CS 337 **Solution to Quiz 1** **2/14/03**

1. (Game of Nim)
   Let the exclusive-or of the piles other than y be s. Then

   $$s \oplus y = u \tag{1}$$

   Replacing y by x results in a losing state. From the definition of losing state

   $$s \oplus x = 0 \tag{2}$$

   Combining (1) and (2),

   $$x \oplus y = u \tag{3}$$

   Since y has a 0 in the bit position where u has its leading 1 bit, and $x \oplus y = u$ (from (3)), x has a 1 in that bit position. Use the hint with y for p, x for q and u for $p \oplus q$ (from (3)), to get $y < x$, or $x > y$.

2. (Error Correction)
   The theorem is correct. Under the condition that we don't know which bit is corrupted, parity code can be used to detect the presence of one error. In the RAID architecture we know which bit is corrupted, i.e., which disk has failed, and then, the corrupted bit can be restored.

3. (Hadamard Matrix)
   The proof is by induction on n.

   - H_2: by inspection of H_2 in the handout.

   - H_{n+1}, $n \geq 2$: From the induction hypothesis, each row of H_n has even parity. Moreover, since each row length is even, each row has an even number of zeroes; therefore, each row of $\overline{H_n}$ has even parity.

   Consider the structure of H_{n+1}.

   $$H_{n+1} = \begin{bmatrix} H_n & H_n \\ H_n & \overline{H_n} \end{bmatrix}$$

   For each row, both its left and right half have even parity. Hence each row has even parity.

4. (Finite State Machine)

(a)

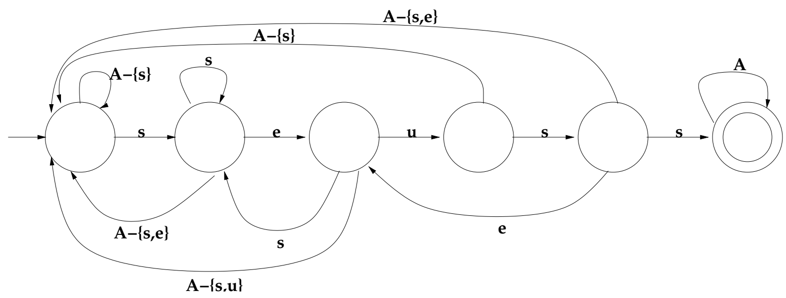

Figure 1: Finite State Machine to recognize "seuss"

(b)

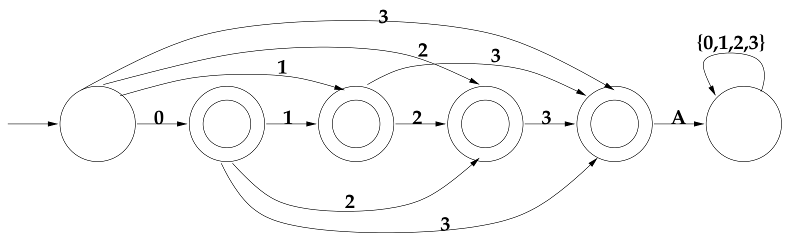

Figure 2: Finite State Machine to recognize sorted strings

(c)

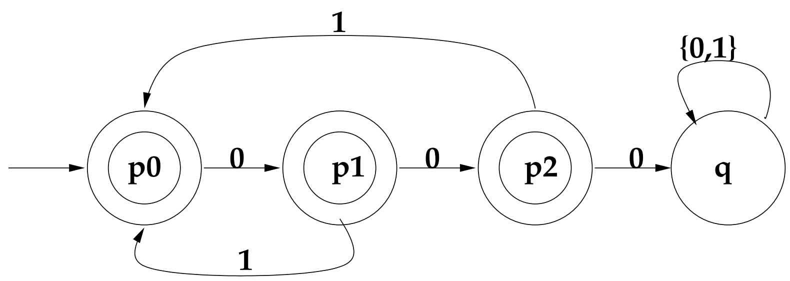

Figure 3: Finite State Machine to reject strings with 3 consecutive 0s

For $0 \leq i \leq 2$, p_i denotes that the input string (seen so far) ends with i consecutive 0s,
q denotes that the input string has 3 consecutive 0s.