) \\
& \wedge \text{at}(l, k, 5) \\
& \rightarrow (\neg \text{at}(l, j, 4))
\end{aligned}$$

THEOREM: nth-k-lt-j-or-eq-j

$$\begin{aligned}
& (\text{molws}(n, l, g, h) \\
& \wedge (k \in \text{nset}(n)) \\
& \wedge (j \in \text{nset}(n)) \\
& \wedge ((j - 1) < \text{nth}(h, k)) \\
& \wedge (j \not< \text{nth}(h, k))) \\
& \rightarrow \text{at}(h, k, j)
\end{aligned}$$

THEOREM: lm-j-not-in-l4

$$\begin{aligned}
& (\text{molws}(n, l, g, h) \\
& \wedge (j \in \text{nset}(n)) \\
& \wedge (k \in \text{nset}(n)) \\
& \wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp) \\
& \wedge \text{lg}(n, l, g) \\
& \wedge \text{b0a}(n, l, h, k, j) \\
& \wedge \text{at}(l, k, 5) \\
& \wedge \text{at}(lp, k, 5) \\
& \wedge ((j - 1) < \text{nth}(h, k))) \\
& \rightarrow (\neg \text{at}(l, j, 4))
\end{aligned}$$

THEOREM: cond-l5
(molws (n, l, g, h)
 $\wedge (k \in \text{nset}(n))$
 $\wedge (j \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{at}(l, k, 5)$
 $\wedge (j < \text{nth}(hp, k))$)
 $\rightarrow ((j - 1) < \text{nth}(h, k))$

THEOREM: j-not-in-l4
(molws (n, l, g, h)
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{b0a}(n, l, h, k, j)$
 $\wedge \text{at}(l, k, 5)$
 $\wedge \text{at}(lp, k, 5)$
 $\wedge (j < \text{nth}(hp, k))$)
 $\rightarrow (\neg \text{at}(l, j, 4))$

THEOREM: k-in-l5
(molws (n, l, g, h)
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{at}(lp, k, 5)$
 $\wedge (j < \text{nth}(hp, k))$)
 $\rightarrow \text{at}(l, k, 5)$

;;;The order of the hints is crucial.

THEOREM: lm-b0a-i-eq-k-j-neq-k
(molws (n, l, g, h)
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{b0a}(n, l, h, k, j)$
 $\wedge \text{at}(lp, k, 5)$
 $\wedge (j < \text{nth}(hp, k))$)
 $\rightarrow (\neg \text{at}(l, j, 4))$

THEOREM: b0a-i-eq-k-j-neq-k
(molws (n, l, g, h)

$$\begin{aligned}
& \wedge (j \in \text{nset}(n)) \\
& \wedge (k \in \text{nset}(n)) \\
& \wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp) \\
& \wedge (j \neq k) \\
& \wedge \text{lg}(n, l, g) \\
& \wedge \text{b0a}(n, l, h, k, j) \\
& \rightarrow \text{b0a}(n, lp, hp, k, j)
\end{aligned}$$

THEOREM: b0a-i-j-eq-k

$$\begin{aligned}
& (\text{molws}(n, l, g, h) \\
& \wedge (k \in \text{nset}(n)) \\
& \wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp) \\
& \wedge \text{b0a}(n, l, h, k, k)) \\
& \rightarrow \text{b0a}(n, lp, hp, k, k)
\end{aligned}$$

THEOREM: b0a-i-eq-k

$$\begin{aligned}
& (\text{molws}(n, l, g, h) \\
& \wedge (j \in \text{nset}(n)) \\
& \wedge (k \in \text{nset}(n)) \\
& \wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp) \\
& \wedge \text{lg}(n, l, g) \\
& \wedge \text{b0a}(n, l, h, k, j)) \\
& \rightarrow \text{b0a}(n, lp, hp, k, j)
\end{aligned}$$

;;;n-not-less-j is necessary.

THEOREM: cond-lp4

$$\begin{aligned}
& (\text{molws}(n, l, g, h) \\
& \wedge (i \in \text{nset}(n)) \\
& \wedge (k \in \text{nset}(n)) \\
& \wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp) \\
& \wedge \text{at}(l, k, 3) \\
& \wedge (i \not\prec \text{nth}(h, k))) \\
& \rightarrow (\neg \text{at}(lp, k, 4))
\end{aligned}$$

THEOREM: not-l3-then-lp4

$$\begin{aligned}
& (\text{molws}(n, l, g, h) \\
& \wedge (k \in \text{nset}(n)) \\
& \wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp) \\
& \wedge (\neg \text{at}(l, k, 3))) \\
& \rightarrow (\neg \text{at}(lp, k, 4))
\end{aligned}$$

THEOREM: i-in-l5

$$\begin{aligned}
& (\text{molws}(n, l, g, h) \\
& \wedge (i \in \text{nset}(n))
\end{aligned}$$

$$\begin{aligned}
& \wedge (k \in \text{nset}(n)) \\
& \wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp) \\
& \wedge \text{b0b}(n, l, h, i, k) \\
& \wedge \text{at}(l, i, 5) \\
& \wedge (k < \text{nth}(h, i)) \\
& \rightarrow (\neg \text{at}(lp, k, 4))
\end{aligned}$$

THEOREM: lm-b0a-i-neq-k-j-eq-k

$$\begin{aligned}
& (\text{molws}(n, l, g, h) \\
& \wedge (i \in \text{nset}(n)) \\
& \wedge (k \in \text{nset}(n)) \\
& \wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp) \\
& \wedge (i \neq k) \\
& \wedge \text{b0b}(n, l, h, i, k) \\
& \wedge \text{at}(lp, i, 5) \\
& \wedge (k < \text{nth}(h, i)) \\
& \rightarrow (\neg \text{at}(lp, k, 4))
\end{aligned}$$

THEOREM: b0a-i-neq-k-j-eq-k

$$\begin{aligned}
& (\text{molws}(n, l, g, h) \\
& \wedge (i \in \text{nset}(n)) \\
& \wedge (k \in \text{nset}(n)) \\
& \wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp) \\
& \wedge (i \neq k) \\
& \wedge \text{b0a}(n, l, h, i, k) \\
& \wedge \text{b0b}(n, l, h, i, k) \\
& \rightarrow \text{b0a}(n, lp, hp, i, k)
\end{aligned}$$

THEOREM: b0a-i-j-neq-k

$$\begin{aligned}
& (\text{molws}(n, l, g, h) \\
& \wedge (i \in \text{nset}(n)) \\
& \wedge (j \in \text{nset}(n)) \\
& \wedge (k \in \text{nset}(n)) \\
& \wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp) \\
& \wedge (i \neq k) \\
& \wedge (j \neq k) \\
& \wedge \text{b0a}(n, l, h, i, j) \\
& \wedge \text{b0b}(n, l, h, i, j) \\
& \rightarrow \text{b0a}(n, lp, hp, i, j)
\end{aligned}$$

THEOREM: b0a-i-neq-k

$$\begin{aligned}
& (\text{molws}(n, l, g, h) \\
& \wedge (i \in \text{nset}(n)) \\
& \wedge (j \in \text{nset}(n)) \\
& \wedge (k \in \text{nset}(n))
\end{aligned}$$

\wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge ($i \neq k$)
 \wedge b0a(n, l, h, i, j)
 \wedge b0b(n, l, h, i, j)
 \rightarrow b0a(n, lp, hp, i, j)

THEOREM: rho-preserves-b0a

(molws(n, l, g, h)
 \wedge ($i \in \text{nset}(n)$)
 \wedge ($j \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge lg(n, l, g)
 \wedge b0a(n, l, h, i, j)
 \wedge b0b(n, l, h, i, j)
 \rightarrow b0a(n, lp, hp, i, j)

;;;;;;;;;;;;; b0b ;;;;;;;;;;;;;;

;;;;;;;;;;;;;Common in mole and atom.

THEOREM: b0b-if1

$((j \in \text{nset}(n)) \wedge \text{lg}(n, l, g) \wedge \text{at}(l, j, 3)) \rightarrow \text{at}(g, j, 1)$

THEOREM: b0b-if3

$((i \in \text{nset}(n)) \wedge \text{lg}(n, l, g) \wedge \text{at}(l, i, 5))$
 $\rightarrow (\neg \text{union-at-n}(g, i, '(0 1 2)))$

;;;;;;;;;;;;;Common in mole and atom end.

THEOREM: lm-j-neq-h-k

(molws(n, l, g, h)
 \wedge ($k \in \text{nset}(n)$)
 \wedge ($j \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge lg(n, l, g)
 \wedge at($l, j, 3$)
 \wedge ($\neg \text{at}(g, \text{nth}(h, k), 1)$)
 \rightarrow ($\neg \text{at}(h, k, j)$)

THEOREM: h-k-not-g1

(molws(n, l, g, h)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)

\wedge at($l, k, 5$)
 \wedge at($lp, k, 5$)
 \rightarrow (\neg at($g, \text{nth}(h, k), 1$))

THEOREM: j-neq-h-k

(molws(n, l, g, h)
 \wedge ($j \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge lg(n, l, g)
 \wedge at($lp, k, 5$)
 \wedge at($l, k, 5$)
 \wedge at($l, j, 3$)
 \rightarrow (\neg at(h, k, j))

THEOREM: n-k-leq-sub1-i

(molws(n, l, g, h)
 \wedge ($k \in \text{nset}(n)$)
 \wedge ($i \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge (\neg at(h, k, i))
 \wedge ($i \not\leq \text{nth}(h, k)$)
 \rightarrow ($(i - 1) \not\leq \text{nth}(h, k)$)

;;;This is proved with help of n-k-leq-sub1-i.

THEOREM: lm1-j-in-l3

(molws(n, l, g, h)
 \wedge ($j \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge lg(n, l, g)
 \wedge b0b(n, l, h, k, j)
 \wedge at($l, k, 5$)
 \wedge at($lp, k, 5$)
 \wedge at($l, j, 3$)
 \wedge ($(j - 1) < \text{nth}(h, k)$)
 \rightarrow ($k \not\leq \text{nth}(h, j)$)

THEOREM: lm-j-in-l3

(molws(n, l, g, h)
 \wedge ($j \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge lg(n, l, g)

\wedge b0b(n, l, h, k, j)
 \wedge at($l, k, 5$)
 \wedge at($lp, k, 5$)
 \wedge at($l, j, 3$)
 \wedge ($j < \text{nth}(hp, k)$)
 \rightarrow ($k \not< \text{nth}(h, j)$)

;;;The order of hints is crucial.

THEOREM: j-in-l3

(molws(n, l, g, h)
 \wedge ($j \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge lg(n, l, g)
 \wedge b0b(n, l, h, k, j)
 \wedge at($lp, k, 5$)
 \wedge at($l, j, 3$)
 \wedge ($j < \text{nth}(hp, k)$)
 \rightarrow ($k \not< \text{nth}(h, j)$)

THEOREM: lm-b0b-i-eq-k-j-neq-k

(molws(n, l, g, h)
 \wedge ($j \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge ($j \neq k$)
 \wedge lg(n, l, g)
 \wedge b0b(n, l, h, k, j)
 \wedge at($lp, k, 5$)
 \wedge at($lp, j, 3$)
 \wedge ($j < \text{nth}(hp, k)$)
 \rightarrow ($k \not< \text{nth}(h, j)$)

THEOREM: b0b-i-eq-k-j-neq-k

(molws(n, l, g, h)
 \wedge ($j \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge ($j \neq k$)
 \wedge lg(n, l, g)
 \wedge b0b(n, l, h, k, j)
 \rightarrow b0b(n, lp, hp, k, j)

THEOREM: b0b-i-j-eq-k

$$\begin{aligned}
& (\text{molws } (n, l, g, h) \\
& \wedge (k \in \text{nset } (n)) \\
& \wedge \text{mrhoi } (n, k, l, g, h, lp, gp, hp) \\
& \wedge \text{lg } (n, l, g) \\
& \rightarrow \text{b0b } (n, lp, hp, k, k)
\end{aligned}$$

THEOREM: b0b-i-eq-k

$$\begin{aligned}
& (\text{molws } (n, l, g, h) \\
& \wedge (j \in \text{nset } (n)) \\
& \wedge (k \in \text{nset } (n)) \\
& \wedge \text{mrhoi } (n, k, l, g, h, lp, gp, hp) \\
& \wedge \text{lg } (n, l, g) \\
& \wedge \text{b0b } (n, l, h, k, j)) \\
& \rightarrow \text{b0b } (n, lp, hp, k, j)
\end{aligned}$$

THEOREM: lm-i-neq-h-k

$$\begin{aligned}
& (\text{molws } (n, l, g, h) \\
& \wedge (k \in \text{nset } (n)) \\
& \wedge (i \in \text{nset } (n)) \\
& \wedge \text{mrhoi } (n, k, l, g, h, lp, gp, hp) \\
& \wedge \text{lg } (n, l, g) \\
& \wedge \text{at } (l, i, 5) \\
& \wedge \text{union-at-n } (g, \text{nth } (h, k), '(0 1 2))) \\
& \rightarrow (\neg \text{at } (h, k, i))
\end{aligned}$$

THEOREM: h-k-g02

$$\begin{aligned}
& (\text{molws } (n, l, g, h) \\
& \wedge (k \in \text{nset } (n)) \\
& \wedge \text{mrhoi } (n, k, l, g, h, lp, gp, hp) \\
& \wedge \text{at } (l, k, 3) \\
& \wedge \text{at } (lp, k, 3)) \\
& \rightarrow \text{union-at-n } (g, \text{nth } (h, k), '(0 1 2))
\end{aligned}$$

THEOREM: i-neq-h-k

$$\begin{aligned}
& (\text{molws } (n, l, g, h) \\
& \wedge (k \in \text{nset } (n)) \\
& \wedge (i \in \text{nset } (n)) \\
& \wedge \text{mrhoi } (n, k, l, g, h, lp, gp, hp) \\
& \wedge \text{lg } (n, l, g) \\
& \wedge \text{at } (l, i, 5) \\
& \wedge \text{at } (l, k, 3) \\
& \wedge \text{at } (lp, k, 3)) \\
& \rightarrow (\neg \text{at } (h, k, i))
\end{aligned}$$

