Consensus and Reliable Broadcast

Broadcast

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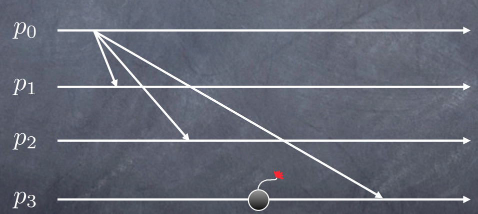

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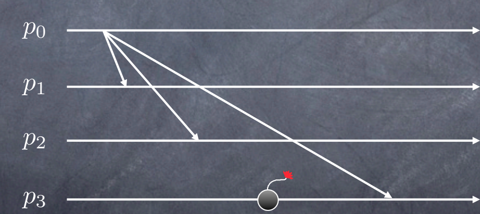
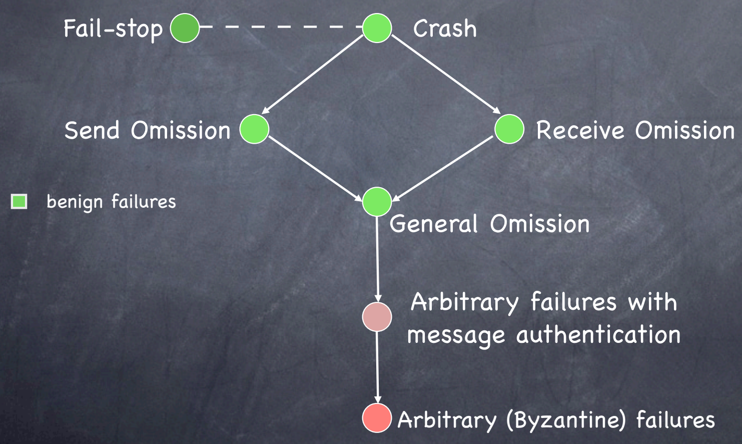


- How can we adapt the spec for an environment where processes can fail? And what does "fail" mean?

A hierarchy of failure models



Reliable Broadcast

Validity If the sender is correct and broadcasts a message m, then all correct processes eventually deliver m

Agreement If a correct process delivers a message m, then all correct processes eventually deliver m

Integrity Every correct process delivers at most one message, and if it delivers m, then some process must have broadcast m

Terminating Reliable Broadcast

Validity If the sender is correct and broadcasts a message m, then all correct processes eventually deliver m

Agreement If a correct process delivers a message m, then all correct processes eventually deliver m

Integrity Every correct process delivers at most one message, and if it delivers $m \neq$ SF, then some process must have broadcast m

Termination Every correct process eventually delivers some message

Consensus

Validity If all processes that propose a value propose v, then all correct processes eventually decide v

Agreement If a correct process decides v, then all correct processes eventually decide v

Integrity Every correct process decides at most one value, and if it decides v, then some process must have proposed v

Termination Every correct process eventually decides some value

Properties of send(m) and receive(m)

Benign failures:

Validity If p sends m to q, and p , q, and the link between them are correct, then q eventually receives m

Uniform* Integrity For any message m, q receives m at most once from p, and only if p sent m to q

* A property is uniform if it applies to both correct and faulty processes

Properties of send(m) and receive(m)

Arbitrary failures:

Integrity For any message m , if p and q are correct then q receives m at most once from p, and only if p sent m to q

Questions, Questions...

- Are these problems solvable at all?
- Can they be solved independent of the failure model?
- Does solvability depend on the ratio between faulty and correct processes?
- Does solvability depend on assumptions about the reliability of the network?
- Are the problems solvable in both synchronous and asynchronous systems?
- If a solution exists, how expensive is it?

