

Open book and notes.

**Problem** Let  $t$  be the string  $abba$ . What is the largest  $i$  such that  $c^i(t^{20}) \neq \epsilon$ ? Justify.

**Solution** First, let us solve the problem for any  $n$  instead of 20. Observe  $c(t^n) = t^{n-1}$ , for  $n > 1$ . Hence,  $c^{n-1}(t^n) = t$ , for  $n > 1$ . And, we see that  $c(t) = c(abba) = a$  and  $c(a) = \epsilon$ . Therefore,  $c^n(t^n) = a$  and  $c^{n+1}(t^n) = \epsilon$ .

We conclude that we can compute cores of  $t^n$  repeatedly  $n$  times without hitting  $\epsilon$ . So, the answer is 20.