THEOREM: lm1-k-in-l3-imp

$$\begin{aligned}
& (\text{molws } (n, l, g, h) \\
& \wedge (k \in \text{nset } (n)) \\
& \wedge (i \in \text{nset } (n)) \\
& \wedge \text{mrhoi } (n, k, l, g, h, lp, gp, hp) \\
& \wedge \text{lg } (n, l, g) \\
& \wedge \text{at } (l, i, 5) \\
& \wedge \text{at } (l, k, 3) \\
& \wedge \text{at } (lp, k, 3) \\
& \wedge (i \not\prec \text{nth } (h, k))) \\
& \rightarrow ((i - 1) \not\prec \text{nth } (h, k))
\end{aligned}$$

THEOREM: lm-k-in-l3-imp

$$\begin{aligned}
& (\text{molws } (n, l, g, h) \\
& \wedge (k \in \text{nset } (n)) \\
& \wedge (i \in \text{nset } (n)) \\
& \wedge \text{mrhoi } (n, k, l, g, h, lp, gp, hp) \\
& \wedge \text{lg } (n, l, g) \\
& \wedge \text{bOb } (n, l, h, i, k) \\
& \wedge \text{at } (l, i, 5) \\
& \wedge \text{at } (l, k, 3) \\
& \wedge \text{at } (lp, k, 3) \\
& \wedge (k < \text{nth } (h, i))) \\
& \rightarrow ((i - 1) \not\prec \text{nth } (h, k))
\end{aligned}$$

THEOREM: cond-l3

$$\begin{aligned}
& (\text{molws } (n, l, g, h) \\
& \wedge (k \in \text{nset } (n)) \\
& \wedge (i \in \text{nset } (n)) \\
& \wedge \text{mrhoi } (n, k, l, g, h, lp, gp, hp) \\
& \wedge \text{at } (l, k, 3) \\
& \wedge (i < \text{nth } (hp, k))) \\
& \rightarrow ((i - 1) < \text{nth } (h, k))
\end{aligned}$$

THEOREM: k-in-l3-imp

$$\begin{aligned}
& (\text{molws } (n, l, g, h) \\
& \wedge (i \in \text{nset } (n)) \\
& \wedge (k \in \text{nset } (n)) \\
& \wedge \text{mrhoi } (n, k, l, g, h, lp, gp, hp) \\
& \wedge \text{lg } (n, l, g) \\
& \wedge \text{bOb } (n, l, h, i, k) \\
& \wedge \text{at } (l, i, 5) \\
& \wedge \text{at } (l, k, 3) \\
& \wedge \text{at } (lp, k, 3) \\
& \wedge (k < \text{nth } (h, i))) \\
& \rightarrow (i \not\prec \text{nth } (hp, k))
\end{aligned}$$

THEOREM: k-in-l2-imp

(molws (n, l, g, h)
 $\wedge (i \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{at}(lp, k, 3)$
 $\wedge \text{at}(l, k, 2))$
 $\rightarrow (i \not\prec \text{nth}(hp, k))$

THEOREM: lp3-then-l2-or-l3

(molws (n, l, g, h)
 $\wedge (i \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{at}(lp, k, 3)$
 $\wedge (\neg \text{at}(l, k, 2)))$
 $\rightarrow \text{at}(l, k, 3)$

THEOREM: b0b-i-in-l5

(molws (n, l, g, h)
 $\wedge (i \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{b0b}(n, l, h, i, k)$
 $\wedge \text{at}(l, i, 5)$
 $\wedge \text{at}(lp, k, 3)$
 $\wedge (k < \text{nth}(h, i)))$
 $\rightarrow (i \not\prec \text{nth}(hp, k))$

THEOREM: lm-b0b-i-neq-k-j-eq-k

(molws (n, l, g, h)
 $\wedge (i \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (i \neq k)$
 $\wedge \text{b0b}(n, l, h, i, k)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{at}(lp, i, 5)$
 $\wedge \text{at}(lp, k, 3)$
 $\wedge (k < \text{nth}(h, i)))$
 $\rightarrow (i \not\prec \text{nth}(hp, k))$

THEOREM: b0b-i-neq-k-j-eq-k

(molws (n, l, g, h)

$\wedge (i \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (i \neq k)$
 $\wedge \text{b0b}(n, l, h, i, k)$
 $\wedge \text{lg}(n, l, g)$
 $\rightarrow \text{b0b}(n, lp, hp, i, k)$

THEOREM: b0b-i-j-neq-k

$(\text{molws}(n, l, g, h)$
 $\wedge (i \in \text{nset}(n))$
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (i \neq k)$
 $\wedge (j \neq k)$
 $\wedge \text{b0b}(n, l, h, i, j))$
 $\rightarrow \text{b0b}(n, lp, hp, i, j)$

THEOREM: b0b-i-neq-k

$(\text{molws}(n, l, g, h)$
 $\wedge (i \in \text{nset}(n))$
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (i \neq k)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{b0b}(n, l, h, i, j))$
 $\rightarrow \text{b0b}(n, lp, hp, i, j)$

THEOREM: rho-preserves-b0b

$(\text{molws}(n, l, g, h)$
 $\wedge (i \in \text{nset}(n))$
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{b0b}(n, l, h, i, j))$
 $\rightarrow \text{b0b}(n, lp, hp, i, j)$

;; **b1.ev** ;;
 ;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;; **b1a** ;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;

THEOREM: lm-h-k-eq-add1-n-nex-hint

$$((n \neq 0) \wedge \text{listp}(h) \wedge (n \not\prec i) \wedge (k \in \text{nset}(n)) \wedge (n < \text{nth}(h, k)))$$

$$\rightarrow (\neg \text{exist-hint-8-12-3-4}(i, l, g, h, k))$$

THEOREM: h-k-eq-add1-n-nex-hint
 $(\text{molws}(n, l, g, h) \wedge (k \in \text{nset}(n)) \wedge \text{at}(h, k, 1 + n))$
 $\rightarrow (\neg \text{exist-hint-8-12-3-4}(n, l, g, h, k))$

THEOREM: h-k-eq-add1-n-k-not-in-l3
 $(\text{molws}(n, l, g, h)$
 $\wedge (i \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{b1b}(n, l, g, h, i, k)$
 $\wedge \text{at}(h, k, 1 + n)$
 $\wedge \text{union-at-n}(l, i, '(8\ 9\ 10\ 11\ 12)))$
 $\rightarrow (\neg \text{at}(l, k, 3))$

THEOREM: not-l3-then-not-lp4
 $(\text{molws}(n, l, g, h)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (\neg \text{at}(l, k, 3)))$
 $\rightarrow (\neg \text{at}(lp, k, 4))$

THEOREM: h-k-eq-add1-n
 $(\text{molws}(n, l, g, h)$
 $\wedge (i \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{b1b}(n, l, g, h, i, k)$
 $\wedge \text{at}(h, k, 1 + n)$
 $\wedge \text{union-at-n}(l, i, '(8\ 9\ 10\ 11\ 12)))$
 $\rightarrow (\neg \text{at}(lp, k, 4))$

THEOREM: h-k-neq-add1-n
 $(\text{molws}(n, l, g, h)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (\neg \text{at}(h, k, 1 + n)))$
 $\rightarrow (\neg \text{at}(lp, k, 4))$

;;;The order of the hints is crucial.

THEOREM: lm-b1a-i-neq-k-j-eq-k
 $(\text{molws}(n, l, g, h)$

$\wedge (i \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{blb}(n, l, g, h, i, k)$
 $\wedge \text{union-at-n}(l, i, '(8\ 9\ 10\ 11\ 12))$
 $\rightarrow (\neg \text{at}(lp, k, 4))$

;;;need m-l-same-lp.

THEOREM: bla-i-neq-k-j-eq-k

$(\text{molws}(n, l, g, h)$
 $\wedge (i \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (i \neq k)$
 $\wedge \text{blb}(n, l, g, h, i, k))$
 $\rightarrow \text{bla}(lp, i, k)$

THEOREM: bla-i-j-neq-k

$(\text{molws}(n, l, g, h)$
 $\wedge (i \in \text{nset}(n))$
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (i \neq k)$
 $\wedge (j \neq k)$
 $\wedge \text{bla}(l, i, j))$
 $\rightarrow \text{bla}(lp, i, j)$

THEOREM: bla-i-neq-k

$(\text{molws}(n, l, g, h)$
 $\wedge (i \in \text{nset}(n))$
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (i \neq k)$
 $\wedge \text{bla}(l, i, j)$
 $\wedge \text{blb}(n, l, g, h, i, j))$
 $\rightarrow \text{bla}(lp, i, j)$

THEOREM: cond-l7

$(\text{molws}(n, l, g, h)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{at}(l, k, 7)$

\wedge union-at-n($lp, k, '(8\ 9\ 10\ 11\ 12)$)
 \rightarrow at($g, \text{nth}(h, k), 4$)

THEOREM: k-in-l7-imp

(molws(n, l, g, h)
 \wedge ($j \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge b1d(n, l, h, k)
 \wedge lg(n, l, g)
 \wedge at($l, k, 7$)
 \wedge b1a($l, \text{nth}(h, k), j$)
 \wedge union-at-n($lp, k, '(8\ 9\ 10\ 11\ 12)$)
 \rightarrow (\neg at($l, j, 4$))

THEOREM: l5-j-lt-h-k

(molws(n, l, g, h)
 \wedge ($j \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge at($l, k, 5$)
 \wedge union-at-n($lp, k, '(8\ 9\ 10\ 11\ 12)$)
 \rightarrow ($j < \text{nth}(h, k)$)

THEOREM: k-in-l5-then-j-not-l4

(molws(n, l, g, h)
 \wedge ($j \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge at($l, k, 5$)
 \wedge union-at-n($lp, k, '(8\ 9\ 10\ 11\ 12)$)
 \wedge b0a(n, l, h, k, j)
 \rightarrow (\neg at($l, j, 4$))

THEOREM: lp9-12-k-in-l8-12

(molws(n, l, g, h)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge union-at-n($lp, k, '(9\ 10\ 11\ 12)$)
 \rightarrow union-at-n($l, k, '(8\ 9\ 10\ 11\ 12)$)

THEOREM: k-in-lp9-12-then-j-not-l4

(molws(n, l, g, h)
 \wedge ($j \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)

```

^ mrhoi (n, k, l, g, h, lp, gp, hp)
^ bla (l, k, j)
^ union-at-n (lp, k, '(9 10 11 12))
→ (¬ at (l, j, 4))

```

THEOREM: k-not-in-l7-then-lp9-12-or-l5

```

(molws (n, l, g, h)
^ (k ∈ nset (n))
^ mrhoi (n, k, l, g, h, lp, gp, hp)
^ union-at-n (lp, k, '(8 9 10 11 12))
^ (¬ at (l, k, 7))
^ (¬ union-at-n (lp, k, '(9 10 11 12))))
→ at (l, k, 5)

```

THEOREM: k-in-not-l7-imp

```

(molws (n, l, g, h)
^ (j ∈ nset (n))
^ (k ∈ nset (n))
^ mrhoi (n, k, l, g, h, lp, gp, hp)
^ (¬ at (l, k, 7))
^ b0a (n, l, h, k, j)
^ bla (l, k, j)
^ union-at-n (lp, k, '(8 9 10 11 12))
→ (¬ at (l, j, 4))

```

```

;;;I wonder why the following two do not imply
;;;lm-b1a-i-eq-k-j-neq-k although those without
;;;(member u (nset n)) are perfectly able to
;;;imply it.
;;;(prove-lemma k-in-lp9-12-then-j-not-l4 (rewrite)
;;; (implies (and (molws n l g h)
;;;               (member j (nset n))
;;;               (member k (nset n))
;;;               (mrhoi n k l g h lp gp hp)
;;;               (bla l k j)
;;;               (union-at-n lp k '(9 10 11 12)))
;;;           (not (at l j 4))))
;;;
;;;(prove-lemma k-in-lp8-then-j-not-l4 (rewrite)
;;; (implies (and (at lp k 8)
;;;               (molws n l g h)
;;;               (member j (nset n))
;;;               (member u (nset n))
;;;               (member k (nset n))
;;;               (mrhoi n k l g h lp gp hp)

```

```

;;;          (lg n l g)
;;;          (b0a n l h k j)
;;;          (b1a l (nth h k) j))
;;;          (not (at l j 4)))

```

THEOREM: lm-b1a-i-eq-k-j-neq-k

```

(molws (n, l, g, h)
  ^ (j ∈ nset (n))
  ^ (k ∈ nset (n))
  ^ mrhoi (n, k, l, g, h, lp, gp, hp)
  ^ b1d (n, l, h, k)
  ^ lg (n, l, g)
  ^ b0a (n, l, h, k, j)
  ^ b1a (l, k, j)
  ^ b1a (l, nth (h, k), j)
  ^ union-at-n (lp, k, '(8 9 10 11 12)))
→ (¬ at (l, j, 4))

```

;;;m-l-same-lp-at-not is used.

