Computation Orchestration

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Computation Orchestration

Given are basic computing elements. How to compose them?

- Computing elements are logic gates: $\wedge$, $\vee$, $\neg$

  Composition is a circuit.

- Computing elements are functions.

  Composition is through higher-order functions.

- Computing elements are processes.

  Composition is through CCS or CSP operators.

Orc

Computing elements are Sites, such as

- function: Compress MPEG file
- method of an object: LogOn procedure at a bank
- monitor procedure: read from a buffer
- web service: get a stock quote
- transaction: check account balance
- distributed transaction: move money from one bank to another

Structure of the Lectures

- Programming Notation: the composition operators, their usage
- Programming Methodology: Parallelism, Synchronization, Interrupt
- Semantics, Implementation
- Site Specification, Commitment, Revocation

Other Possible Topics:

- Program Structuring
- Concurrency

Lecture Material

Computation Orchestration: A Basis for Wide-Area Computing

http://www.cs.utexas.edu/users/psp/Wide-area.pdf

Exercises in your Handouts

I will give additional exercises during the lecture.

Example: Airline

- Contact two airlines simultaneously for price quotes.
- Buy ticket from either airline if its quote is at most $300.
- Buy the cheapest ticket if both quotes are above $300.
- Buy any ticket if the other airline does not provide a timely quote.
- Notify client if neither airline provides a timely quote.

Example: workflow

- An office assistant contacts a potential visitor.
- The visitor responds, sends the date of her visit.
- The assistant books an airline ticket and
  contacts two hotels for reservation.
- After hearing from the airline and any of the hotels:
  he tells the visitor about the airline and the hotel.
- The visitor sends a confirmation which the assistant notes.

Example: workflow, contd.

After receiving the confirmation, the assistant

- confirms hotel and airline reservations.
- reserves a room for the lecture.
- announces the lecture by posting it at a web-site.
- requests a technician to check the equipment in the room.

Wide-area Computing

Acquire data from remote services.

Calculate with these data.

Invoke yet other remote services with the results.

Additionally

Invoke alternate services for failure tolerance.

Repeatedly poll a service.

Ask a service to notify the user when it acquires the appropriate data.

Download an application and invoke it locally.

Have a service call another service on behalf of the user.

The Nature of Distributed Applications

Three major components in distributed applications:

Persistent storage management

databases by the airline and the hotels.

Specification of sequential computational logic

does ticket price exceed $300?

Methods for orchestrating the computations

contact the visitor for a second time only after hearing from the airline and one of the hotels.

We look at only the third problem.

Orc

A new kind of assignment

$x :\in f$

where x is a variable and f is an Orc expression.

Evaluation of f yields zero or more values.

Assign the first value to x.

An Orc expression is

- Simple: Site (Function call, method, web service, transaction)
- Compound: $f \mid g$, $f \gg g$, $f * g$, $\{\ f\ \mathbf{where}\ x :\in g\ \}$

Simple Orc Expression

- M is a news service, d a date. Download the news page for d.

  $x :\in M(d)$

- Side-effect: Book ticket at airline A for a flight described by c.

  $x :\in A(c)$

The returned value is the price and the confirmation number.

Properties of Sites

- A site may not respond.

  Its response at different times (for the same input) may be different.

- A site call may change states (of external servers) tentatively or permanently.

  Tentative state changes are made permanent by explicit commitment.

Structure of response

- The response from a site has:
  value, which the programmer can manipulate, and
  pledge, which the programmer cannot manipulate.

- Pledge is used to commit this site call.
  Pledge is valid for some time period.
  Value is meaningful during then.

- By committing a valid pledge (during the given period), the programmer establishes some fact.

Nesting

- (Data Piping) Retrieve a news page for date d from M and email it to address a. Here, $Email$ is a site.

$$Email(a, M(d))$$

- (Higher-order site) Call discovery service D with parameter x to locate a site; call that site with parameter y.

$$Apply(D(x), y)$$

Simple Orc Expression: Sequencing

M, N, R are sites for 3 professors.

s is a set of possible meeting times.

$M(s)$ is a subset of s, the times when M can meet.

$M(N(R(s)))$ is the possible meeting times of all three professors.

Parallel, Strict evaluation

Arguments of a site call are evaluated in parallel.

A site is called only after all its arguments have been evaluated.

Fork-join parallelism

$A(c)$ and $B(c)$ return ticket prices from airlines A and B.