Plan

- Synchronous Systems
  - Consensus for synchronous systems with crash failures
  - Lower bound on the number of rounds
  - Reliable Broadcast for arbitrary failures with message authentication
  - Lower bound on the ratio of faulty processes for Consensus with arbitrary failures
  - Reliable Broadcast for arbitrary failures
- Asynchronous Systems
  - Impossibility of Consensus for crash failures
  - Failure detectors
  - PAXOS

Model

- Synchronous Message Passing
  - Execution is a sequence of rounds
  - In each round every process takes a step
    - sends messages to neighbors
    - receives messages sent in that round
    - changes its state
- Network is fully connected (an n-clique)
- No communication failures

A simple Consensus algorithm

Process p_i:

Initially $V = \{v_i\}$

To execute propose(v_i)

send $\{v_i\}$ to all

decide(x) occurs as follows:

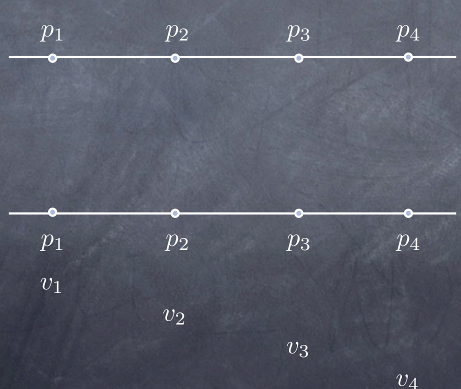
for all j, $0 \le j \le n-1$, $j \neq i$ do

receive S_j from p_j

$V := V \cup S_j$

decide min(V)

An execution



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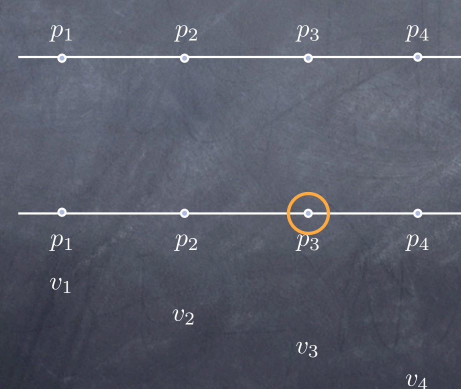
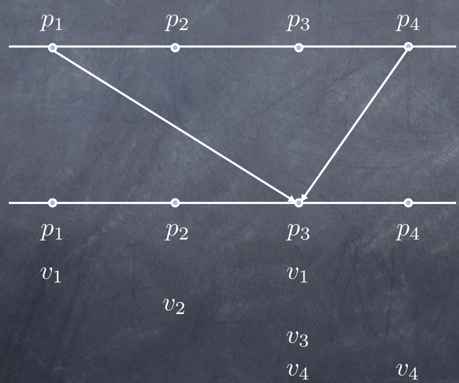


An execution

Suppose $v_1 = v_3 = v_4$ at the end of round 1
Can p_3 decide?



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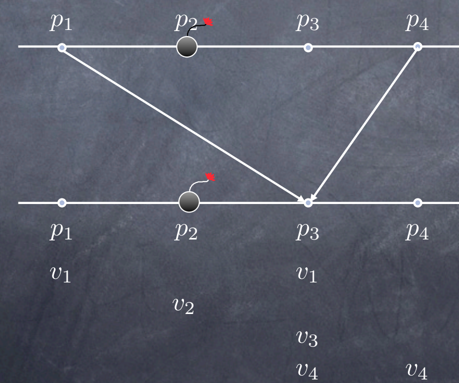

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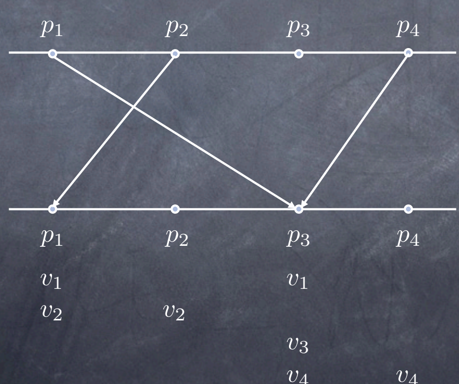

