CS 337 **Test 3** **5/5/06**

Open book and notes.

Max points = 50 Time = 50 min Do all questions.

1. (Relational Algebra; 15 points)

   (a) (5 points) You are given relations CT and CR in Table 1 and Table 2 respectively. Compute their (natural) join, $CT \bowtie CR$.

   (b) (5 points) Write a query for (but don't compute) the Day-Hour pairs during which Tay 2.106 is occupied by some course, given only the tables CT and CR. Use the π and σ notation.

   (c) (5 points) Write a query for (but don't compute) the courses which meet on Friday(F) in Tay 2.106. Use the π and σ notation.

Course	Day	Hour
Phy313K	T	9AM
Phy313K	Th	9AM
CS380D	F	9AM
CS337	M	2PM
CS337	W	2PM
CS337	F	2PM

Table 1: CT: Course Timings

Course	Room
Phy313K	Phi 1.021
CS380D	Wel 2.304
CS337	Tay 2.106

Table 2: CR: Course Room

2. (String Matching; 17 points)

   (a) (4 points) Show a string s of length $2n$ such that $c^n(s) = \epsilon$. Recall that $c(u)$ is defined if and only if $u \neq \epsilon$.

   (b) (6 points) A *palindrome* is a string that is equal to its reverse. Let $pal(v)$ denote that v is a palindrome, that is

   $pal(v) \;\equiv\; (v = rev(v))$, where rev is the reverse function.

   Given that $u \preceq v$ means (as in the class notes) "u is both a prefix and a suffix of v", prove that

   $(pal(v) \;\wedge\; u \preceq v) \;\Rightarrow\; pal(u)$.

   Hint: Write $u \sqsubseteq v$ to denote that u is a prefix of v.

i. A formal definition of $u \preceq v$ is given by

$$u \preceq v \;\equiv\; (u \sqsubseteq v \;\wedge\; rev(u) \sqsubseteq rev(v))$$

ii. If strings x and y are both prefixes of z and they have the same length, then $x = y$.

iii. x and $rev(x)$ are of the same length.

(c) (7 points) Using

$$\textbf{(core definition):}\; x \preceq c(y) \;\equiv\; x \prec y$$

show that the core function is monotonic, that is,

$$u \preceq v \;\Rightarrow\; c(u) \preceq c(v)$$

You may assume that $\preceq$ is a partial order.

3. (Powerlist; 18 points)

(a) (10 points) Let p be a powerlist of even length (so it can not be a singleton list) and ex a powerlist which consists of 0s and 1s and is half the length of p. Write a function f over p and ex, using the notation for powerlists, that permutes the items of p as follows: if the i^{th} element of ex is 1 then exchange the $2i^{th}$ and $(2i+1)^{th}$ elements of p, and if it is 0 leave them as they are. (The items in a powerlist are indexed starting at 0). So,

$$f\langle 0\ 1\ 2\ 3\rangle\langle 1\ 0\rangle \;=\; \langle 1\ 0\ 2\ 3\rangle, \text{ and}$$
$$f\langle 0\ 1\ 2\ 3\rangle\langle 0\ 1\rangle \;=\; \langle 0\ 1\ 3\ 2\rangle$$

(b) (8 points) You are given the following definition of rev over powerlists (reproduced from Page 185 of Class notes).

$$rev\langle x\rangle = \langle x\rangle$$
$$rev(p \mid q) = (rev\ q) \mid (rev\ p)$$

Prove that

$$rev(p \bowtie q) = (rev\ q) \bowtie (rev\ p)$$