(a) (4 points)
Write a MATLAB function `[L,U] = MyLU(A)` which computes the LU factorization of the input matrix A. Write it as 3 nested loops.

(b) (2 points)
Write the function `y = UTriSol(U,x)` which solves for y such that $Uy = x$ (given U and x); write the function `x = LTriSol(L, b)` which solves for x such that $Lx = b$ (given L and b).

(c) (5 points)
Write a function `x = LUSolve(A, b)`. You should use `MyLU`, `UTriSol` and `LTriSol`.

(d) (6 points)
Use the above to solve for x when

(i)

$$A = \begin{bmatrix} -2 & 4 & -1 & -1 & 3 \\ 4 & -9 & 0 & 5 & 3 \\ -4 & 5 & -5 & 5 & 3 \\ -8 & 8 & -23 & 20 & 3 \\ -1 & 1 & 2 & 3 & 3 \end{bmatrix} \qquad b = \begin{bmatrix} 12 \\ -32 \\ 3 \\ -13 \\ -8 \end{bmatrix} \qquad (1)$$

(ii)

$$A = \begin{bmatrix} 10^{-16} & 1 \\ 1 & 1 \end{bmatrix} \qquad b = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad (2)$$

(iii)

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 2 \\ 2 & 2 & 6 \end{bmatrix} \qquad b = \begin{bmatrix} 2 \\ 4 \\ 10 \end{bmatrix} \qquad (3)$$

(e) (2 points)
Comment on the accuracy of x in (1), (2) and (3).

(f) (5 points)
Incorporate partial pivoting in `MyLU`, i.e., write the function `[P,L,U] = MyPLU(A)`. Note that PA must be equal to LU.

(g) (6 points)
Use `MyPLU` to solve $Ax = b$ when A and b are as in (1), (2) and (3). Comment on the accuracy of the solution.