Event: Start with the initial **nqthm** theory.

Definition:
delete (x, l)
= **if** listp (l)
then if $x = \text{car}(l)$ **then** cdr (l)
else cons (car (l), delete $(x, \text{cdr}(l))$) **endif**
else l **endif**

Definition:
bagdiff (x, y)
= **if** listp (y)
then if car $(y) \in x$ **then** bagdiff (delete (car (y), x), cdr (y))
else bagdiff $(x, \text{cdr}(y))$ **endif**
else x **endif**

Definition:
bagint (x, y)
= **if** listp (x)
then if car $(x) \in y$
then cons (car (x), bagint (cdr (x), delete (car (x), y)))
else bagint (cdr (x), y) **endif**
else nil endif

Definition:
occurrences (x, l)
= **if** listp (l)
then if $x = \text{car}(l)$ **then** $1 +$ occurrences $(x, \text{cdr}(l))$
else occurrences $(x, \text{cdr}(l))$ **endif**
else 0 **endif**

Definition:
subbagp (x, y)
= **if** listp (x)
then if car $(x) \in y$ **then** subbagp (cdr (x), delete (car (x), y))
else f endif
else t endif

Theorem: listp-delete
listp (delete (x, l))
= **if** listp (l) **then** $(x \neq \text{car}(l)) \lor$ listp (cdr (l))
else f endif

Event: Disable listp-delete.

THEOREM: delete-non-member
$(x \notin y) \rightarrow (\text{delete}(x, y) = y)$

THEOREM: delete-delete
$\text{delete}(y, \text{delete}(x, z)) = \text{delete}(x, \text{delete}(y, z))$

THEOREM: equal-occurrences-zero
$(\text{occurrences}(x, l) = \mathtt{0}) = (x \notin l)$

THEOREM: member-non-list
$(\neg \text{listp}(l)) \rightarrow (x \notin l)$

THEOREM: member-delete
$(x \in \text{delete}(y, l))$
$= \ \ \textbf{if } x \in l$
$\quad \textbf{then if } x = y \ \textbf{ then } \mathtt{1} < \text{occurrences}(x, l)$
$\quad\quad \textbf{else } \mathbf{t} \textbf{ endif}$
$\quad \textbf{else } \mathbf{f} \textbf{ endif}$

THEOREM: member-delete-implies-membership
$(x \in \text{delete}(y, l)) \rightarrow (x \in l)$

THEOREM: occurrences-delete
$\text{occurrences}(x, \text{delete}(y, l))$
$= \ \ \textbf{if } x = y$
$\quad \textbf{then if } x \in l \ \textbf{ then } \text{occurrences}(x, l) - 1$
$\quad\quad \textbf{else } \mathtt{0} \textbf{ endif}$
$\quad \textbf{else } \text{occurrences}(x, l) \textbf{ endif}$

THEOREM: member-bagdiff
$(x \in \text{bagdiff}(a, b)) = (\text{occurrences}(x, b) < \text{occurrences}(x, a))$

THEOREM: bagdiff-delete
$\text{bagdiff}(\text{delete}(e, x), y) = \text{delete}(e, \text{bagdiff}(x, y))$

THEOREM: subbagp-delete
$\text{subbagp}(x, \text{delete}(u, y)) \rightarrow \text{subbagp}(x, y)$

THEOREM: subbagp-cdr1
$\text{subbagp}(x, y) \rightarrow \text{subbagp}(\text{cdr}(x), y)$

THEOREM: subbagp-cdr2
$\text{subbagp}(x, \text{cdr}(y)) \rightarrow \text{subbagp}(x, y)$

THEOREM: subbagp-bagint1
$\text{subbagp}(\text{bagint}(x, y), x)$

THEOREM: subbagp-bagint2
subbagp (bagint $(x,\ y),\ y)$

THEOREM: occurrences-bagint
occurrences $(x,$ bagint $(a,\ b))$
$=$ **if** occurrences $(x,\ a) <$ occurrences $(x,\ b)$ **then** occurrences $(x,\ a)$
else occurrences $(x,\ b)$ **endif**

THEOREM: occurrences-bagdiff
occurrences $(x,$ bagdiff $(a,\ b)) =$ (occurrences $(x,\ a) -$ occurrences $(x,\ b))$

THEOREM: member-bagint
$(x \in$ bagint $(a,\ b)) = ((x \in a) \land (x \in b))$

EVENT: Let us define the theory *bags* to consist of the following events: occurrences-bagint, bagdiff-delete, occurrences-bagdiff, member-bagint, member-bagdiff, subbagp-bagint2, subbagp-bagint1, subbagp-cdr2, subbagp-cdr1, subbagp-delete.

EVENT: Make the library `"bags"`.

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