CS 337 **Solution to Quiz 3** **12/10/03**

Open book and notes.
Max points = 50 Time = 50 min

1. (String Matching)

   (a) (Rabin-Karp algorithm) First create a list of 2 digit numbers n where $val(n) = 5$. The list is 05 12 19 26 33 40 47 54 61 68 75 82 89 96. Then create the desired string as follows: start with any digit x, find a y so that xy is in the list, and repeat this step with x set to y. The result is not unique, because there is sometime more than one choice for y. Starting with 1, I get 1 2 6 8 9 6 1 9.

   (b) Proof is by induction on i.

   • $i = 1$: We have to show that $c(ab)^n a \;=\; (ab)^{n-1}a$, for $n \geq 1$. This follows from: (1) $(ab)^{n-1}a$ is both a prefix and a suffix of $(ab)^n a$, and (2) the only longer proper prefix of $(ab)^n a$ is $(ab)^n$, which is not a suffix.

   • $i+1$: Assume $c^i((ab)^n a) \;=\; (ab)^{n-i}a$, where $i < n$. We show that $c^{i+1}(ab)^n a \;=\; (ab)^{n-i-1}a$.

$$
\begin{array}{ll}
 & c^{i+1}(ab)^n a \\
= & \{\text{definition of } c^{i+1}\} \\
 & c(c^i((ab)^n a)) \\
= & \{\text{induction hypothesis}\} \\
 & c((ab)^{n-i}a) \\
= & \{\text{from the first proof}\} \\
 & (ab)^{n-i-1}a
\end{array}
$$

   (c) Yes. Suppose v = "ab'' and v' = "aba''. Then $c(v) = \epsilon$, and u has been set to $c(v)$, i.e., ϵ, before this portion of the code is executed. Because $p[\overline{u}]$ and $p[\overline{v}]$ are both "a'', $c(v')$ will be set "a''.

   (d) (KMP Algorithm)

l	*index*	0	1	2	3	4	5	6	7	8	9	10	11		
	text	a	b	a	b	a	a	b	b	a	b	a	b		
0	*pattern*	a	b	a	b	a	b								
2	*pattern*			a	b	a	b	-	-						
4	*pattern*					a	b	-	-	-	-				
5	*pattern*						a	b	a	-	-	-			
7	*pattern*								a	b	-	-	-	-	
8	*pattern*									a	b	a	b	-	-

   It is possible to terminate the algorithm when $l = 7$, because the text cannot possibly match the pattern.

2. (Data Parallel Programming)

(a) We show that for $n \geq 1$, $rev(f\ n) = (f\ n)$. The required result follows because

$$\begin{array}{ll} & f(n+1) \\ = & \{\text{definition of } f\} \\ & (f\ n) \mid rev(f\ n) \\ = & \{rev(f\ n) = (f\ n)\} \\ & (f\ n) \mid (f\ n) \end{array}$$

The proof is by induction on n.

• $n = 1$: We show $rev(f\ 1) = (f\ 1)$. From the definition of f, $(f\ 1) = \langle 0\ 1\ 1\ 0 \rangle$. And from the definition of rev, $rev(f\ 1) = \langle 0\ 1\ 1\ 0 \rangle$. Hence $rev(f\ 1) = (f\ 1)$.

• $n + 1$: Assume by induction hypothesis that $rev(f\ n) = (f\ n)$. Then,

$$\begin{array}{ll} & rev(f(n+1)) \\ = & \{\text{definition of } f\} \\ & rev((f\ n) \mid rev(f\ n)) \\ = & \{\text{induction hypothesis applied to the second term}\} \\ & rev((f\ n) \mid (f\ n)) \\ = & \{\text{definition of } rev\} \\ & rev(f\ n) \mid rev(f\ n) \\ = & \{\text{induction hypothesis applied to the first term}\} \\ & (f\ n) \mid rev(f\ n) \\ = & \{\text{definition of } f\} \\ & f(n+1) \end{array}$$

(b) i. $(u \sim r) \wedge (v \sim s)$

ii. $gr\ u = u \sim (rr\ u)$, where rr is right-rotate.

iii.

$$\begin{array}{ll} & gr(u \bowtie v) \\ = & \{\text{definition of } gr\} \\ & (u \bowtie v) \sim rr(u \bowtie v) \\ = & \{\text{definition of } rr\} \\ & (u \bowtie v) \sim ((rr\ v) \bowtie u) \\ = & \{\text{from (i)}\} \\ & (u \sim (rr\ v)) \wedge (v \sim u) \\ = & \{\text{rewrite the second term}\} \\ & (u \sim (rr\ v)) \wedge (u \sim v) \end{array}$$