Proving the existence of the Euler line.

<u>Theorem</u>. The orthocenter, the centroid, and the circumcenter of a triangle are collinear.

<u>Proof</u>. The altitudes and the perpendicular bisectors of the sides of a triangle are pairwise parallel. A multiplication with respect to a point transforms a line into a parallel one, and the centroid divides each median in the ratio 1:2. Multiplication with a factor -2 with respect to the centroid, therefore, maps each midpoint of a side on the opposite vertex, hence each perpendicular bisector on the parallel altitude, hence the circumcenter on the orthocenter. (End of Proof.)

<u>Acknowledgement</u>. We gratefully acknowledge the assistance from the Tuesday Afternoon Club, and W.H.J. Feijen's inspiration in particular.

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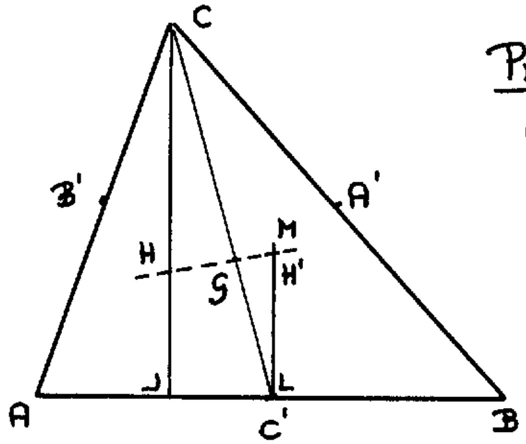
The above was triggered by W.H.J. Feijen during a discussion about the presentation of proofs. He invited us to try to improve upon his own presentation, which he had written down a few years ago because he liked the proof so much when it was shown to him. For the sake of comparison, we copy Feijen's presentation.

"*Theorem*: The orthocenter, the center of gravity, and the circumcenter of a triangle all lie on a single line: the Euler line.



Proof: Let H be the orthocenter, G the center of gravity, and M the circumcenter of triangle ABC. Multiply the whole figure with respect to G with a factor $-\frac{1}{2}$, so that C is mapped onto the midpoint of AB, and cyclically A onto the midpoint of BC and B onto the midpoint of CA. Of course the images C', A', and B' respectively are such that A'B' // AB, etc., so that M is the orthocenter of triangle A'B'C', or M = H'. □"

The above proof is hardly longer than ours, but is rather implicit about a few things. It does not mention the convention of using primes to distinguish the image from the original. It does not mention the theorem that the centroid divides each median in the ratio 1:2 and takes for granted that the mapping is such that when ΔABC is mapped on $\Delta A'B'C'$, the altitudes of the original are mapped onto the altitudes of the image.

Its statement that each vertex is mapped onto the midpoint of the opposite side is repetitious;

the statement of the parallelism is elliptical —in the meantime we have grown suspicious of "etc." and of "and so on"— ; finally, it is not entirely satisfactory to need 3 primes to denote the image of △ABC. Remembering from recent experiences that "naming" might destroy symmetry - see EWD772 - we decided to try to leave the vertices anonymous.

Following Lagrange we decided to dispense of a picture.

Finally, Fejes's proof suggests an almost false symmetry by multiplying "the whole figure", whereas only the image of the altitudes enters the argument.

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