

EVENT: Start with the library "subset".

DEFINITION:  $\text{set-equal}(x, y) = (\text{subsetp}(x, y) \wedge \text{subsetp}(y, x))$

THEOREM: set-equal-char  
 $\text{set-equal}(x, y) \rightarrow ((a \in x) = (a \in y))$

DEFINITION:  
 $\text{diff}(x, y)$   
 $=$  **if**  $\text{listp}(x)$   
    **then if**  $\text{car}(x) \in y$  **then**  $\text{diff}(\text{cdr}(x), y)$   
        **else**  $\text{cons}(\text{car}(x), \text{diff}(\text{cdr}(x), y))$  **endif**  
    **else**  $x$  **endif**

DEFINITION:  $\text{sym-diff}(x, y) = (\text{diff}(x, y) \cup \text{diff}(y, x))$

THEOREM: diff-with-nil  
 $(\neg \text{listp}(x)) \rightarrow (\text{diff}(y, x) = y)$

DEFINITION:  
 $\text{not-subsetp-skolem}(x, y)$   
 $=$  **if**  $\neg \text{listp}(x)$  **then nil**  
    **elseif**  $\text{car}(x) \in y$  **then**  $\text{not-subsetp-skolem}(\text{cdr}(x), y)$   
    **else**  $\text{car}(x)$  **endif**

THEOREM: subsetp-witness  
 $((\text{not-subsetp-skolem}(x, y) \notin x) \vee (\text{not-subsetp-skolem}(x, y) \in y))$   
 $\rightarrow \text{subsetp}(x, y)$

THEOREM: diff-non-member  
 $(a \in \text{diff}(x, y)) \rightarrow (a \notin y)$

THEOREM: member-union  
 $(a \in (x \cup y)) = ((a \in x) \vee (a \in y))$

THEOREM: sym-diff-char-1  
 $(a \in \text{sym-diff}(x, y))$   
 $\rightarrow$  **if**  $a \in x$  **then**  $a \notin y$   
    **else**  $a \in y$  **endif**

THEOREM: sym-diff-char-2  
**if**  $a \in x$  **then**  $a \notin y$   
    **else**  $a \in y$  **endif**  
 $\rightarrow (a \in \text{sym-diff}(x, y))$

EVENT: Disable sym-diff.

THEOREM: sym-diff-char  
( $a \in \text{sym-diff}(x, y)$ )  
= **if**  $a \in x$  **then**  $a \notin y$   
  **else**  $a \in y$  **endif**

THEOREM: sym-diff-assoc  
set-equal (sym-diff (sym-diff ( $x, y$ ),  $z$ ), sym-diff ( $x, \text{sym-diff}(y, z)$ ))  
  
; (PROVE-LEMMA SYM-DIFF-ASSOC (REWRITE)  
;     (SET-EQUAL (SYM-DIFF (SYM-DIFF X Y) Z)  
; (SYM-DIFF X (SYM-DIFF Y Z))))

EVENT: Enable sym-diff.

THEOREM: sym-diff-comm  
set-equal (sym-diff ( $x, y$ ), sym-diff ( $y, x$ ))

## Index

diff, 1  
diff-non-member, 1  
diff-with-nil, 1  
  
member-union, 1  
  
not-subsetp-skolem, 1  
  
set-equal, 1, 2  
set-equal-char, 1  
subsetp, 1  
subsetp-witness, 1  
sym-diff, 1, 2  
sym-diff-assoc, 2  
sym-diff-char, 2  
sym-diff-char-1, 1  
sym-diff-char-2, 1  
sym-diff-comm, 2