A very first introductory example

Each morning a plant manager, who comes to work in a combined train/car ride, arrives by train at the railway station at 8:00 in the morning, where he is picked up by the company driver. Not wanting to keep his boss waiting nor liking to wait himself, the driver leaves the plant each morning so as to arrive exactly at 8:00 at the station.

One morning the manager happens to catch an earlier train, which arrives at the station at 7:00 . Instead of waiting an hour for his driver to arrive, he starts walking to the plant. When he meets his driver on the road, he steps into the car and together they return to the plant. That day, the manager arrives at the plant 20 minutes earlier than usual. For how long has the manager been walking? (As usual in such problems, each person or object moves at its own constant speed, events such as leaving the train or turning the car, are instantaneous, etc.)

Before reading on, you might want to tackle this problem yourself. If so, look now at your watch.

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The driver saved 20 minutes on the round trip, so he saved 10 minutes on his way out. He had planned to turn at 8:00, so this day he turned at 7:50. Hence the manager, who started at 7:00, walked for 50 minutes.

Some may think this solution cute, surprising, or ingenious, but I would like to point out that there is nothing ingenious about it, because the argument is all but forced. Let me show you how.

Since we know when the manager started walking, we have to determine when he stopped doing so. The only other definitions of that moment are the moment when manager and driver met and the moment the driver turned. Moreover we know that we have to relate this moment to 8:00, the only other moment in the data. That day, the only significance of 8:00 is that it was the moment at which the driver had planned to turn. So the question has become: by how much was the driver's trip to the station shortened? This interval has to be related to the only interval in the data, the 20 minutes the manager arrived earlier. This is also the shortening of the driver's round trip, which is twice the amount by which his trip out was shortened, and now we have the solution. Experience has shown that people used to the above type of problem analysis solve the problem within 2 to 60 seconds.

There is another approach. One just introduces variables for all unknown quantities (car speed, train speed, walking speed of the manager, distance between plant and station, etc.), writes down as many equations as one can think of, and solves those equations, which are all linear, so that should be no problem. In practice it does.

Firstly, almost all our training in solving systems of equations dealt with systems in which all roots could be determined. Secondly, we have not been trained in how to be sure that we have written down all equations relating the unknowns.

The person following this approach typically grabs pencil and paper within 1 minute and complains a few minutes later that he has been given insufficient data. It is then very tempting —I never did— to give him more data like "Oh yes, sorry! I forgot to tell you that the manager's train ride takes 30 minutes." or "The distance between station and plant is 15 miles.", thereby only adding to the confusion. His

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problem is, firstly, that by his random start he has lost track of what he really needs and, secondly, that he is under the misapprehension that, the more he knows, the better off he is.

In the latter approach, the solution requires at least 10 minutes, if reached at all.

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You may feel tempted to complain that this problem was too simple to deserve a whole chapter of what purports to be a *serious* book on mathematical methodology. Yet its inclusion is justified, for the simpler this problem, the more alarming the following observation: of the scientists this problem is posed to, the fraction that solves it within 1 minute is no more than a few percent. Evidently this little problem embodies some trap and we have not learned how to avoid being trapped by it; perhaps we have even been conditioned to fall into the trap. In any case it presents a reason why a book like this one seems so needed.

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