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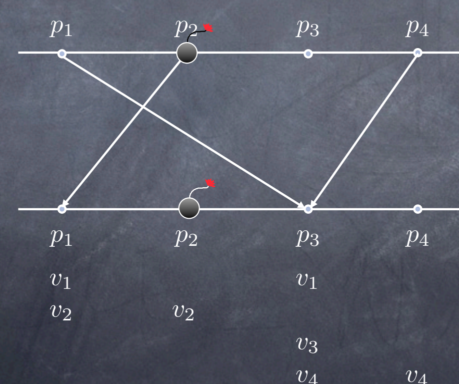

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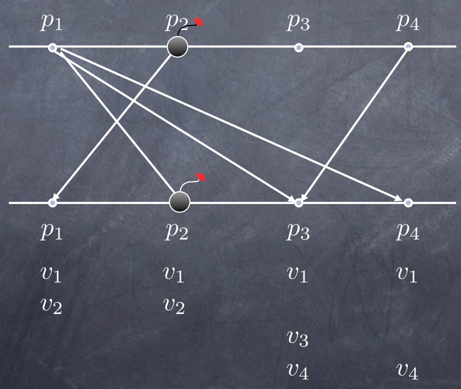

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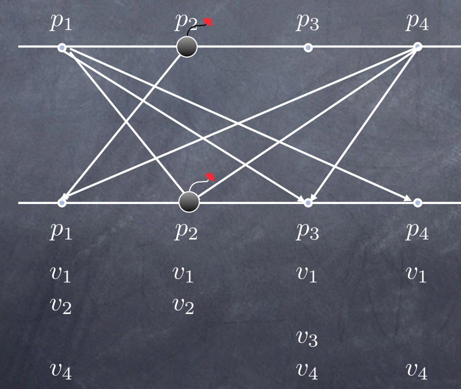

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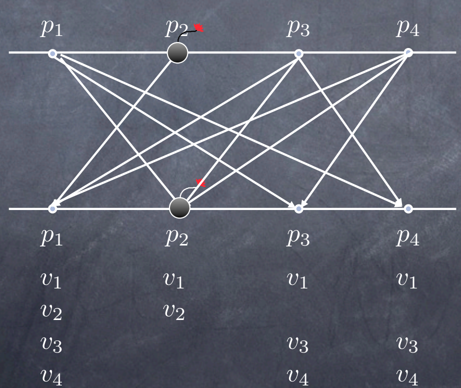

Echoing values

- A process that receives a proposal in round 1, relays it to others during round 2.

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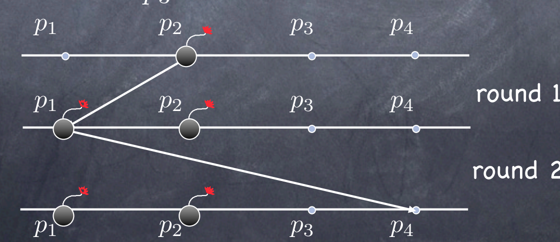


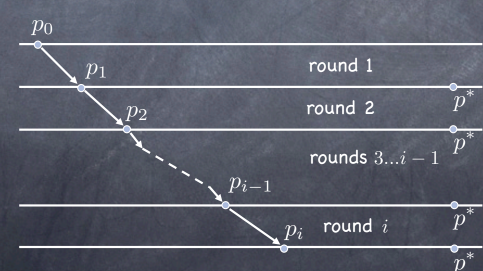
What is going on

- A correct process p^* has not received all proposals by the end of round i. Can p^* decide?
- Another process may have received the missing proposal at the end of round i and be ready to relay it in round $i+1$

Dangerous Chains

Dangerous chain

The last process in the chain is correct, all others are faulty



Living dangerously

How many rounds can a dangerous chain span?

- f faulty processes
- at most $f+1$ nodes in the chain
- spans at most f rounds

It is safe to decide by the end of round $f+1$!