THEOREM: b1a-i-eq-k-j-neq-k

```

(molws (n, l, g, h)
  ^ (j ∈ nset (n))
  ^ (k ∈ nset (n))
  ^ mrhoi (n, k, l, g, h, lp, gp, hp)
  ^ (j ≠ k)
  ^ b1d (n, l, h, k)
  ^ lg (n, l, g)
  ^ b0a (n, l, h, k, j)
  ^ b1a (l, k, j)
  ^ b1a (l, nth (h, k), j))
→ b1a (lp, k, j)

```

THEOREM: b1a-i-j-eq-k

```

(molws (n, l, g, h)
  ^ (k ∈ nset (n))
  ^ mrhoi (n, k, l, g, h, lp, gp, hp)
  ^ b1a (l, k, k))
→ b1a (lp, k, k)

```

THEOREM: b1a-i-eq-k

```

(molws (n, l, g, h)
  ^ (j ∈ nset (n))
  ^ (k ∈ nset (n))

```

\wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge bld(n, l, h, k)
 \wedge lg(n, l, g)
 \wedge b0a(n, l, h, k, j)
 \wedge b1a(l, k, j)
 \wedge b1a($l, nth(h, k), j$)
 \rightarrow b1a(lp, k, j)

THEOREM: mrho-preserves-b1a

(molws(n, l, g, h)
 \wedge ($i \in \text{nset}(n)$)
 \wedge ($j \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge bld(n, l, h, i)
 \wedge lg(n, l, g)
 \wedge b0a(n, l, h, i, j)
 \wedge b1a(l, i, j)
 \wedge b1a($l, nth(h, i), j$)
 \wedge b1b(n, l, g, h, i, j)
 \rightarrow b1a(lp, i, j)

;;;;;;;;;;;;; b1b ;;;;;;;;;;;;;;

;;;;;;;;;;;;;common in mole and atom.

THEOREM: un8-11-then-un8-12

union-at-n($lp, r, '(8\ 9\ 10\ 11)$) \rightarrow union-at-n($lp, r, '(8\ 9\ 10\ 11\ 12)$)

THEOREM: l8-11-k-in-gp34

$((r \in \text{nset}(n)) \wedge \text{lg}(n, l, g) \wedge \text{union-at-n}(l, r, '(8\ 9\ 10\ 11)))$
 \rightarrow union-at-n($gp, r, '(3\ 4)$)

THEOREM: u-if4

$((u \in \text{nset}(n)) \wedge \text{lg}(n, l, g) \wedge \text{at}(g, u, 4)) \rightarrow (\neg \text{at}(l, u, 2))$

THEOREM: l12-then-un10-12

at($l, u, 12$) \rightarrow union-at-n($l, u, '(10\ 11\ 12)$)

THEOREM: r-neq-k

union-at-n($l, k, '(8\ 9\ 10\ 11)$) \wedge at($l, r, 12$) $\rightarrow (k \neq r)$

;;;;;;;;;;;;;common in mole and atom end.

;;;;;;;;;;;;;Lemmas on hints.

THEOREM: ex-hint-in-l8-12
 exist-hint-8-12-3-4 (n, l, g, h, j)
 \rightarrow union-at-n ($l, \text{exist-hint-8-12-3-4}(n, l, g, h, j), '(8\ 9\ 10\ 11\ 12)$)

THEOREM: ex-hint-in-g34
 exist-hint-8-12-3-4 (n, l, g, h, k)
 \rightarrow union-at-n ($g, \text{exist-hint-8-12-3-4}(n, l, g, h, k), '(3\ 4)$)

THEOREM: ex-hint-l-g-h
 exist-hint-8-12-3-4 (n, l, g, h, j)
 \rightarrow ($\text{exist-hint-8-12-3-4}(n, l, g, h, j) \not\leq \text{nth}(h, j)$)

THEOREM: ex-hint-lp-gp-h-in-int-8-12-3-4
 exist-hint-8-12-3-4 (n, lp, gp, h, j)
 \rightarrow intersect-8-12-3-4-at-n ($\text{exist-hint-8-12-3-4}(n, lp, gp, h, j), lp, gp$)

THEOREM: ex-hint-lp-gp-h-leq-h-j
 exist-hint-8-12-3-4 (n, lp, gp, h, j)
 \rightarrow ($\text{exist-hint-8-12-3-4}(n, lp, gp, h, j) \not\leq \text{nth}(h, j)$)

THEOREM: ex-hint-not-in-g02
 exist-hint-8-12-3-4 (n, l, g, h, k)
 \rightarrow (\neg union-at-n ($g, \text{exist-hint-8-12-3-4}(n, l, g, h, k), '(0\ 1\ 2)$))

THEOREM: hint-wtn
 $((r \in \text{nset}(n)) \wedge \text{intersect-8-12-3-4-at-n}(r, lp, gp) \wedge (r \not\leq \text{nth}(h, j)))$
 \rightarrow exist-hint-8-12-3-4 (n, lp, gp, h, j)

;;;;;;;;;;;;;Lemmas on hints end.

THEOREM: lm-k-in-l7-imp
 (molws (n, l, g, h)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi ($n, k, l, g, h, lp, gp, hp$)
 \wedge bld (n, l, h, k)
 \wedge lg (n, l, g)
 \wedge at ($l, k, 7$)
 \wedge b1b ($n, l, g, h, \text{nth}(h, k), j$)
 \wedge at ($l, j, 3$)
 \wedge union-at-n ($lp, k, '(8\ 9\ 10\ 11\ 12)$))
 \rightarrow exist-hint-8-12-3-4 (n, l, g, h, j)

THEOREM: ex-hint-neq-k-imp
 (molws (n, l, g, h)
 \wedge ($k \in \text{nset}(n)$)

\wedge $\text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 \wedge $\text{exist-hint-8-12-3-4}(n, l, g, h, j)$
 \wedge $(k \neq \text{exist-hint-8-12-3-4}(n, l, g, h, j))$
 \rightarrow $\text{intersect-8-12-3-4-at-n}(\text{exist-hint-8-12-3-4}(n, l, g, h, j), lp, gp)$

THEOREM: ex-hint-neq-k-in-l7
 $(\text{at}(l, k, 7) \wedge \text{exist-hint-8-12-3-4}(n, l, g, h, j))$
 $\rightarrow (k \neq \text{exist-hint-8-12-3-4}(n, l, g, h, j))$

THEOREM: ex-hint-in-int-8-12-3-4-l7
 $(\text{molws}(n, l, g, h)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{at}(l, k, 7)$
 $\wedge \text{exist-hint-8-12-3-4}(n, l, g, h, j))$
 $\rightarrow \text{intersect-8-12-3-4-at-n}(\text{exist-hint-8-12-3-4}(n, l, g, h, j), lp, gp)$

THEOREM: ex-hint-wtn-l7
 $(\text{molws}(n, l, g, h)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{at}(l, k, 7)$
 $\wedge \text{exist-hint-8-12-3-4}(n, l, g, h, j))$
 $\rightarrow \text{exist-hint-8-12-3-4}(n, lp, gp, h, j)$

THEOREM: b1b-k-in-l7-imp
 $(\text{molws}(n, l, g, h)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{b1d}(n, l, h, k)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{at}(l, k, 7)$
 $\wedge \text{b1b}(n, l, g, h, \text{nth}(h, k), j)$
 $\wedge \text{at}(l, j, 3)$
 $\wedge \text{union-at-n}(lp, k, '(8\ 9\ 10\ 11\ 12)))$
 $\rightarrow \text{exist-hint-8-12-3-4}(n, lp, gp, h, j)$

THEOREM: h-j-leq-k
 $(\text{molws}(n, l, g, h)$
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{at}(l, j, 3)$
 $\wedge \text{at}(l, k, 5)$
 $\wedge \text{b0b}(n, l, h, k, j)$

\wedge union-at-n($lp, k, '(8\ 9\ 10\ 11\ 12))$)
 \rightarrow ($k \notin \text{nth}(h, j)$)

THEOREM: lm-lp8-then-k-in-g34

$(\text{listp}(l)$
 \wedge ($n \in \mathbf{N}$)
 \wedge ($\text{length}(l) = n$)
 \wedge $\text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 \wedge ($k \in \text{nset}(n)$)
 \wedge $\text{lg}(n, l, g)$
 \wedge $\text{at}(l, k, 5)$
 \wedge $\text{at}(lp, k, 8)$
 \rightarrow union-at-n($gp, k, '(3\ 4)$)

EVENT: Disable lm-lp8-then-k-in-g34.

THEOREM: lp8-then-k-in-g34

$(\text{molws}(n, l, g, h)$
 \wedge ($k \in \text{nset}(n)$)
 \wedge $\text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 \wedge $\text{lg}(n, l, g)$
 \wedge $\text{at}(l, k, 5)$
 \wedge $\text{at}(lp, k, 8)$
 \rightarrow union-at-n($gp, k, '(3\ 4)$)

THEOREM: lm-k-in-g34

$(\text{molws}(n, l, g, h)$
 \wedge ($k \in \text{nset}(n)$)
 \wedge $\text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 \wedge $\text{lg}(n, l, g)$
 \wedge $\text{lg}(n, lp, gp)$
 \wedge $\text{at}(l, k, 5)$
 \wedge union-at-n($lp, k, '(8\ 9\ 10\ 11\ 12))$)
 \rightarrow union-at-n($gp, k, '(3\ 4)$)

THEOREM: k-in-g34

$(\text{molws}(n, l, g, h)$
 \wedge ($k \in \text{nset}(n)$)
 \wedge $\text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 \wedge $\text{lg}(n, l, g)$
 \wedge $\text{at}(l, k, 5)$
 \wedge union-at-n($lp, k, '(8\ 9\ 10\ 11\ 12))$)
 \rightarrow union-at-n($gp, k, '(3\ 4)$)

THEOREM: k-in-int
(molws (n, l, g, h)
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{at}(l, k, 5)$
 $\wedge \text{union-at-n}(lp, k, '(8\ 9\ 10\ 11\ 12))$)
 $\rightarrow \text{intersect-8-12-3-4-at-n}(k, lp, gp)$

THEOREM: k-in-l5-imp
(molws (n, l, g, h)
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{at}(l, j, 3)$
 $\wedge \text{at}(l, k, 5)$
 $\wedge \text{b0b}(n, l, h, k, j)$
 $\wedge \text{union-at-n}(lp, k, '(8\ 9\ 10\ 11\ 12))$)
 $\rightarrow \text{exist-hint-8-12-3-4}(n, lp, gp, h, j)$

;;;This is slow.

THEOREM: ex-hint-in-l12
(molws (n, l, g, h)
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{union-at-n}(l, k, '(8\ 9\ 10\ 11))$
 $\wedge \text{exist-hint-8-12-3-4}(n, l, g, h, j)$
 $\wedge \text{at}(l, \text{exist-hint-8-12-3-4}(n, l, g, h, j), 12)$)
 $\rightarrow \text{intersect-8-12-3-4-at-n}(\text{exist-hint-8-12-3-4}(n, l, g, h, j), lp, gp)$

THEOREM: r-neq-k-l8-11-k-in-lp8-12
(molws (n, l, g, h)
 $\wedge (r \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (r \neq k)$
 $\wedge \text{union-at-n}(l, r, '(8\ 9\ 10\ 11))$)
 $\rightarrow \text{union-at-n}(lp, r, '(8\ 9\ 10\ 11))$

THEOREM: r-eq-k-l8-11-k-in-lp8-12
(molws (n, l, g, h)
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$

\wedge union-at-n($l, k, \text{'(8 9 10 11)}$)
 \rightarrow union-at-n($lp, k, \text{'(8 9 10 11 12)}$)

THEOREM: l8-11-k-in-lp8-12

(molws(n, l, g, h)
 \wedge ($r \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge union-at-n($l, r, \text{'(8 9 10 11)}$)
 \rightarrow union-at-n($lp, r, \text{'(8 9 10 11 12)}$)

THEOREM: hint-in-l8-11

(molws(n, l, g, h)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge lg(n, l, g)
 \wedge exist-hint-8-12-3-4(n, l, g, h, j)
 \wedge union-at-n($l, \text{exist-hint-8-12-3-4}(n, l, g, h, j), \text{'(8 9 10 11)}$)
 \rightarrow intersect-8-12-3-4-at-n($\text{exist-hint-8-12-3-4}(n, l, g, h, j), lp, gp$)

THEOREM: ex-hint-not-in-l12

(molws(n, l, g, h)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge lg(n, l, g)
 \wedge exist-hint-8-12-3-4(n, l, g, h, j)
 \wedge (\neg at($l, \text{exist-hint-8-12-3-4}(n, l, g, h, j), 12$))
 \rightarrow intersect-8-12-3-4-at-n($\text{exist-hint-8-12-3-4}(n, l, g, h, j), lp, gp$)

THEOREM: ex-hint-in-int-8-12-3-4-l8-11

(molws(n, l, g, h)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge lg(n, l, g)
 \wedge union-at-n($l, k, \text{'(8 9 10 11)}$)
 \wedge exist-hint-8-12-3-4(n, l, g, h, j)
 \rightarrow intersect-8-12-3-4-at-n($\text{exist-hint-8-12-3-4}(n, l, g, h, j), lp, gp$)

THEOREM: ex-hint-wtn-l8-11

(molws(n, l, g, h)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge lg(n, l, g)
 \wedge union-at-n($l, k, \text{'(8 9 10 11)}$)
 \wedge exist-hint-8-12-3-4(n, l, g, h, j)
 \rightarrow exist-hint-8-12-3-4(n, lp, gp, h, j)

THEOREM: k-in-l8-11-imp

(molws (n, l, g, h)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi ($n, k, l, g, h, lp, gp, hp$)
 \wedge lg (n, l, g)
 \wedge at ($l, j, 3$)
 \wedge b1b (n, l, g, h, k, j)
 \wedge union-at-n ($l, k, '(8\ 9\ 10\ 11)$))
 \rightarrow exist-hint-8-12-3-4 (n, lp, gp, h, j)

THEOREM: m-lp9-12-k-in-l8-11

(molws (n, l, g, h)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi ($n, k, l, g, h, lp, gp, hp$)
 \wedge union-at-n ($lp, k, '(9\ 10\ 11\ 12)$))
 \rightarrow union-at-n ($l, k, '(8\ 9\ 10\ 11)$)

THEOREM: k-in-lp9-12-imp

(molws (n, l, g, h)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi ($n, k, l, g, h, lp, gp, hp$)
 \wedge lg (n, l, g)
 \wedge at ($l, j, 3$)
 \wedge b1b (n, l, g, h, k, j)
 \wedge union-at-n ($lp, k, '(9\ 10\ 11\ 12)$))
 \rightarrow exist-hint-8-12-3-4 (n, lp, gp, h, j)

