An educational stupidity.

The following two divisibility tests are widely taught at Dutch schools.

The 9-test. To reduce a number modulo 9, add its decimal digits. E.g., with $N = 15098$,

$$15098 \equiv 8+9+0+5+1 = 23 \equiv 3+2 = 5 \quad .$$

It is proved by observing that, modulo 9, $10 \equiv 1$, hence $10^k \equiv 1$. As a derivative we are taught the 3-test: $N \,\underline{\mathrm{mod}}\, 3 = 2$ because $5 \,\underline{\mathrm{mod}}\, 3 = 2$.

The 11-test. To reduce a number modulo 11, add its decimal digits with alternating signs, e.g.

$$15098 \equiv 8-9+0-5+1 = -5 \equiv 6 \quad .$$

It is proved by observing that, modulo 11, $10 \equiv -1$, hence $10^k \equiv (-1)^k$.

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Here is another approach to the 11-test. Interpret the number, by pairing the digits, as written in base 100 and start with the 99-test

$$15098 \equiv 1+50+98 = 149 \equiv 1+49 = 50 \equiv 6 \quad ,$$

where the last congruence is only modulo 11.

The fact that the last test manipulates only natural numbers could be viewed as a minor advantage. It is more important to see that the 11-test is related to the 99-test in exactly the same way as the 3-test is related to the 9-test. It is now immediately obvious how to reduce N modulo 111, viz. via a reduction modulo 999:

$$15098 \equiv 15 + 098 = 113 \equiv 2 \quad ,$$

where the last congruence is only modulo 111. The possibility of this generalization is well-hidden in the traditional formulation of the 11-test.

Two instances of a general method are being taught as isolated tricks. I know that the mathematics involved is absolutely trivial; so much the worse that it isn't taught in all clarity!

<u>Exercise</u>. Prove that <u>none</u> of the decimal numbers 1001, 1001001, 1001001001, 1001001001001, ... is prime.

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