Counting characters

In [0], "factors" are introduced in relation algebra in the usual fashion. I quote the definition of the binary operator $\backslash$ (pronounced "under"):

(7) $\quad R \sqsubseteq S\backslash T \;\equiv\; S\circ R \sqsubseteq T$.

Instantiating (7) with $R := S\backslash T$ gives —since $\sqsubseteq$ is reflexive— the "rule of cancelation"

(9) $\quad S\circ S\backslash T \sqsubseteq T$.

(In [0], other identifiers are used; I have followed its convention of giving $\backslash$ a higher binding power than $\circ$.)

Four pages later, the operator $\backslash$ is used to prove —about the reflexive transitive closure— $R\circ R^* = R^*\circ R$. The ping-pong argument starts with

$$\begin{array}{ll} & R\circ R^* \sqsubseteq R^*\circ R \\ \equiv & \quad \{ \text{ factors: } (7) \ \} \\ & R^* \sqsubseteq R\backslash(R^*\circ R) \\ \Leftarrow & \quad \ldots\ldots\ldots\ldots \end{array}$$

The proof contains a second reference to (7) and 2 references to (9).

* * *

Instead of constructing the expression

$R\backslash(R^*\circ R)$, we could have <u>named</u> it, say K ; defining K would have required

(7') $\quad X \sqsubseteq K \;\equiv\; R\circ X \sqsubseteq R^*\circ R$,

and instantiating this with $X := K$ gives the tailored "rule of cancelation"

(9') $\quad R\circ K \sqsubseteq R^*\circ R$.

The ping-pong argument would then start

$$
\begin{array}{ll}
 & R\circ R^* \sqsubseteq R^*\circ R \\
\equiv & \quad \{ \text{(7') with } X := R^* \} \\
 & R^* \sqsubseteq K \\
\Leftarrow & \quad \ldots\ldots\ldots\ldots
\end{array}
$$

* * *

The introduction of a new identifier K and the 20 characters needed to formulate (7') and (9') are an investment, but worth the price, because each time we replace $R\backslash(R^*\circ R)$ by K , we save 7 characters, and the complete proof in [0] presents that opportunity 13 times, and after all $13 \cdot 7 - 20 = 71$. It <u>is</u> significant:

$$I \sqcup R \sqcup R\backslash(R^*\circ R)\circ R\backslash(R^*\circ R) \sqsubseteq R\backslash(R^*\circ R)$$

is <u>much</u> less pleasant to deal with than

$$I \sqcup R \sqcup K\circ K \sqsubseteq K \quad .$$

Excluding the hints, the formulae of the proof offering the 13 opportunities consist of 195 characters. (And after all, $195-91=104$)

*   *
*

Why do I spend an evening writing a technical note whose mathematical content is absolutely nil? I think because I wholeheartedly agree with the authors of [0] when they write on pg 1:

> "Second, it is not sufficiently recognised that formal methods must combine <u>precision</u> with <u>conciseness</u>."

Amen.

[0] R.C. Backhouse and H. Doornbos "Mathematical Induction Made Calculational". Computing Science Notes 94/16, Department of Mathematics and Computing Science, Eindhoven University of Technology

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