THEOREM: k-not-in-l7-imp

(molws (n, l, g, h)
 \wedge ($j \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi ($n, k, l, g, h, lp, gp, hp$)
 \wedge lg (n, l, g)
 \wedge at ($l, j, 3$)
 \wedge (\neg at ($l, k, 7$))
 \wedge b0b (n, l, h, k, j)
 \wedge b1b (n, l, g, h, k, j)
 \wedge union-at-n ($lp, k, '(8\ 9\ 10\ 11\ 12)$))
 \rightarrow exist-hint-8-12-3-4 (n, lp, gp, h, j)

THEOREM: lm1-b1b-i-eq-k-j-neq-k

(molws (n, l, g, h)
 \wedge ($j \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi ($n, k, l, g, h, lp, gp, hp$)

$$\begin{aligned}
& \wedge \text{b1d}(n, l, h, k) \\
& \wedge \text{lg}(n, l, g) \\
& \wedge \text{at}(l, j, 3) \\
& \wedge \text{b0b}(n, l, h, k, j) \\
& \wedge \text{b1b}(n, l, g, h, k, j) \\
& \wedge \text{b1b}(n, l, g, h, \text{nth}(h, k), j) \\
& \wedge \text{union-at-n}(lp, k, '(8\ 9\ 10\ 11\ 12))) \\
\rightarrow & \text{exist-hint-8-12-3-4}(n, lp, gp, h, j)
\end{aligned}$$

THEOREM: ex-hint-lp-gp-h-leq-hp-j

$$\begin{aligned}
& (\text{molws}(n, l, g, h) \\
& \wedge (j \in \text{nset}(n)) \\
& \wedge (k \in \text{nset}(n)) \\
& \wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp) \\
& \wedge (j \neq k) \\
& \wedge \text{exist-hint-8-12-3-4}(n, lp, gp, h, j)) \\
\rightarrow & (\text{exist-hint-8-12-3-4}(n, lp, gp, h, j) \not\prec \text{nth}(hp, j))
\end{aligned}$$

THEOREM: j-neq-k-then-hp-eq-h

$$\begin{aligned}
& (\text{molws}(n, l, g, h) \\
& \wedge (j \in \text{nset}(n)) \\
& \wedge (k \in \text{nset}(n)) \\
& \wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp) \\
& \wedge (j \neq k) \\
& \wedge \text{exist-hint-8-12-3-4}(n, lp, gp, h, j)) \\
\rightarrow & \text{exist-hint-8-12-3-4}(n, lp, gp, hp, j)
\end{aligned}$$

THEOREM: lm-b1b-i-eq-k-j-neq-k

$$\begin{aligned}
& (\text{molws}(n, l, g, h) \\
& \wedge (j \in \text{nset}(n)) \\
& \wedge (k \in \text{nset}(n)) \\
& \wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp) \\
& \wedge \text{b1d}(n, l, h, k) \\
& \wedge (j \neq k) \\
& \wedge \text{lg}(n, l, g) \\
& \wedge \text{b0b}(n, l, h, k, j) \\
& \wedge \text{b1b}(n, l, g, h, k, j) \\
& \wedge \text{b1b}(n, l, g, h, \text{nth}(h, k), j) \\
& \wedge \text{at}(l, j, 3) \\
& \wedge \text{union-at-n}(lp, k, '(8\ 9\ 10\ 11\ 12))) \\
\rightarrow & \text{exist-hint-8-12-3-4}(n, lp, gp, hp, j)
\end{aligned}$$

THEOREM: b1b-i-eq-k-j-neq-k

$$\begin{aligned}
& (\text{molws}(n, l, g, h) \\
& \wedge (j \in \text{nset}(n))
\end{aligned}$$

$\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{b1d}(n, l, h, k)$
 $\wedge (j \neq k)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{b0b}(n, l, h, k, j)$
 $\wedge \text{b1b}(n, l, g, h, k, j)$
 $\wedge \text{b1b}(n, l, g, h, \text{nth}(h, k), j))$
 $\rightarrow \text{b1b}(n, lp, gp, hp, k, j)$

THEOREM: b1b-i-j-eq-k

$(\text{molws}(n, l, g, h)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{b1b}(n, l, g, h, k, k))$
 $\rightarrow \text{b1b}(n, lp, gp, hp, k, k)$

;;;I wonder if (b1d n l h i) is
 ;;;better than (b1d n l h k).

THEOREM: b1b-i-eq-k

$(\text{molws}(n, l, g, h)$
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{b1d}(n, l, h, k)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{b0b}(n, l, h, k, j)$
 $\wedge \text{b1b}(n, l, g, h, k, j)$
 $\wedge \text{b1b}(n, l, g, h, \text{nth}(h, k), j))$
 $\rightarrow \text{b1b}(n, lp, gp, hp, k, j)$

;;;I wonder why the following two do not imply
 ;;;lm-b1b-i-neq-k-j-eq-k.
 ;;;(prove-lemma k-not-in-l3 (rewrite)
 ;;; (implies (and (molws n l g h)
 ;;; (member i (nset n))
 ;;; (member k (nset n))
 ;;; (mrhoi n k l g h lp gp hp)
 ;;; (not (equal i k))
 ;;; (lg n l g)
 ;;; (at lp k 3)
 ;;; (not (at l k 3))
 ;;; (union-at-n l i '(8 9 10 11 12)))
 ;;; (exist-hint-8-12-3-4 n lp gp hp k)))

```

;;;
;;;(prove-lemma k-in-l3 (rewrite)
;;;  (implies (and (molws n l g h)
;;;                (member i (nset n))
;;;                (member k (nset n))
;;;                (mrhoi n k l g h lp gp hp)
;;;                (at l k 3)
;;;                (at lp k 3)
;;;                (union-at-n l i '(8 9 10 11 12))))
;;;  (exist-hint-8-12-3-4 n lp gp hp k)))
;;;
;;;(prove-lemma lm-b1b-i-neq-k-j-eq-k (rewrite)
;;;  (implies (and (molws n l g h)
;;;                (member i (nset n))
;;;                (member k (nset n))
;;;                (mrhoi n k l g h lp gp hp)
;;;                (not (equal i k))
;;;                (lg n l g)
;;;                (at lp k 3)
;;;                (union-at-n l i '(8 9 10 11 12))))
;;;  (exist-hint-8-12-3-4 n lp gp hp k)))

```

THEOREM: ex-hint-leq-h-k
 $\text{exist-hint-8-12-3-4}(n, l, g, h, k)$
 $\rightarrow (\text{exist-hint-8-12-3-4}(n, l, g, h, k) \not\leq \text{nth}(h, k))$

THEOREM: h-k-leq-sub1-ex-hint
 $(\text{exist-hint-8-12-3-4}(n, l, g, h, k)$
 $\wedge (\text{nth}(h, k) \neq \text{exist-hint-8-12-3-4}(n, l, g, h, k)))$
 $\rightarrow ((\text{exist-hint-8-12-3-4}(n, l, g, h, k) - 1) \not\leq \text{nth}(h, k))$

THEOREM: ex-hint-neq-h-k
 $(\text{molws}(n, l, g, h)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{exist-hint-8-12-3-4}(n, l, g, h, k)$
 $\wedge \text{at}(l, k, 3)$
 $\wedge \text{at}(lp, k, 3))$
 $\rightarrow (\text{nth}(h, k) \neq \text{exist-hint-8-12-3-4}(n, l, g, h, k))$

THEOREM: lm-hp-k-leq-ex-l-g-h
 $(\text{molws}(n, l, g, h)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$

\wedge at($l, k, 3$)
 \wedge at($lp, k, 3$)
 \wedge exist-hint-8-12-3-4(n, l, g, h, k)
 \rightarrow ((exist-hint-8-12-3-4(n, l, g, h, k) - 1) $\not\prec$ nth(h, k))

THEOREM: ex-cond-l3

(molws(n, l, g, h)
 \wedge ($k \in$ nset(n))
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge at($l, k, 3$)
 \wedge exist-hint-8-12-3-4(n, l, g, h, k)
 \wedge ((exist-hint-8-12-3-4(n, l, g, h, k) - 1) $\not\prec$ nth(h, k)))
 \rightarrow (exist-hint-8-12-3-4(n, l, g, h, k) $\not\prec$ nth(hp, k))

THEOREM: hp-k-leq-ex-l-g-h

(molws(n, l, g, h)
 \wedge ($k \in$ nset(n))
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge at($l, k, 3$)
 \wedge at($lp, k, 3$)
 \wedge exist-hint-8-12-3-4(n, l, g, h, k)
 \rightarrow (exist-hint-8-12-3-4(n, l, g, h, k) $\not\prec$ nth(hp, k))

THEOREM: ex-hint-neq-k-in-l3

(at($l, k, 3$)
 \wedge union-at-n($l, \text{exist-hint-8-12-3-4}(n, l, g, h, k), '8 9 10 11 12$))
 \rightarrow ($k \neq$ exist-hint-8-12-3-4(n, l, g, h, k))

;;;This is successfully proved
;;;by m-gp-same-g and m-lp-same-l.
;;;This is successfully proved by ex-hint-neq-k-imp,
;;;ex-hint-neq-k-in-l3 and ex-hint-in-l8-12.

THEOREM: ex-hint-l-g-h-in-int-8-12-3-4

(molws(n, l, g, h)
 \wedge ($k \in$ nset(n))
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge at($l, k, 3$)
 \wedge exist-hint-8-12-3-4(n, l, g, h, k)
 \rightarrow intersect-8-12-3-4-at-n(exist-hint-8-12-3-4(n, l, g, h, k), lp, gp)

THEOREM: ex-l-g-h-k-in-l3

(molws(n, l, g, h)
 \wedge ($k \in$ nset(n))
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)

\wedge at($l, k, 3$)
 \wedge at($lp, k, 3$)
 \wedge exist-hint-8-12-3-4(n, l, g, h, k)
 \rightarrow exist-hint-8-12-3-4(n, lp, gp, hp, k)

THEOREM: lm-k-in-l3

(molws(n, l, g, h)
 \wedge ($i \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge at($l, k, 3$)
 \wedge at($lp, k, 3$)
 \wedge union-at-n($l, i, '(8\ 9\ 10\ 11\ 12)$)
 \wedge b1b(n, l, g, h, i, k)
 \rightarrow exist-hint-8-12-3-4(n, lp, gp, hp, k)

THEOREM: k-in-l3

(molws(n, l, g, h)
 \wedge ($i \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge ($i \neq k$)
 \wedge at($l, k, 3$)
 \wedge b1b(n, l, g, h, i, k)
 \rightarrow b1b(n, lp, gp, hp, i, k)

THEOREM: hp-k-leq-i

(molws(n, l, g, h)
 \wedge ($i \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge at($l, k, 2$)
 \wedge at($lp, k, 3$)
 \rightarrow ($i \not\prec \text{nth}(hp, k)$)

THEOREM: b1b-u-neq-k

($u \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge lg(n, l, g)
 \wedge at($g, u, 4$)
 \wedge at($l, k, 2$)
 \rightarrow ($k \neq u$)

THEOREM: lm-u-in-int-8-12-3-4

(molws(n, l, g, h))

```

^ (u ∈ nset (n))
^ (k ∈ nset (n))
^ mrhoi (n, k, l, g, h, lp, gp, hp)
^ lg (n, l, g)
^ at (l, k, 2)
^ at (g, u, 4)
^ union-at-n (lp, u, '(8 9 10 11 12))
→ intersect-8-12-3-4-at-n (u, lp, gp)

```

THEOREM: k-neq-u-in-lp8-12

```

(molws (n, l, g, h)
^ (u ∈ nset (n))
^ (k ∈ nset (n))
^ mrhoi (n, k, l, g, h, lp, gp, hp)
^ (u ≠ k)
^ lg (n, l, g)
^ at (g, u, 4))
→ union-at-n (lp, u, '(8 9 10 11 12))

```

THEOREM: lm1-u-in-int-8-12-3-4

```

(molws (n, l, g, h)
^ (u ∈ nset (n))
^ (k ∈ nset (n))
^ mrhoi (n, k, l, g, h, lp, gp, hp)
^ lg (n, l, g)
^ at (l, k, 2)
^ at (g, u, 4))
→ union-at-n (lp, u, '(8 9 10 11 12))

```

THEOREM: u-in-int-8-12-3-4

```

(molws (n, l, g, h)
^ (u ∈ nset (n))
^ (k ∈ nset (n))
^ mrhoi (n, k, l, g, h, lp, gp, hp)
^ lg (n, l, g)
^ at (l, k, 2)
^ at (g, u, 4))
→ intersect-8-12-3-4-at-n (u, lp, gp)

```

```

;;;I wonder why the following does not trigger
;;;molws-ln-lp, molws-ln-gp.
;;;(prove-lemma h-i-in-g34-imp (rewrite)
;;;  (implies (and (molws n l g h)
;;;                (member k (nset n))
;;;                (mrhoi n k l g h lp gp hp)

```

```

;;;          (member (nth h i) (nset n))
;;;          (lg n l g)
;;;          (at l k 2)
;;;          (at lp k 3)
;;;          (at g (nth h i) 4))
;;;          (exist-hint-8-12-3-4 n lp gp hp k)))
;;;although
;;;(prove-lemma h-i-in-g34-imp (rewrite)
;;;  (implies (and (member (nth h i) (nset n))
;;;                (molws n l g h)
;;;                (member k (nset n))
;;;                (mrhoi n k l g h lp gp hp)
;;;                (lg n l g)
;;;                (at l k 2)
;;;                (at lp k 3)
;;;                (at g (nth h i) 4))
;;;                (exist-hint-8-12-3-4 n lp gp hp k)))
;;;does.

