

Open book and notes.

Max points = 75

Time = 75 min

Do all questions.

1. (Relational Databases; 15 points) You are given relations  $SL$  (denoting Stores and Locations),  $IT$  (Items and Types) and  $SIP$  (Stores, Items and Prices) in Table 1, Table 2 and Table 3, respectively.

Store	Location
Amazon	WA
Fry's	CA
Fry's	TX
Best Buy	TX
Olde Tire	TX

Table 1:  $SL$ : Stores and Locations

Item	Type
Nikon Cool-Pix	Camera
Sony Cybershot	Camera
Dell Inspiron	Computer
Firestone	Tire

Table 2:  $IT$ : Items and Types

Store	Item	Price
Amazon	Nikon Cool-Pix	240
Amazon	Dell Inspiron	1200
Fry's	Nikon Cool-Pix	250
Fry's	Sony Cybershot	310
Best Buy	HP Laptop	1300
Best Buy	Sony Cybershot	280
Olde Tire	Firestone	500

Table 3:  $SIP$ : Stores, Items and Prices

- (a) (7 points) Compute their (natural) join,  $SL \bowtie IT \bowtie SIP$ .
- (b) (4 points) Write a query for (but don't compute) the stores in TX that sell a Camera for less than 300.
- (c) (4 points) Write a query for (but don't compute) the stores, locations, items and prices for all computers that are being sold.

2. (Rabin-Karp String Matching; 12 points)

- (8 points) You are given the text string 01110011010001100111, and you are matching the pattern 1100. Use the hash function  $m \bmod 3$ , where  $m$  is 4-bit binary number. Show the values of the hash function, and the true matches. Repeat using  $\bmod 5$  as the hash function.

Hint: You are not required to apply the modular simplification rule to compute the hash function values; you may use simpler methods if you find them more convenient.

- (4 points) What are the merits and demerits of using exclusive-or as the hash function in Rabin-Karp string matching over binary strings?

3. (KMP String Matching; 18 points)

- (3 points) In doing string matching, which is better: prefixes of the pattern string have short cores or long cores? Write one sentence in explanation.
- (9 points) It is given that cores of all prefixes of some string  $s$  are the empty string. State and briefly explain the necessary and sufficient conditions for  $s$  to satisfy this requirement (no formal proof needed).
- (6 points) Find a shortest string whose core is “ababa”. Argue that your answer is correct.

4. (Parallel Recursion; 30 points)

- (a) (4 points) Let function  $h$  be defined by

$$\begin{aligned} h\langle x \rangle &= \langle x \rangle \\ h(p \bowtie q) &= p \mid q \end{aligned}$$

What is  $h\langle 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \rangle$ ?

- (b) (8 points) Use the following definitions of right rotate ( $rr$ ) and reverse ( $rev$ ).

$$\begin{aligned} rr\langle x \rangle &= \langle x \rangle & rr(p \bowtie q) &= q \bowtie rr(p) \\ rev\langle x \rangle &= \langle x \rangle & rev(p \bowtie q) &= rev(q) \bowtie rev(p) \end{aligned}$$

Prove that for any powerlist  $u$ ,  $rev(rr(rev(rr u))) = u$ . You may assume that  $rev(rev q) = q$ , for any powerlist  $q$ .

- (c) (10 points) You are given two infinite families of powerlists,  $u$  and  $v$ , where  $u_i$  is the  $i^{th}$  powerlist in the  $u$  family, and similarly  $v_i$ . They are defined by,

$$\begin{aligned} u_0 &= \langle 0 \rangle, & v_0 &= \langle 1 \rangle \\ u_{i+1} &= u_i \mid v_i, & v_{i+1} &= v_i \mid u_i \end{aligned}$$

Show that for all  $i$ ,  $i \geq 0$ ,

- $u_{i+1} = u_i \bowtie v_i$ , and  $v_{i+1} = v_i \bowtie u_i$ .
  - $u_i$  is the bit-wise complement of  $v_i$ .
- (d) (8 points) It is required to compute the prefix-sum of  $\langle 3 \ 0 \ 4 \ 6 \ 0 \ 1 \ 6 \ 0 \rangle$  using addition over integers as the (associative) operator. The result is the list  $\langle 3 \ 3 \ 7 \ 13 \ 13 \ 14 \ 20 \ 20 \rangle$ . Use the list function  $sps$  defined in your notes in Page 226, and graphically show the movement of data in computing this prefix-sum.