

Open book and notes.

Max points = 75

Time = 75 min

Do all questions.

## 1. (Finite State Machine; 15 points)

- (a) A turnstile is either *locked* or *unlocked*. When the turnstile is locked, a customer can drop a coin into its slot. This causes the turnstile to show a green signal and become unlocked. If the turnstile is unlocked, then a customer can pass through it; then the turnstile becomes locked and a red signal is displayed. Assume that the turnstile begins in the locked state.

Draw a finite state transducer to specify the behavior of the turnstile. Identify the states, input alphabet and the output alphabet clearly.

- (b) The previous description gives the turnstile behavior in normal cases. Suppose that a customer passes the turnstile when it is locked without dropping a coin. Then the turnstile becomes *superlocked* and an alarm starts sounding. The only way out of this state is (1) the customer drops a coin which causes the turnstile to become locked and a red signal to be displayed, or (2) a technician presses a reset button which causes the turnstile to become locked.

Redraw the machine.

2. (Finite State Machine; 15 points) The machine in Figure 1 accepts only and all binary strings in which any occurrence of 0 is immediately followed by a 1. Write predicates for each state to prove this claim. Show the theorems that have to be proved. You are not required to prove the theorems.

Hint: You may use English to describe your predicates, but be unusually precise.

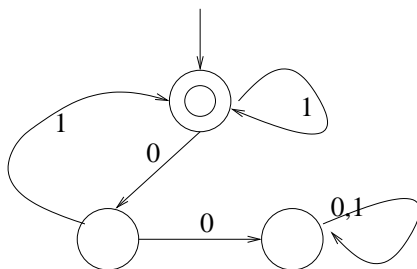


Figure 1: Accept binary strings in which a 0 is immediately followed by a 1

3. (Regular Expressions; 15 points) We would like to specify strings over the alphabet  $\{0, 1, 2\}$  which are strictly increasing; thus, 02 is acceptable but 021 and 022 are not.

(a) What is wrong with this solution?

```
0* 1* 2*
```

(b) Let *zero* denote the set  $\{\epsilon, 0\}$ . Write a regular expression for *zero*. Similarly define *one* and *two*. Solve the problem in the first part using these additional symbols in your regular expression.

4. (Types; 15 points) What are the types of the functions defined below? Below `++` concatenates two lists (whose elements are of the same type) to form a single list. So, `[2,3] ++ [4,5,6] = [2,3,4,5,6]`.

Hint: Do *not* try to understand what each function is computing.

(a) `charVal n = chr(n + (ord '0'))`. Assume *n* is of type `Int`.

(b) `parallel ((a,b),(c,d)) ((u,v),(x,y)) =  
(d-b) * (x-u) == (y - v) * (c - a)`  
Assume that *a, b, c, d, u, v, x, y* are of type `Int`.

(c) `test f r = (f r) || not (f r)`

(d) Assume *n* is of type `Int`.

```
tower n a b c  
  | n == 0    = []  
  | otherwise = (tower (n-1) a c b)  
                ++ [(n,a,b)]  
                ++ (tower (n-1) c b a)
```

(e) `flatten [] = []`  
`flatten (xs : xss) = xs ++ (flatten xss)`

5. (Haskell Programming; 15 points)

(a) Define a function whose type is `[(a,b)] -> ([a],[b])`, where *a* and *b* are polymorphic type variables.

(b) Consider the following variation, *newfib*, of the Fibonacci sequence. The first three items of the sequence are 0, 1 and 2. Any other item in the sequence is a sum of its last three items. Define an efficient procedure for computing *newfib n*, for any *n*,  $n \geq 0$ .

(c) Define a function which takes as input a list of integers with at least two elements, and computes the smallest difference  $x - y$ , where *x* and *y* are adjacent elements. Thus, in the list `[3,7,2,4,8,11,9]`, the smallest difference is  $-4$ , for `(3,7)` and `(4,8)`.