The Algorithm

Code for process p_i:

Initially $V = \{v_i\}$
To execute propose(v_i)
round k, $1 \le k \le f+1$
1: send $\{v \in V : p_i$ has not already sent $v\}$ to all
2: for all j, $0 \le j \le n-1$, $j \ne i$ do
3: receive S_j from p_j
4: $V := V \cup S_j$
decide(x) occurs as follows:
5: if $k = f+1$ then
6: decide min(V)

Termination and Integrity

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Every correct process
- reaches round f + 1
- Decides on min(V) --- which is well defined

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At most one value:
- one decide, and min(V) is unique

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Integrity

At most one value:
- one decide, and min(V) is unique

Only if it was proposed:
- To be decided upon, must be in V at round $f+1$
- if value = v_i, then it is proposed in round 1
- else, suppose received in round k. By induction:
- $k = 1$:
  - by Uniform Integrity of underlying send and receive, it must have been sent in round 1
  - by the protocol and because only crash failures, it must have been proposed
- Induction Hypothesis: all values received up to round k = j have been proposed
- $k = j+1$:
  - sent in round j+1 (Uniform Integrity of send and synchronous model)
  - must have been part of V of sender at end of round j
  - by protocol, must have been received by sender by end of round j
  - by induction hypothesis, must have been proposed

Termination

Every correct process
- reaches round $f+1$
- Decides on min(V) --- which is well defined

Validity

Initially $V = \{v_i\}$

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- Suppose every process proposes v^*
- Since only crash model, only v^* can be sent
- By Uniform Integrity of send and receive, only v^* can be received
- By protocol, $V = \{ v^* \}$
- min(V) = v^*
- decide(v^*)

Agreement

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Lemma 1

For any $r \ge 1$, if a process p receives a value v in round r, then there exists a sequence of processes $p_0, p_1, \ldots, p_r$ such that $p_r = p$, p_0 is v's proponent, and in each round p_{k-1} sends v and p_k receives it. Furthermore, all processes in the sequence are distinct.

Proof

By induction on the length of the sequence

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Lemma 2:

In every execution, at the end of round $f+1$, $V_i = V_j$ for every correct processes p_i and p_j

Agreement follows from Lemma 2, since min is a deterministic function

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Proof:

- Show that if a correct p has x in its V at the end of round $f+1$, then every correct p has x in its V at the end of round $f+1$

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Initially V = {v_i}

To execute propose(v_i)
  round k, 1 ≤ k ≤ f+1
    send {v ∈ V : p_i has not already sent v} to all
    for all j, 0 ≤ j ≤ n−1, j ≠ i do
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Proof:

- Show that if a correct p has x in its V at the end of round $f+1$, then every correct p has x in its V at the end of round $f+1$
- Let r be earliest round x is added to the V of a correct p. Let that process be p^*
- If $r \leq f$, then p^* sends x in round $r+1 \leq f+1$; every correct process receives x and adds x to its V in round $r+1$

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- What if $r = f+1$?

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- What if $r = f+1$?
- By Lemma 1, there exists a sequence of distinct processes $p_0, \ldots, p_{f+1} = p^*$
- Consider processes $p_0, \ldots, p_f$
- $f+1$ processes; only f faulty
- one of $p_0, \ldots, p_f$ is correct, and adds x to its V before p^* does it in round r

CONTRADICTION!

Lemma 2:

In every execution, at the end of round $f+1$, $V_i = V_j$ for every correct processes p_i and p_j

Agreement follows from Lemma 2, since min is a deterministic function

Terminating Reliable Broadcast

Validity	If the sender is correct and broadcasts a message m, then all correct processes eventually deliver m
Agreement	If a correct process delivers a message m, then all correct processes eventually deliver m
Integrity	Every correct process delivers at most one message, and if it delivers $m \neq$ SF, then some process must have broadcast m
Termination	Every correct process eventually delivers some message

TRB for benign failures

Sender in round 1:
send m to all

Process p in round k, 1 ≤ k ≤ f+1
if delivered m in round k-1 and p ≠ sender then
  send m to all
  halt
receive round k messages
if received m then
  deliver(m)
  if k = f+1 then halt
else if k = f+1
  deliver(SF)
  halt