```

THEOREM: h-i-in-g34-imp
 $((\text{nth}(h, i) \in \text{nset}(n))$
 $\wedge \text{molws}(n, l, g, h)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{at}(l, k, 2)$
 $\wedge \text{at}(lp, k, 3)$
 $\wedge \text{at}(g, \text{nth}(h, i), 4))$
 $\rightarrow \text{exist-hint-8-12-3-4}(n, lp, gp, hp, k)$

THEOREM: i-not-in-g34
 $((\neg \text{union-at-n}(g, i, '(3 4)))$
 $\wedge \text{molws}(n, l, g, h)$
 $\wedge (i \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{at}(l, k, 2)$
 $\wedge \text{at}(lp, k, 3)$
 $\wedge \text{blc}(n, l, g, h, i)$
 $\wedge \text{union-at-n}(l, i, '(8 9 10 11 12)))$
 $\rightarrow \text{exist-hint-8-12-3-4}(n, lp, gp, hp, k)$

THEOREM: i-in-int-8-12-3-4

(union-at-n($g, i, \text{'(3 4)}$)
 \wedge molws(n, l, g, h)
 \wedge ($i \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge ($i \neq k$)
 \wedge union-at-n($l, i, \text{'(8 9 10 11 12)}$))
 \rightarrow intersect-8-12-3-4-at-n(i, lp, gp)

THEOREM: i-in-g34

(union-at-n($g, i, \text{'(3 4)}$)
 \wedge molws(n, l, g, h)
 \wedge ($i \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge ($i \neq k$)
 \wedge at($l, k, 2$)
 \wedge at($lp, k, 3$)
 \wedge union-at-n($l, i, \text{'(8 9 10 11 12)}$))
 \rightarrow exist-hint-8-12-3-4(n, lp, gp, hp, k)

THEOREM: k-in-l2

(molws(n, l, g, h)
 \wedge ($i \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge bld(n, l, h, k)
 \wedge ($i \neq k$)
 \wedge lg(n, l, g)
 \wedge at($l, k, 2$)
 \wedge at($lp, k, 3$)
 \wedge b1c(n, l, g, h, i)
 \wedge union-at-n($l, i, \text{'(8 9 10 11 12)}$))
 \rightarrow exist-hint-8-12-3-4(n, lp, gp, hp, k)

THEOREM: lp3-then-l3-or-l2

(molws(n, l, g, h)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge (\neg at($l, k, 3$))
 \wedge at($lp, k, 3$)
 \rightarrow at($l, k, 2$)

THEOREM: lm-k-not-in-l3

(molws(n, l, g, h))

$\wedge (i \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{bld}(n, l, h, k)$
 $\wedge (i \neq k)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge (\neg \text{at}(l, k, 3))$
 $\wedge \text{at}(lp, k, 3)$
 $\wedge \text{b1c}(n, l, g, h, i)$
 $\wedge \text{union-at-n}(l, i, '(8\ 9\ 10\ 11\ 12))$
 $\rightarrow \text{exist-hint-8-12-3-4}(n, lp, gp, hp, k)$

THEOREM: k-not-in-l3

$(\text{molws}(n, l, g, h)$
 $\wedge (i \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{bld}(n, l, h, k)$
 $\wedge (i \neq k)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge (\neg \text{at}(l, k, 3))$
 $\wedge \text{b1b}(n, l, g, h, i, k)$
 $\wedge \text{b1c}(n, l, g, h, i)$
 $\rightarrow \text{b1b}(n, lp, gp, hp, i, k)$

THEOREM: b1b-i-neq-k-j-eq-k

$(\text{molws}(n, l, g, h)$
 $\wedge (i \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{bld}(n, l, h, k)$
 $\wedge (i \neq k)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{b1b}(n, l, g, h, i, k)$
 $\wedge \text{b1c}(n, l, g, h, i)$
 $\rightarrow \text{b1b}(n, lp, gp, hp, i, k)$

THEOREM: lm-i-neq-k-in-int-8-12-3-4

$(\text{molws}(n, l, g, h)$
 $\wedge (i \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (i \neq k)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{at}(g, i, 4)$

→ intersect-8-12-3-4-at-n (i, lp, gp)

THEOREM: i-neq-k-in-int-8-12-3-4

(molws (n, l, g, h)
∧ ($i \in \text{nset}(n)$)
∧ ($k \in \text{nset}(n)$)
∧ mrhoi ($n, k, l, g, h, lp, gp, hp$)
∧ ($i \neq k$)
∧ union-at-n ($l, i, '(8\ 9\ 10\ 11\ 12)$)
∧ lg (n, l, g)
∧ at ($l, \text{exist-hint-8-12-3-4}(n, l, g, h, j), 12$)
∧ b3a ($l, g, \text{exist-hint-8-12-3-4}(n, l, g, h, j), i$)
→ intersect-8-12-3-4-at-n (i, lp, gp))

THEOREM: h-j-leq-i

(union-at-n ($l, i, '(8\ 9\ 10\ 11\ 12)$)
∧ exist-hint-8-12-3-4 (n, l, g, h, j)
∧ at ($l, \text{exist-hint-8-12-3-4}(n, l, g, h, j), 12$)
∧ b2a ($l, \text{exist-hint-8-12-3-4}(n, l, g, h, j), i$)
→ ($i \not\leq \text{nth}(h, j)$))

THEOREM: i-neq-k-ex-hint-in-l12

(molws (n, l, g, h)
∧ ($i \in \text{nset}(n)$)
∧ ($k \in \text{nset}(n)$)
∧ mrhoi ($n, k, l, g, h, lp, gp, hp$)
∧ ($i \neq k$)
∧ lg (n, l, g)
∧ union-at-n ($l, i, '(8\ 9\ 10\ 11\ 12)$)
∧ exist-hint-8-12-3-4 (n, l, g, h, j)
∧ at ($l, \text{exist-hint-8-12-3-4}(n, l, g, h, j), 12$)
∧ b2a ($l, \text{exist-hint-8-12-3-4}(n, l, g, h, j), i$)
∧ b3a ($l, g, \text{exist-hint-8-12-3-4}(n, l, g, h, j), i$)
→ exist-hint-8-12-3-4 (n, lp, gp, h, j))

THEOREM: i-neq-k-ex-hint-not-in-l12

(molws (n, l, g, h)
∧ ($k \in \text{nset}(n)$)
∧ mrhoi ($n, k, l, g, h, lp, gp, hp$)
∧ lg (n, l, g)
∧ exist-hint-8-12-3-4 (n, l, g, h, j)
∧ ($\neg \text{at}(l, \text{exist-hint-8-12-3-4}(n, l, g, h, j), 12)$))
→ exist-hint-8-12-3-4 (n, lp, gp, h, j))

THEOREM: lm1-b1b-i-j-neq-k

$(\text{molws}(n, l, g, h)$
 $\wedge (i \in \text{nset}(n))$
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (i \neq k)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{union-at-n}(l, i, '(8\ 9\ 10\ 11\ 12))$
 $\wedge \text{exist-hint-8-12-3-4}(n, l, g, h, j)$
 $\wedge \text{b2a}(l, \text{exist-hint-8-12-3-4}(n, l, g, h, j), i)$
 $\wedge \text{b3a}(l, g, \text{exist-hint-8-12-3-4}(n, l, g, h, j), i))$
 $\rightarrow \text{exist-hint-8-12-3-4}(n, lp, gp, h, j)$

THEOREM: lm-b1b-i-j-neq-k

$(\text{molws}(n, l, g, h)$
 $\wedge (i \in \text{nset}(n))$
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (i \neq k)$
 $\wedge (j \neq k)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{union-at-n}(l, i, '(8\ 9\ 10\ 11\ 12))$
 $\wedge \text{b2a}(l, \text{exist-hint-8-12-3-4}(n, l, g, h, j), i)$
 $\wedge \text{b3a}(l, g, \text{exist-hint-8-12-3-4}(n, l, g, h, j), i)$
 $\wedge \text{exist-hint-8-12-3-4}(n, l, g, h, j))$
 $\rightarrow \text{exist-hint-8-12-3-4}(n, lp, gp, hp, j)$

THEOREM: b1b-i-j-neq-k

$(\text{molws}(n, l, g, h)$
 $\wedge (i \in \text{nset}(n))$
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (i \neq k)$
 $\wedge (j \neq k)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{b1b}(n, l, g, h, i, j)$
 $\wedge \text{b2a}(l, \text{exist-hint-8-12-3-4}(n, l, g, h, j), i)$
 $\wedge \text{b3a}(l, g, \text{exist-hint-8-12-3-4}(n, l, g, h, j), i))$
 $\rightarrow \text{b1b}(n, lp, gp, hp, i, j)$

THEOREM: b1b-i-neq-k

$(\text{molws}(n, l, g, h)$
 $\wedge (i \in \text{nset}(n))$

$\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{b1d}(n, l, h, k)$
 $\wedge (i \neq k)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{b0b}(n, l, h, i, j)$
 $\wedge \text{b1b}(n, l, g, h, i, j)$
 $\wedge \text{b1c}(n, l, g, h, i)$
 $\wedge \text{b2a}(l, \text{exist-hint-8-12-3-4}(n, l, g, h, j), i)$
 $\wedge \text{b3a}(l, g, \text{exist-hint-8-12-3-4}(n, l, g, h, j), i)$
 $\rightarrow \text{b1b}(n, lp, gp, hp, i, j)$

;;;I wonder if (b1d n l h i) and
 ;;;(b1d n l h j) are better than
 ;;;(b1d n l h k).
 ;;;(b1b n l g h (nth h i) j) and (b1b n l g h (nth h k) j).
 ;;;What about (b2a l (exist-hint-8-12-3-4 n l g h j) i)
 ;;; (b3a l g (exist-hint-8-12-3-4 n l g h j) i) ?

THEOREM: mrho-preserves-b1b

$(\text{molws}(n, l, g, h)$
 $\wedge (i \in \text{nset}(n))$
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{b1d}(n, l, h, k)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{b0b}(n, l, h, i, j)$
 $\wedge \text{b1b}(n, l, g, h, \text{nth}(h, k), j)$
 $\wedge \text{b1b}(n, l, g, h, i, j)$
 $\wedge \text{b1c}(n, l, g, h, i)$
 $\wedge \text{b2a}(l, \text{exist-hint-8-12-3-4}(n, l, g, h, j), i)$
 $\wedge \text{b3a}(l, g, \text{exist-hint-8-12-3-4}(n, l, g, h, j), i)$
 $\rightarrow \text{b1b}(n, lp, gp, hp, i, j)$

;;;;;;;;;;;;; b1c ;;;;;;;;;;;;;;

;;;;;;;;;;;;;common in mole and atom.

THEOREM: not-g34-then-not-g4

$(\neg \text{union-at-n}(g, i, ' (3 4))) \rightarrow (\neg \text{at}(g, i, 4))$

THEOREM: contra-if4

$((j \in \text{nset}(n)) \wedge \text{lg}(n, l, g) \wedge \text{at}(g, j, 4))$
 $\rightarrow \text{union-at-n}(l, j, '(9\ 10\ 11\ 12))$

;;;;;;;;;;;;;common in mole and atom end.

THEOREM: lp8-not-l5-then-l7

$(\text{molws}(n, l, g, h)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (\neg \text{at}(l, k, 5))$
 $\wedge \text{at}(lp, k, 8))$
 $\rightarrow \text{at}(l, k, 7)$

THEOREM: lp8-not-g34-then-k-in-l7

$(\text{molws}(n, l, g, h)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{at}(lp, k, 8)$
 $\wedge \text{union-at-n}(lp, k, '(8\ 9\ 10\ 11\ 12))$
 $\wedge (\neg \text{union-at-n}(gp, k, '(3\ 4))))$
 $\rightarrow \text{at}(l, k, 7)$

THEOREM: lm-k-in-l7

$(\text{molws}(n, l, g, h)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{lg}(n, lp, gp)$
 $\wedge \text{union-at-n}(lp, k, '(8\ 9\ 10\ 11\ 12))$
 $\wedge (\neg \text{union-at-n}(gp, k, '(3\ 4))))$
 $\rightarrow \text{at}(l, k, 7)$

THEOREM: k-in-l7

$(\text{molws}(n, l, g, h)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{union-at-n}(lp, k, '(8\ 9\ 10\ 11\ 12))$
 $\wedge (\neg \text{union-at-n}(gp, k, '(3\ 4))))$
 $\rightarrow \text{at}(l, k, 7)$

THEOREM: h-k-cond-l7

$(\text{molws}(n, l, g, h)$

$\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{at}(l, k, 7)$
 $\wedge \text{union-at-n}(lp, k, '(8\ 9\ 10\ 11\ 12))$
 $\rightarrow (\text{nth}(hp, k) = \text{nth}(h, k))$

THEOREM: lm-h-k-g4

$(\text{molws}(n, l, g, h)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{bid}(n, l, h, k)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{at}(l, k, 7)$
 $\wedge \text{union-at-n}(lp, k, '(8\ 9\ 10\ 11\ 12))$
 $\rightarrow ((\text{nth}(hp, k) \in \text{nset}(n)) \wedge \text{at}(g, \text{nth}(hp, k), 4))$

THEOREM: h-k-g4

$(\text{molws}(n, l, g, h)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{bid}(n, l, h, k)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{union-at-n}(lp, k, '(8\ 9\ 10\ 11\ 12))$
 $\wedge (\neg \text{union-at-n}(gp, k, '(3\ 4)))$
 $\rightarrow ((\text{nth}(hp, k) \in \text{nset}(n)) \wedge \text{at}(g, \text{nth}(hp, k), 4))$

THEOREM: lm1-i-eq-k-then-h-k-neq-k

$(\text{molws}(n, l, g, h)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{bid}(n, l, h, k)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{at}(l, k, 7)$
 $\wedge \text{at}(g, \text{nth}(hp, k), 4)$
 $\rightarrow (\neg \text{at}(hp, k, k))$

;;;Need k-in-17.

