CS396M: Homework #1

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Due Date: February 7, 2017, Tuesday

Problem 1 *(20 points total)*

Consider a packet source that generates Poisson arrivals to a single server queue at the rate of 500 packets per second. Packet sizes have the following distribution (Note: convert bytes to bits).

packet size	probability
500 bytes	0.5
1,000 bytes	0.5

The single server is a communication channel with transmission rate 4 Megabits/second (Note: 1 Megabit is 10^6 bits).

(a) Calculate the average waiting time in queue (W) when FCFS scheduling is used. (5 points)

(b) Calculate the average waiting time in queue for packets of each size (W_1 and W_2) when shortest processing time first scheduling is used; that is, HOL with all 500-byte packets in priority class 1 and all 1000-byte packets in priority class 2. (10 points)

(c) Check to see whether the Conservation Law holds for result in (a) and (b). (5 points)

Note: For Problems 2 and 3 below, assume that packet lengths have an Exponential distribution so that average delay formulas for the $M/M/1$ special case are applicable.

Problem 2 *(20 points total)*

Consider two single-server systems. System A has an arrival rate of 60 packets/second and system B has an arrival rate of 80 packets/second. Each server has a transmission rate of 100 kilobits/second, and an average packet length of 1000 bits.

(a) Calculate the average delay (waiting plus service) for packets of system A.

(b) Repeat the calculation in part (a) for system B.

(c) What is the average number of packets (queueing and being served) in both systems A and B.

(d) Apply Little's Law to calculate the average delay over all packets served by both systems A and B. (You are required to apply Little's Law, instead of using some other approach. This requires some thinking.)

HINT: Think of a boundary that surrounds both system A and system B. You make observations about all arrivals and all departures in and out of this boundary.

Problem 3 *(10 points)*

Suppose the two packet sources of systems A and B are combined and served by a single queue and a single server. What is the transmission rate of this server that will provide an average delay equal to that in part (d) of Problem 2 ?

Problem 4 *(40 points)*

Queueing Disciplines Slide #7 $M/G/1$ Head-of-the-line nonpreemptive

For Case 1, given that

$$\sigma_r = \sum_{k=1}^{r} \rho_k \tag{1}$$

$$W_1 = \frac{U_s}{1 - \rho_1} \tag{2}$$

$$W_r = \frac{U_s + \sum_{k=1}^{r} \rho_k W_k}{1 - \sigma_{r-1}}, \quad r = 2, 3, \ldots, R \tag{3}$$

Prove that

$$W_r = \frac{U_s}{(1 - \sigma_r)(1 - \sigma_{r-1})}, \quad r = 2, 3, \ldots, R \tag{4}$$