On equality of propositions.

After the observation that equality of propositions seems to be underused in mathematical reasoning, I discovered that I myself was - perhaps not surprisingly - only vaguely familiar with all sorts of equalities. This note -which is not deep at all- is written to remedy that situation. I shall not repeat that $\wedge$ and $\vee$ are symmetric, associative and mutually distributive; neither shall I repeat de Morgan's Laws.

The following expressions (E0.i) are all equal.

(E0.0)	P
(E0.1)	$\neg\neg P$
(E0.2)	$P \vee P$
(E0.3)	$P \wedge P$
(E0.4)	$P \vee F$
(E0.5)	$P \wedge T$
(E0.6)	$P \vee (P \wedge Q)$
(E0.7)	$P \wedge (P \vee Q)$

With the possible exception of the last two, (the so-called "Laws of Absorption") this is very familiar ground. Also expressions (E1.i) are all equal.

In the following, $=$ has the lowest binding power

(E1.0)	$P = Q$
(E1.1)	$\neg P = \neg Q$
(E1.2)	$(P \vee \neg Q) \wedge (\neg P \vee Q)$
(E1.3)	$(P \wedge Q) \vee (\neg P \wedge \neg Q)$

Also expressions (E2.i) are all equal; since (E2.0) is symmetric, they occur in pairs.

(E2.0)	$P \vee Q$
(E2.1)	$P \vee \neg Q = P$
(E2.2)	$\neg P \vee Q = Q$
(E2.3)	$P \wedge \neg Q = \neg Q$
(E2.4)	$\neg P \wedge Q = \neg P$
(E2.5)	$P \wedge (R \vee \neg Q) = (P \wedge R) \vee \neg Q$
(E2.6)	$(\neg P \vee R) \wedge Q = \neg P \vee (R \wedge Q)$

The last two were absolutely new for me; their discovery —in a completely different context— provided the incentive to write this note.

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