THEOREM: lm-i-eq-k-then-h-k-neq-k

$(\text{molws}(n, l, g, h)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{bid}(n, l, h, k)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{at}(l, k, 7)$

$$\begin{aligned}
& \wedge \text{union-at-n}(lp, k, '(8\ 9\ 10\ 11\ 12)) \\
& \wedge (\neg \text{union-at-n}(gp, k, '(3\ 4))) \\
& \rightarrow (\neg \text{at}(hp, k, k))
\end{aligned}$$

THEOREM: i-eq-k-then-h-k-neq-k

$$\begin{aligned}
& (\text{molws}(n, l, g, h) \\
& \wedge (k \in \text{nset}(n)) \\
& \wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp) \\
& \wedge \text{bld}(n, l, h, k) \\
& \wedge \text{lg}(n, l, g) \\
& \wedge \text{union-at-n}(lp, k, '(8\ 9\ 10\ 11\ 12)) \\
& \wedge (\neg \text{union-at-n}(gp, k, '(3\ 4)))) \\
& \rightarrow (\neg \text{at}(hp, k, k))
\end{aligned}$$

THEOREM: b1c-i-eq-k-hp-k-neq-k

$$\begin{aligned}
& (\text{molws}(n, l, g, h) \\
& \wedge (k \in \text{nset}(n)) \\
& \wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp) \\
& \wedge \text{bld}(n, l, h, k) \\
& \wedge (\neg \text{at}(hp, k, k)) \\
& \wedge \text{lg}(n, l, g) \\
& \wedge \text{union-at-n}(lp, k, '(8\ 9\ 10\ 11\ 12)) \\
& \wedge (\neg \text{union-at-n}(gp, k, '(3\ 4)))) \\
& \rightarrow ((\text{nth}(hp, k) \in \text{nset}(n)) \wedge \text{at}(gp, \text{nth}(hp, k), 4))
\end{aligned}$$

THEOREM: b1c-i-eq-k

$$\begin{aligned}
& (\text{molws}(n, l, g, h) \\
& \wedge (k \in \text{nset}(n)) \\
& \wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp) \\
& \wedge \text{bld}(n, l, h, k) \\
& \wedge \text{lg}(n, l, g)) \\
& \rightarrow \text{b1c}(n, lp, gp, hp, k)
\end{aligned}$$

THEOREM: l9-11-then-in-lp9-12

$$\begin{aligned}
& (\text{molws}(n, l, g, h) \\
& \wedge (k \in \text{nset}(n)) \\
& \wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp) \\
& \wedge (\neg \text{at}(l, k, 12)) \\
& \wedge \text{union-at-n}(l, k, '(9\ 10\ 11\ 12))) \\
& \rightarrow \text{union-at-n}(lp, k, '(9\ 10\ 11\ 12))
\end{aligned}$$

THEOREM: k-in-lp9-12

$$\begin{aligned}
& (\text{molws}(n, l, g, h) \\
& \wedge (k \in \text{nset}(n)) \\
& \wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)
\end{aligned}$$

$$\begin{aligned}
& \wedge \text{lg}(n, l, g) \\
& \wedge \text{at}(g, k, 4) \\
& \wedge (\neg \text{at}(l, k, 12)) \\
& \rightarrow \text{union-at-n}(lp, k, '(9\ 10\ 11\ 12))
\end{aligned}$$

THEOREM: lm-k-not-in-l12-imp

$$\begin{aligned}
& (\text{molws}(n, l, g, h) \\
& \wedge (k \in \text{nset}(n)) \\
& \wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp) \\
& \wedge \text{lg}(n, l, g) \\
& \wedge \text{lg}(n, lp, gp) \\
& \wedge \text{at}(g, k, 4) \\
& \wedge (\neg \text{at}(l, k, 12))) \\
& \rightarrow \text{at}(gp, k, 4)
\end{aligned}$$

THEOREM: k-not-in-l12-imp

$$\begin{aligned}
& (\text{molws}(n, l, g, h) \\
& \wedge (k \in \text{nset}(n)) \\
& \wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp) \\
& \wedge \text{lg}(n, l, g) \\
& \wedge \text{at}(g, k, 4) \\
& \wedge (\neg \text{at}(l, k, 12))) \\
& \rightarrow \text{at}(gp, k, 4)
\end{aligned}$$

THEOREM: k-not-in-l12

$$\begin{aligned}
& (\text{molws}(n, l, g, h) \\
& \wedge (i \in \text{nset}(n)) \\
& \wedge (k \in \text{nset}(n)) \\
& \wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp) \\
& \wedge \text{b3a}(l, g, k, i) \\
& \wedge \text{union-at-n}(l, i, '(8\ 9\ 10\ 11\ 12)) \\
& \wedge (\neg \text{union-at-n}(g, i, '(3\ 4)))) \\
& \rightarrow (\neg \text{at}(l, k, 12))
\end{aligned}$$

THEOREM: lm1-b1c-i-neq-k-h-i-eq-k

$$\begin{aligned}
& (\text{molws}(n, l, g, h) \\
& \wedge (i \in \text{nset}(n)) \\
& \wedge (k \in \text{nset}(n)) \\
& \wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp) \\
& \wedge (\text{nth}(h, i) = k) \\
& \wedge \text{lg}(n, l, g) \\
& \wedge \text{b1c}(n, l, g, h, i) \\
& \wedge \text{b3a}(l, g, \text{nth}(h, i), i) \\
& \wedge \text{union-at-n}(l, i, '(8\ 9\ 10\ 11\ 12)) \\
& \wedge (\neg \text{union-at-n}(g, i, '(3\ 4)))) \\
& \rightarrow \text{at}(gp, k, 4)
\end{aligned}$$

THEOREM: lm-b1c-i-neq-k-h-i-eq-k

(molws (n, l, g, h)
 $\wedge (i \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{at}(h, i, k)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{b1c}(n, l, g, h, i)$
 $\wedge \text{b3a}(l, g, \text{nth}(h, i), i)$
 $\wedge \text{union-at-n}(l, i, '(8\ 9\ 10\ 11\ 12))$
 $\wedge (\neg \text{union-at-n}(g, i, '(3\ 4)))$
 $\rightarrow ((\text{nth}(h, i) \in \text{nset}(n)) \wedge \text{at}(gp, \text{nth}(h, i), 4))$

THEOREM: b3a-h-rholemma

(molws (n, l, g, h)
 $\wedge (i \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (i \neq k)$
 $\wedge \text{b3a}(l, g, \text{nth}(h, i), i)$
 $\rightarrow \text{b3a}(l, g, \text{nth}(hp, i), i)$

;;m-l-same-lp and m-gp-same-g are used.

THEOREM: b1c-i-neq-k-h-i-eq-k

(molws (n, l, g, h)
 $\wedge (i \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (i \neq k)$
 $\wedge \text{at}(h, i, k)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{b1c}(n, l, g, h, i)$
 $\wedge \text{b3a}(l, g, \text{nth}(h, i), i)$
 $\rightarrow \text{b1c}(n, lp, gp, hp, i)$

THEOREM: lm-b1c-i-h-i-neq-k

(molws (n, l, g, h)
 $\wedge (i \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (\neg \text{at}(h, i, k))$
 $\wedge \text{b1c}(n, l, g, h, i)$
 $\wedge \text{union-at-n}(l, i, '(8\ 9\ 10\ 11\ 12))$
 $\wedge (\neg \text{union-at-n}(g, i, '(3\ 4)))$
 $\rightarrow ((\text{nth}(h, i) \in \text{nset}(n)) \wedge \text{at}(gp, \text{nth}(h, i), 4))$

THEOREM: b1c-i-h-i-neq-k

(molws (n, l, g, h)
 $\wedge (i \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (i \neq k)$
 $\wedge (\neg \text{at}(h, i, k))$
 $\wedge \text{b1c}(n, l, g, h, i)$
 $\rightarrow \text{b1c}(n, lp, gp, hp, i)$

THEOREM: b1c-i-neq-k

(molws (n, l, g, h)
 $\wedge (i \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (i \neq k)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{b1c}(n, l, g, h, i)$
 $\wedge \text{b3a}(l, g, \text{nth}(h, i), i)$
 $\rightarrow \text{b1c}(n, lp, gp, hp, i)$

THEOREM: mrho-preserves-b1c

(molws (n, l, g, h)
 $\wedge (i \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{b1d}(n, l, h, k)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{b1c}(n, l, g, h, i)$
 $\wedge \text{b3a}(l, g, \text{nth}(h, i), i)$
 $\rightarrow \text{b1c}(n, lp, gp, hp, i)$

;;;;;;;;;;;;; b1d ;;;;;;;;;;;;;;

THEOREM: remainder-quotient

($x \bmod (1 + x) = \text{fix}(x)$)

THEOREM: lm1-member-remainder

($x \not\prec n \rightarrow ((1 + x) \notin \text{nset}(n - 1))$)

THEOREM: lm-member-remainder

(($1 + x \in \text{nset}(n - 1)$) $\rightarrow ((1 + (x \bmod n)) \in \text{nset}(n - 1))$)

THEOREM: member-remainder

($j \in \text{nset}(n) \rightarrow ((1 + ((j - 1) \bmod n)) \in \text{nset}(n))$)

THEOREM: one-nset
 $(n \neq 0) \rightarrow (1 \in \text{nset}(n))$

THEOREM: lm-b1d-i-eq-k
 $(\text{listp}(l)$
 $\wedge \text{listp}(h)$
 $\wedge (n \in \mathbf{N})$
 $\wedge (\text{nth}(h, k) \in \mathbf{N})$
 $\wedge (\text{length}(l) = n)$
 $\wedge (\text{length}(h) = n)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{b1d}(n, l, h, k))$
 $\rightarrow \text{b1d}(n, lp, hp, k)$

THEOREM: b1d-i-eq-k
 $(\text{molws}(n, l, g, h)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{b1d}(n, l, h, k))$
 $\rightarrow \text{b1d}(n, lp, hp, k)$

THEOREM: b1d-neq-k
 $(\text{molws}(n, l, g, h)$
 $\wedge (i \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (i \neq k)$
 $\wedge \text{b1d}(n, l, h, i))$
 $\rightarrow \text{b1d}(n, lp, hp, i)$

THEOREM: mrhoi-preserves-b1d
 $(\text{molws}(n, l, g, h)$
 $\wedge (i \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{b1d}(n, l, h, i))$
 $\rightarrow \text{b1d}(n, lp, hp, i)$

;; b2.ev ;;
 ;;;;;;;;;;;;;;;;;;;;;; b2a ;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;

;* i-eq-k-j-neq-k

THEOREM: j-lt-h-k
(molws (n, l, g, h)
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (j < k)$
 $\wedge \text{at}(l, k, 9)$
 $\wedge \text{union-at-n}(lp, k, '(10\ 11\ 12))$)
 $\rightarrow (j < \text{nth}(h, k))$

THEOREM: lm-case-k-in-l9
(molws (n, l, g, h)
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (j \neq k)$
 $\wedge (j < k)$
 $\wedge \text{b2b}(l, h, k, j)$
 $\wedge \text{at}(l, k, 9)$
 $\wedge (j < \text{nth}(h, k))$
 $\wedge \text{union-at-n}(lp, k, '(10\ 11\ 12))$)
 $\rightarrow (\neg \text{union-at-n}(l, j, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)))$

;;;In the rewrite rule a set of hypotheses is replaced
;;;by another set of formulas. Thus if in a proof
;;;intended beforehand there is a formula belonging to
;;;more than one set of hypotheses which are expected to
;;;be replaced, Bmp is very likely to be unsuccessful.
;;;If j is not equal to k and the k's entry in l is
;;;between 10 and 12, then the j's entry in lp is not
;;;between 5 and 12.

THEOREM: case-k-in-l9
(molws (n, l, g, h)
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (j \neq k)$
 $\wedge (j < k)$
 $\wedge \text{b2b}(l, h, k, j)$
 $\wedge \text{at}(l, k, 9)$
 $\wedge \text{union-at-n}(lp, k, '(10\ 11\ 12))$)
 $\rightarrow (\neg \text{union-at-n}(lp, j, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)))$

;;;need un10-11-then-un10-12.

THEOREM: case-k-in-l10-11

```
(molws (n, l, g, h)
  ^ (j ∈ nset (n))
  ^ (k ∈ nset (n))
  ^ mrhoi (n, k, l, g, h, lp, gp, hp)
  ^ (j ≠ k)
  ^ (j < k)
  ^ b2a (l, k, j)
  ^ union-at-n (l, k, '(10 11)))
→ (¬ union-at-n (lp, j, '(5 6 7 8 9 10 11 12)))
```

THEOREM: k-in-l10-11-or-l9

```
(molws (n, l, g, h)
  ^ (k ∈ nset (n))
  ^ mrhoi (n, k, l, g, h, lp, gp, hp)
  ^ union-at-n (lp, k, '(10 11 12))
  ^ (¬ union-at-n (l, k, '(10 11))))
→ at (l, k, 9)
```

THEOREM: lm-b2a-i-eq-k-j-neq-k

```
(molws (n, l, g, h)
  ^ (j ∈ nset (n))
  ^ (k ∈ nset (n))
  ^ mrhoi (n, k, l, g, h, lp, gp, hp)
  ^ (j ≠ k)
  ^ (j < k)
  ^ lg (n, l, g)
  ^ b2a (l, k, j)
  ^ b2b (l, h, k, j)
  ^ union-at-n (lp, k, '(10 11 12)))
→ (¬ union-at-n (lp, j, '(5 6 7 8 9 10 11 12)))
```

;;;I proved

```
;;;(prove-lemma lm-b2a-i-eq-k-j-neq-k (rewrite)
;;;  (implies (and (molws n l g h)
;;;                (member j (nset n))
;;;                (member k (nset n))
;;;                (mrhoi n k l g h lp gp hp)
;;;                (not (equal j k))
;;;                (lessp j k)
;;;                (lg n l g)
;;;                (b2a l k j)
;;;                (b2b l h k j)
;;;                (union-at-n lp k '(10 11 12))))
;;;    (not (union-at-n l j '(5 6 7 8 9 10 11 12))))))
```

;;;and tried to prove the following lemma counting on
 ;;;m-lp-same-1, but it was unsuccessful.

THEOREM: b2a-i-eq-k-j-neq-k

(molws (n, l, g, h)
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (j \neq k)$
 $\wedge (j < k)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{b2b}(l, h, k, j)$
 $\wedge \text{b2a}(l, k, j)$
 $\rightarrow \text{b2a}(lp, k, j)$

;* i-neq-k-j-eq-k

;;;If the k's entry in l is not 4 and
 ;;;the k's entry in lp is between 5 and 7, then
 ;;;the k's entry in l is between 5 and 7.