Terminates in $f+1$ rounds

How can we do better?

find a protocol whose round complexity is proportional to t –the number of failures that actually occurred– rather than to f–the max number of failures that may occur

Early stopping: the idea

- Suppose processes can detect the set of processes that have failed by the end of round i
- Call that set $faulty(p, i)$
- If $|faulty(p,i)| < i$ there can be no active dangerous chains, and p can safely deliver SF

Early Stopping: The Protocol

Let $faulty(p,k)$ be the set of processes that have failed to send a message to p in any round $1, \ldots, k$

if p = sender then value := m else value:= ?

Process p in round $k, 1 \leq k \leq f+1$

send value to all
if value ≠ ? and delivered m in round $k-1$ then halt
receive round k values from all
$faulty(p,k) := faulty(p,k-1) \cup \{q \mid p$ received no value from q in round $k\}$
if received value v ≠ ? then
  value := v
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else if $k = f+1$ or $|faulty(p,k)| < k$ then
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    value := SF
    deliver value
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- If in any round a process receives a value, then it delivers the value in that round
- If a process has received only "?" for $f+1$ rounds, then it delivers SF in round $f+1$

Validity

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    value := SF
    deliver value
    if $k = f+1$ then halt

- If the sender is correct then it sends m to all in round 1
- By Validity of the underlying send and receive, every correct process will receive m by the end of round 1
- By the protocol, every correct process will deliver m by the end of round 1

Agreement - 1

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Lemma 1

For any $r \geq 1$, if a process p delivers m ≠ SF in round r, then there exists a sequence of processes $p_0, p_1, \ldots, p_r$ such that p_0 = sender, $p_r = p$, and in each round $k, 1 \leq k \leq r$, p_{k-1} sent m and p_k received it. Furthermore, all processes in the sequence are distinct, unless $r = 1$ and $p_0 = p_1 =$ sender

Lemma 2:

For any $r \geq 1$, if a process p sets value to SF in round r, then there exist some $j \leq r$ and a sequence of distinct processes $q_j, q_{j+1}, \ldots, q_r = p$ such that q_j only receives "?" in rounds 1 to j, $|faulty(q_j, j)| < j$, and in each round $k, j+1 \leq k \leq r$, q_{k-1} sends SF to q_k and q_k receives SF

Agreement - 2

Let $faulty(p,k)$ be the set of processes that have failed to send a message to p in any round $1,\ldots,k$

```
if p = sender then value := m else value:= ?
```

Process p in round $k, 1 \leq k \leq f+1$

```
send value to all
if value ≠ ?  and delivered m in round k−1 then halt
receive round k values from all
faulty(p,k) := faulty(p,k−1) ∪ {q | p
received no value from q in round k}
if received value v ≠ ?  then
    value := v
    deliver value
else if k = f+1 or |faulty(p,k)| < k then
    value := SF
    deliver value
    if k = f+1 then halt
```

Lemma 3:

It is impossible for p and q, not necessarily correct or distinct, to set value in the same round r to m and SF, respectively

Agreement - 2

Let $faulty(p,k)$ be the set of processes that have failed to send a message to p in any round $1,\ldots,k$

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received no value from q in round k}
if received value v ≠ ?  then
    value := v
    deliver value
else if k = f+1 or |faulty(p,k)| < k then
    value := SF
    deliver value
    if k = f+1 then halt
```

Lemma 3:

It is impossible for p and q, not necessarily correct or distinct, to set value in the same round r to m and SF, respectively

Proof

By contradiction

Suppose p sets value = m and q sets value = SF

By Lemmas 1 and 2 there exist

$$p_0, \ldots, p_r$$

$$q_j, \ldots, q_r$$

with the appropriate characteristics

Since q_j did not receive m from process p_{k-1} $1 \leq k \leq j$ in round k

q_j must conclude that $p_0, \ldots, p_{j-1}$ are all faulty processes

But then, $|faulty(q_j, j)| \geq j$

CONTRADICTION