THEOREM: m-k-in-lp5-7-not-l4-then-l5-7

(molws (n, l, g, h)
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (\neg \text{at}(l, k, 4))$
 $\wedge \text{union-at-n}(lp, k, '(5 6 7))$
 $\rightarrow \text{union-at-n}(l, k, '(5 6 7))$

;;;If the k's entry in lp is between 5 and 7 then
 ;;;the k's entry in l is certainly between 5 and 12.

THEOREM: m-k-in-lp5-7-then-l5-11

(molws (n, l, g, h)
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (\neg \text{at}(l, k, 4))$
 $\wedge \text{union-at-n}(lp, k, '(5 6 7))$
 $\rightarrow \text{union-at-n}(l, k, '(5 6 7 8 9 10 11))$

;;;If the k's entry in lp is 8 then k's entry
 ;;;in l is either 5 or 7.

THEOREM: m-lp8-k-in-l57

```

(molws (n, l, g, h)
  ∧ (k ∈ nset (n))
  ∧ mrhoi (n, k, l, g, h, lp, gp, hp)
  ∧ at (lp, k, 8))
→ union-at-n (l, k, '(5 7))

```

```

;;;If the k's entry in lp is 8,
;;;then the k's entry in l is between 5 and 11.

```

THEOREM: m-k-in-lp8-then-l5-11

```

(molws (n, l, g, h)
  ∧ (k ∈ nset (n))
  ∧ mrhoi (n, k, l, g, h, lp, gp, hp)
  ∧ at (lp, k, 8))
→ union-at-n (l, k, '(5 6 7 8 9 10 11))

```

```

;;;If the k's entry in lp is between 9 and 12,
;;;then the k's entry in l is between 5 and 12.

```

THEOREM: m-k-in-lp9-12-then-l5-11

```

(molws (n, l, g, h)
  ∧ (k ∈ nset (n))
  ∧ mrhoi (n, k, l, g, h, lp, gp, hp)
  ∧ union-at-n (lp, k, '(9 10 11 12)))
→ union-at-n (l, k, '(5 6 7 8 9 10 11))

```

```

;;;If the k's entry in l is 4 an the k's entry in lp is
;;;between 5 and 12, then the k's entry in l is
;;;between 5 and 11.

```

THEOREM: m-k-in-l5-11

```

(molws (n, l, g, h)
  ∧ (k ∈ nset (n))
  ∧ mrhoi (n, k, l, g, h, lp, gp, hp)
  ∧ (¬ at (l, k, 4))
  ∧ union-at-n (lp, k, '(5 6 7 8 9 10 11 12)))
→ union-at-n (l, k, '(5 6 7 8 9 10 11))

```

THEOREM: m-k-not-in-l4

```

(molws (n, l, g, h)
  ∧ (k ∈ nset (n))
  ∧ mrhoi (n, k, l, g, h, lp, gp, hp)
  ∧ (¬ at (l, k, 4))
  ∧ (¬ union-at-n (l, k, '(5 6 7 8 9 10 11 12))))
→ (¬ union-at-n (lp, k, '(5 6 7 8 9 10 11 12)))

```

THEOREM: m-k-not-in-lp5-12

(molws (n, l, g, h)
 $\wedge (i \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{b1a}(l, i, k)$
 $\wedge \text{union-at-n}(l, i, '(10\ 11\ 12))$
 $\wedge (\neg \text{union-at-n}(l, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)))$
 $\rightarrow (\neg \text{union-at-n}(lp, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)))$)

THEOREM: lm-b2a-i-neq-k-j-eq-k

(molws (n, l, g, h)
 $\wedge (i \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (i \neq k)$
 $\wedge (k < i)$
 $\wedge \text{b1a}(l, i, k)$
 $\wedge \text{b2a}(l, i, k)$
 $\wedge \text{union-at-n}(lp, i, '(10\ 11\ 12))$
 $\rightarrow (\neg \text{union-at-n}(lp, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)))$)

THEOREM: b2a-i-neq-k-j-eq-k

(molws (n, l, g, h)
 $\wedge (i \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (i \neq k)$
 $\wedge (k < i)$
 $\wedge \text{b1a}(l, i, k)$
 $\wedge \text{b2a}(l, i, k)$
 $\rightarrow \text{b2a}(lp, i, k)$)

;* i-j-neq-k-neq-k

THEOREM: b2a-i-j-neq-k

(molws (n, l, g, h)
 $\wedge (i \in \text{nset}(n))$
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (i \neq k)$
 $\wedge (j \neq k)$
 $\wedge (j < i)$)

\wedge b2a(l, i, j)
 \rightarrow b2a(lp, i, j)

THEOREM: b2a-i-neq-k

(molws(n, l, g, h)
 \wedge ($i \in \text{nset}(n)$)
 \wedge ($j \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge ($i \neq k$)
 \wedge ($j < i$)
 \wedge b1a(l, i, j)
 \wedge b2a(l, i, j)
 \wedge b2b(l, h, i, j)
 \rightarrow b2a(lp, i, j)

THEOREM: b2a-i-eq-k

(molws(n, l, g, h)
 \wedge ($j \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge ($j < k$)
 \wedge lg(n, l, g)
 \wedge b2a(l, k, j)
 \wedge b2b(l, h, k, j)
 \rightarrow b2a(lp, k, j)

THEOREM: mrho-preserves-b2a

(molws(n, l, g, h)
 \wedge ($i \in \text{nset}(n)$)
 \wedge ($j \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge ($j < i$)
 \wedge lg(n, l, g)
 \wedge b1a(l, i, j)
 \wedge b2a(l, i, j)
 \wedge b2b(l, h, i, j)
 \rightarrow b2a(lp, i, j)

;;;;;;;;;;;;; b2b ;;;;;;;;;;;;;;

;;;;;;;;;;;;;Common in atom and mole.

THEOREM: l9-then-un8-12

at($l, i, 9$) \rightarrow union-at-n($l, i, '(8 9 10 11 12)$)

;;;;;;;;;;;;;Common in atom and mole end.

THEOREM: lg-nth-h-k

(molws (n, l, g, h)
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{at}(h, k, j)$
 $\wedge \text{union-at-n}(g, \text{nth}(h, k), '(0 1))$
 $\rightarrow (\neg \text{union-at-n}(l, j, '(5 6 7 8 9 10 11 12)))$)

THEOREM: l9-g01

(molws (n, l, g, h)
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{at}(h, k, j)$
 $\wedge \text{at}(l, k, 9)$
 $\wedge \text{at}(lp, k, 9)$
 $\rightarrow \text{union-at-n}(g, \text{nth}(h, k), '(0 1))$)

THEOREM: l9-nth-h-k-eq-j

(at (h, k, j)
 $\wedge \text{molws}(n, l, g, h)$
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{at}(l, k, 9)$
 $\wedge \text{at}(lp, k, 9)$
 $\rightarrow (\neg \text{union-at-n}(l, j, '(5 6 7 8 9 10 11 12)))$)

THEOREM: lm-j-not-in-l5-12

(molws (n, l, g, h)
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (j < k)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{b2b}(l, h, k, j)$
 $\wedge \text{at}(l, k, 9)$
 $\wedge \text{at}(lp, k, 9)$
 $\wedge ((j - 1) < \text{nth}(h, k))$
 $\rightarrow (\neg \text{union-at-n}(l, j, '(5 6 7 8 9 10 11 12)))$)

THEOREM: cond-l9
(molws (n, l, g, h)
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{at}(l, k, 9)$
 $\wedge (j < \text{nth}(hp, k))$
 $\rightarrow ((j - 1) < \text{nth}(h, k))$

THEOREM: j-not-in-l5-12
(molws (n, l, g, h)
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (j < k)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{b2b}(l, h, k, j)$
 $\wedge \text{at}(l, k, 9)$
 $\wedge \text{at}(lp, k, 9)$
 $\wedge (j < \text{nth}(hp, k))$
 $\rightarrow (\neg \text{union-at-n}(l, j, '(5 6 7 8 9 10 11 12)))$

THEOREM: k-in-l9
(molws (n, l, g, h)
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{at}(lp, k, 9)$
 $\wedge (j < \text{nth}(hp, k))$
 $\rightarrow \text{at}(l, k, 9)$

;;;The order of the hints is crucial.

THEOREM: lm-b2b-i-eq-k-j-neq-k
(molws (n, l, g, h)
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (j \neq k)$
 $\wedge (j < k)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{b2b}(l, h, k, j)$
 $\wedge \text{at}(lp, k, 9)$
 $\wedge (j < \text{nth}(hp, k))$
 $\rightarrow (\neg \text{union-at-n}(lp, j, '(5 6 7 8 9 10 11 12)))$

THEOREM: b2b-i-eq-k-j-neq-k

(molws (n, l, g, h)
 \wedge ($j \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi ($n, k, l, g, h, lp, gp, hp$)
 \wedge ($j \neq k$)
 \wedge ($j < k$)
 \wedge lg (n, l, g)
 \wedge b2b (l, h, k, j)
 \rightarrow b2b (lp, hp, k, j))

THEOREM: b2b-i-eq-k

(molws (n, l, g, h)
 \wedge ($j \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi ($n, k, l, g, h, lp, gp, hp$)
 \wedge ($j < k$)
 \wedge lg (n, l, g)
 \wedge b2b (l, h, k, j)
 \rightarrow b2b (lp, hp, k, j))

THEOREM: not-k-in-l5-12-imp

(molws (n, l, g, h)
 \wedge ($i \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi ($n, k, l, g, h, lp, gp, hp$)
 \wedge b1a (l, i, k)
 \wedge at ($l, i, 9$)
 \wedge (\neg union-at-n ($l, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)$)))
 \rightarrow (\neg union-at-n ($lp, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)$)))

;;;The order of hypotheses is crucial.

THEOREM: lm-b2b-i-neq-k-j-eq-k

(molws (n, l, g, h)
 \wedge ($i \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi ($n, k, l, g, h, lp, gp, hp$)
 \wedge ($i \neq k$)
 \wedge ($k < i$)
 \wedge b1a (l, i, k)
 \wedge b2b (l, h, i, k)
 \wedge at ($l, i, 9$)
 \wedge ($k < \text{nth}(h, i)$)
 \rightarrow (\neg union-at-n ($lp, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)$)))

THEOREM: b2b-i-neq-k-j-eq-k

(molws (n, l, g, h)
 \wedge ($i \in \text{nset } (n)$)
 \wedge ($k \in \text{nset } (n)$)
 \wedge mrhoi ($n, k, l, g, h, lp, gp, hp$)
 \wedge ($i \neq k$)
 \wedge ($k < i$)
 \wedge b1a (l, i, k)
 \wedge b2b (l, h, i, k)
 \rightarrow b2b (lp, hp, i, k)

;;;The position of (member k (nset n)) is
;;;crucial to trigger rholemmas.

THEOREM: b2b-i-j-neq-k

(molws (n, l, g, h)
 \wedge ($i \in \text{nset } (n)$)
 \wedge ($j \in \text{nset } (n)$)
 \wedge ($k \in \text{nset } (n)$)
 \wedge mrhoi ($n, k, l, g, h, lp, gp, hp$)
 \wedge ($i \neq k$)
 \wedge ($j \neq k$)
 \wedge ($j < i$)
 \wedge b2b (l, h, i, j)
 \rightarrow b2b (lp, hp, i, j)

THEOREM: b2b-i-neq-k

(molws (n, l, g, h)
 \wedge ($i \in \text{nset } (n)$)
 \wedge ($j \in \text{nset } (n)$)
 \wedge ($k \in \text{nset } (n)$)
 \wedge mrhoi ($n, k, l, g, h, lp, gp, hp$)
 \wedge ($i \neq k$)
 \wedge ($j < i$)
 \wedge b1a (l, i, j)
 \wedge b2b (l, h, i, j)
 \rightarrow b2b (lp, hp, i, j)

THEOREM: mrho-preserves-b2b

(molws (n, l, g, h)
 \wedge ($i \in \text{nset } (n)$)
 \wedge ($j \in \text{nset } (n)$)
 \wedge ($k \in \text{nset } (n)$)
 \wedge mrhoi ($n, k, l, g, h, lp, gp, hp$)
 \wedge ($j < i$)

\wedge lg(n, l, g)
 \wedge b1a(l, i, j)
 \wedge b2b(l, h, i, j)
 \rightarrow b2b(lp, hp, i, j)

;; b3.ev ;;
 ;;;;;;;;;;;;;;;;;; b3a ;;;;;;;;;;;;;;;;;;

;* i-neq-k-j-eq-k

THEOREM: lm-b3a-k-in-l9-11

(molws(n, l, g, h)
 \wedge ($i \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge lg(n, l, g)
 \wedge b3a(l, g, i, k)
 \wedge at($l, i, 12$)
 \wedge union-at-n($l, k, '(5 6 7 8 9 10 11)$))
 \rightarrow union-at-n($l, k, '(9 10 11)$)

THEOREM: b3a-k-in-l9-11

(molws(n, l, g, h)
 \wedge ($i \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge lg(n, l, g)
 \wedge b1a(l, i, k)
 \wedge b3a(l, g, i, k)
 \wedge at($l, i, 12$)
 \wedge union-at-n($lp, k, '(5 6 7 8 9 10 11 12)$))
 \rightarrow union-at-n($l, k, '(9 10 11)$)

THEOREM: m-k-in-lp9-12

(molws(n, l, g, h)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge union-at-n($l, k, '(9 10 11)$))
 \rightarrow union-at-n($lp, k, '(9 10 11 12)$)

THEOREM: lm-b3a-i-neq-k-j-eq-k

(molws(n, l, g, h)
 \wedge ($i \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)

\wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge lg(n, l, g)
 \wedge lg(n, lp, gp)
 \wedge b1a(l, i, k)
 \wedge b3a(l, g, i, k)
 \wedge at($l, i, 12$)
 \wedge union-at-n($lp, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)$)
 \rightarrow at($gp, k, 4$)

THEOREM: b3a-i-neq-k-j-eq-k

(molws(n, l, g, h)
 \wedge ($i \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge ($i \neq k$)
 \wedge lg(n, l, g)
 \wedge b1a(l, i, k)
 \wedge b3a(l, g, i, k)
 \rightarrow b3a(lp, gp, i, k)

;* i-eq-k-j-neq-k

THEOREM: cond-lp12

(molws(n, l, g, h)
 \wedge ($k \in \text{nset}(n)$)
 \wedge ($j \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge at($lp, k, 12$)
 \wedge at($l, k, 11$)
 \rightarrow ($j < \text{nth}(h, k)$)

THEOREM: b3a-j-in-l5-12

(molws(n, l, g, h)
 \wedge ($j \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge lg(n, l, g)
 \wedge b3b(l, g, h, k, j)
 \wedge at($l, k, 11$)
 \wedge at($lp, k, 12$)
 \wedge union-at-n($l, j, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)$)
 \rightarrow at($g, j, 4$)

THEOREM: m-k-in-l11

(molws(n, l, g, h))

$\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{at}(lp, k, 12)$
 $\rightarrow \text{at}(l, k, 11)$

THEOREM: lm-b3a-i-eq-k-j-neq-k

$(\text{molws}(n, l, g, h)$
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (j \neq k)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{b3b}(l, g, h, k, j)$
 $\wedge \text{at}(lp, k, 12)$
 $\wedge \text{union-at-n}(lp, j, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)))$
 $\rightarrow \text{at}(g, j, 4)$

THEOREM: b3a-i-eq-k-j-neq-k

$(\text{molws}(n, l, g, h)$
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (j \neq k)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{b3b}(l, g, h, k, j))$
 $\rightarrow \text{b3a}(lp, gp, k, j)$

;* i-j-neq-k

THEOREM: b3a-i-j-neq-k

$(\text{molws}(n, l, g, h)$
 $\wedge (i \in \text{nset}(n))$
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (i \neq k)$
 $\wedge (j \neq k)$
 $\wedge \text{b3a}(l, g, i, j))$
 $\rightarrow \text{b3a}(lp, gp, i, j)$

THEOREM: b3a-i-neq-k

$(\text{molws}(n, l, g, h)$
 $\wedge (i \in \text{nset}(n))$
 $\wedge (j \in \text{nset}(n))$

$\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (i \neq k)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{b1a}(l, i, j)$
 $\wedge \text{b3a}(l, g, i, j)$
 $\rightarrow \text{b3a}(lp, gp, i, j)$

;* i-j-eq-k

THEOREM: b3a-i-j-eq-k

$(\text{molws}(n, l, g, h)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{b3a}(l, g, k, k)$
 $\wedge \text{b3b}(l, g, h, k, k))$
 $\rightarrow \text{b3a}(lp, gp, k, k)$

THEOREM: b3a-i-eq-k

$(\text{molws}(n, l, g, h)$
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{b3a}(l, g, k, j)$
 $\wedge \text{b3b}(l, g, h, k, j))$
 $\rightarrow \text{b3a}(lp, gp, k, j)$

THEOREM: mrho-preserves-b3a

$(\text{molws}(n, l, g, h)$
 $\wedge (i \in \text{nset}(n))$
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{b1a}(l, i, j)$
 $\wedge \text{b3a}(l, g, i, j)$
 $\wedge \text{b3b}(l, g, h, i, j))$
 $\rightarrow \text{b3a}(lp, gp, i, j)$

;;;;;;;;;;;;; b3b ;;;;;;;;;;;;;;

;;;;;;;;;;;;;common in atom and mole.

THEOREM: l10-then-un10-12
at ($l, k, 10$) \rightarrow union-at-n ($l, k, '(10\ 11\ 12)$)

THEOREM: l11-then-un9-12
at ($lp, k, 11$) \rightarrow union-at-n ($lp, k, '(9\ 10\ 11\ 12)$)

THEOREM: l11-then-un8-12
at ($l, i, 11$) \rightarrow union-at-n ($l, i, '(8\ 9\ 10\ 11\ 12)$)

;;;;;;;;;;;;;common in atom and mole end.

;* i-neq-k-j-eq-k

THEOREM: lm-b3b-k-in-l9-11
(molws (n, l, g, h)
 $\wedge (i \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{b3b}(l, g, h, i, k)$
 $\wedge \text{at}(l, i, 11)$
 $\wedge (k < \text{nth}(h, i))$
 $\wedge \text{union-at-n}(l, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11))$)
 $\rightarrow \text{union-at-n}(l, k, '(9\ 10\ 11))$

THEOREM: b3b-k-in-l9-11
(molws (n, l, g, h)
 $\wedge (i \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{b1a}(l, i, k)$
 $\wedge \text{b3b}(l, g, h, i, k)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{at}(l, i, 11)$
 $\wedge (k < \text{nth}(h, i))$
 $\wedge \text{union-at-n}(lp, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12))$)
 $\rightarrow \text{union-at-n}(l, k, '(9\ 10\ 11))$

THEOREM: lm-b3b-i-neq-k-j-eq-k
(molws (n, l, g, h)
 $\wedge (i \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{lg}(n, l, g)$

\wedge lg(n, lp, gp)
 \wedge b1a(l, i, k)
 \wedge b3b(l, g, h, i, k)
 \wedge at($l, i, 11$)
 \wedge ($k < \text{nth}(h, i)$)
 \wedge union-at-n($lp, k, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)$)
 \rightarrow at($gp, k, 4$)

THEOREM: b3b-i-neq-k-j-eq-k

(molws(n, l, g, h)
 \wedge ($i \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge ($i \neq k$)
 \wedge lg(n, l, g)
 \wedge b1a(l, i, k)
 \wedge b3b(l, g, h, i, k)
 \rightarrow b3b(lp, gp, hp, i, k)

;* i-eq-k-j-neq-k

THEOREM: j-in-g4

(molws(n, l, g, h)
 \wedge ($j \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge lg(n, l, g)
 \wedge at(h, k, j)
 \wedge (\neg union-at-n($g, \text{nth}(h, k), '(2\ 3)$))
 \wedge union-at-n($l, j, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)$)
 \rightarrow at($g, j, 4$)

THEOREM: l11-g14

(molws(n, l, g, h)
 \wedge ($j \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge at(h, k, j)
 \wedge at($l, k, 11$)
 \wedge at($lp, k, 11$)
 \rightarrow (\neg union-at-n($g, \text{nth}(h, k), '(2\ 3)$))

THEOREM: l11-nth-h-k-eq-j

(at(h, k, j))

\wedge molws(n, l, g, h)
 \wedge ($j \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge lg(n, l, g)
 \wedge at($l, k, 11$)
 \wedge at($lp, k, 11$)
 \wedge union-at-n($l, j, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)$))
 \rightarrow at($g, j, 4$)

THEOREM: lm-j-in-l5-12

(molws(n, l, g, h)
 \wedge ($j \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge lg(n, l, g)
 \wedge b3b(l, g, h, k, j)
 \wedge at($l, k, 11$)
 \wedge at($lp, k, 11$)
 \wedge ($(j - 1) < \text{nth}(h, k)$)
 \wedge union-at-n($l, j, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)$))
 \rightarrow at($g, j, 4$)

THEOREM: cond-l11

(molws(n, l, g, h)
 \wedge ($j \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge at($l, k, 11$)
 \wedge ($j < \text{nth}(hp, k)$)
 \rightarrow ($(j - 1) < \text{nth}(h, k)$)

THEOREM: j-in-l5-12

(molws(n, l, g, h)
 \wedge ($j \in \text{nset}(n)$)
 \wedge ($k \in \text{nset}(n)$)
 \wedge mrhoi($n, k, l, g, h, lp, gp, hp$)
 \wedge ($j \neq k$)
 \wedge lg(n, l, g)
 \wedge b3b(l, g, h, k, j)
 \wedge at($l, k, 11$)
 \wedge at($lp, k, 11$)
 \wedge ($j < \text{nth}(hp, k)$)
 \wedge union-at-n($l, j, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)$))
 \rightarrow at($g, j, 4$)

THEOREM: j-leq-add1k-then-k-not-in-l10
(molws (n, l, g, h)
 \wedge ($k \in \text{nset } (n)$)
 \wedge ($j \in \text{nset } (n)$)
 \wedge $\text{mrhoi } (n, k, l, g, h, lp, gp, hp)$
 \wedge $\text{b2a } (l, k, j)$
 \wedge $\text{union-at-n } (l, j, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12))$
 \wedge ($j \neq k$)
 \wedge ($j < (1 + k)$)
 \rightarrow ($\neg \text{at } (l, k, 10)$)

THEOREM: not-j-leq-add1k-then-k-not-in-l10
(molws (n, l, g, h)
 \wedge ($k \in \text{nset } (n)$)
 \wedge ($j \in \text{nset } (n)$)
 \wedge $\text{mrhoi } (n, k, l, g, h, lp, gp, hp)$
 \wedge $\text{at } (lp, k, 11)$
 \wedge ($j < \text{nth } (hp, k)$)
 \wedge ($j \not< (1 + k)$)
 \rightarrow ($\neg \text{at } (l, k, 10)$)

;;;The order of (member k (nset n)) and
;;;(member j (nset n)) are switched deliberately.

THEOREM: k-not-in-l10
(molws (n, l, g, h)
 \wedge ($k \in \text{nset } (n)$)
 \wedge ($j \in \text{nset } (n)$)
 \wedge $\text{mrhoi } (n, k, l, g, h, lp, gp, hp)$
 \wedge ($j \neq k$)
 \wedge $\text{b2a } (l, k, j)$
 \wedge $\text{union-at-n } (l, j, '(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12))$
 \wedge $\text{at } (lp, k, 11)$
 \wedge ($j < \text{nth } (hp, k)$)
 \rightarrow ($\neg \text{at } (l, k, 10)$)

THEOREM: lp11-then-l11-or-l10
(molws (n, l, g, h)
 \wedge ($k \in \text{nset } (n)$)
 \wedge $\text{mrhoi } (n, k, l, g, h, lp, gp, hp)$
 \wedge $\text{at } (lp, k, 11)$
 \wedge ($\neg \text{at } (l, k, 10)$)
 \rightarrow $\text{at } (l, k, 11)$

;;;When the order of (member j (nset n)) and

```

;;;(member k (nset n)) is switched, the order of
;;;hints must be switched, in order to make the proof
;;;successful.

```

THEOREM: b3b-k-in-l11

```

(molws (n, l, g, h)
  ^ (j ∈ nset (n))
  ^ (k ∈ nset (n))
  ^ mrhoi (n, k, l, g, h, lp, gp, hp)
  ^ (j ≠ k)
  ^ b2a (l, k, j)
  ^ union-at-n (l, j, '(5 6 7 8 9 10 11 12))
  ^ at (lp, k, 11)
  ^ (j < nth (hp, k)))
→ at (l, k, 11)

```

```

;;;When the order of (member j (nset n)) and
;;;(member k (nset n)) is switched then the order of
;;;hints must be switched in order to make the proof
;;;successful.

```

THEOREM: lm-b3b-i-eq-k-j-neq-k

```

(molws (n, l, g, h)
  ^ (j ∈ nset (n))
  ^ (k ∈ nset (n))
  ^ mrhoi (n, k, l, g, h, lp, gp, hp)
  ^ (j ≠ k)
  ^ lg (n, l, g)
  ^ b2a (l, k, j)
  ^ b3b (l, g, h, k, j)
  ^ at (lp, k, 11)
  ^ (j < nth (hp, k))
  ^ union-at-n (l, j, '(5 6 7 8 9 10 11 12)))
→ at (g, j, 4)

```

THEOREM: b3b-i-eq-k-j-neq-k

```

(molws (n, l, g, h)
  ^ (j ∈ nset (n))
  ^ (k ∈ nset (n))
  ^ mrhoi (n, k, l, g, h, lp, gp, hp)
  ^ (j ≠ k)
  ^ lg (n, l, g)
  ^ b3b (l, g, h, k, j)
  ^ b2a (l, k, j)
→ b3b (lp, gp, hp, k, j)

```

THEOREM: b3b-i-j-neq-k
 $(\text{molws}(n, l, g, h)$
 $\wedge (i \in \text{nset}(n))$
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (i \neq k)$
 $\wedge (j \neq k)$
 $\wedge \text{b3b}(l, g, h, i, j))$
 $\rightarrow \text{b3b}(lp, gp, hp, i, j)$

THEOREM: b3b-i-neq-k
 $(\text{molws}(n, l, g, h)$
 $\wedge (i \in \text{nset}(n))$
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge (i \neq k)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{b1a}(l, i, j)$
 $\wedge \text{b3b}(l, g, h, i, j))$
 $\rightarrow \text{b3b}(lp, gp, hp, i, j)$

THEOREM: b3b-i-j-eq-k
 $(\text{molws}(n, l, g, h)$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{b3b}(l, g, h, k, k))$
 $\rightarrow \text{b3b}(lp, gp, hp, k, k)$

THEOREM: b3b-i-eq-k
 $(\text{molws}(n, l, g, h)$
 $\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{b1a}(l, k, j)$
 $\wedge \text{b3b}(l, g, h, k, j)$
 $\wedge \text{b2a}(l, k, j))$
 $\rightarrow \text{b3b}(lp, gp, hp, k, j)$

THEOREM: mrho-preserves-b3b
 $(\text{molws}(n, l, g, h)$
 $\wedge (i \in \text{nset}(n))$

$\wedge (j \in \text{nset}(n))$
 $\wedge (k \in \text{nset}(n))$
 $\wedge \text{mrhoi}(n, k, l, g, h, lp, gp, hp)$
 $\wedge \text{lg}(n, l, g)$
 $\wedge \text{b1a}(l, i, j)$
 $\wedge \text{b3b}(l, g, h, i, j)$
 $\wedge \text{b2a}(l, i, j)$
 $\rightarrow \text{b3b}(lp, gp, hp, i, j)$

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