Min returns the minimum of its arguments.

$Min(A(c), B(c))$:

Compute $A(c)$ and $B(c)$ in parallel.

Call Min when both quotes are available.

Predefined sites

- $Fail$ never responds.
- $let(x, y, \cdots)$ returns a tuple of argument values as soon they are available. $let(\theta)$ is $skip$.
- $random$ returns a random number (in a specified range), instantaneously.
- fst returns the value of the first argument as soon all argument values are available.
- $timer(t)$, where t is a non-negative integer, returns a signal exactly after t time units.
- $timer(t, x)$ is $fst(x, timer(t))$; returns x after t time units.

Composing Expressions

- (Alternation) $f \mid g$: evaluate f and g in parallel;
  values of $f \mid g$ are those from f and from g.

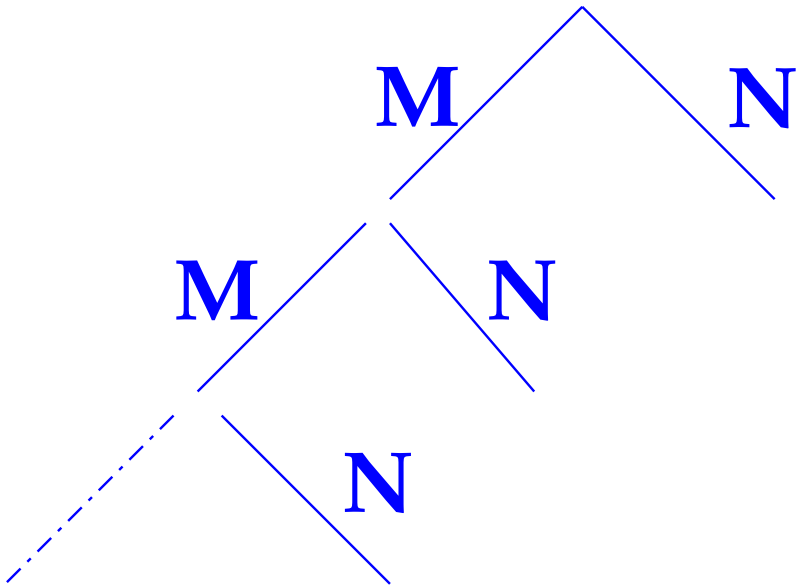
- (Piping) $f \gg g$: Evaluate g for all values of f;
  values of $f \gg g$ are those from g.

- (Iteration) $f * g$: values from g after zero or more piping steps of f.

$$
\begin{aligned}
& f * g \\
= \; & g \mid (f \gg (f * g)) \\
= \; & g \mid (f \gg (g \mid f \gg (g \mid f \gg \cdots)))
\end{aligned}
$$

- (Definition) $\{ f \ \mathbf{where}\ x{:}\in g \}$

Pictorial Depiction of $M * N$



Values of $M * N$ are the ones returned by N.

Binding power

$|$ has the lowest binding power.

$\gg$ and $*$ have equal binding powers.

$$f * g \mid h \gg g \equiv (f * g) \mid (h \gg g)$$

Example of Orc expression:

$$G(q) \gg (\langle M(q) \mid R(\theta, q) \gg G(\theta) \rangle * S(\theta))$$

Default Parameter

- $M \gg N(x, \theta)$
- $(M \mid S) \gg (N(x, \theta) \mid R(\theta))$
- Start computation of f with value v for θ:
  $let(v) \gg f$.
- Start an iteration where $x_0 = v$ and $x_{i+1} = M(x_i)$.
  Values returned are $N(x_i)$, for $i \geq 0$.

$$let(v) \gg (M(\theta) * N(\theta))$$

Properties of the timer

$x :\in timer(t) \mid timer(u) \quad \equiv \quad x :\in timer(t)$, given $t \leq u$,

$timer(t) \gg timer(u) \quad \equiv \quad timer(t + u)$

Alternation, Piping

- Assign the first value from $M(c)$ or $N(d)$ to z.

  $z :\in M(c) \mid N(d)$

- assign to z the value from M if it arrives before t, 0 otherwise.

  $z :\in M \mid timer(t, 0)$

- Interruption

  $f \mid Interrupt.get$

- Make four requests to site M, in intervals of one time unit each.

  $M \mid timer(1) \gg M \mid timer(2) \gg M \mid timer(3) \gg M$

Priority

Request M and N for values. Give priority to M.

- Allocate one extra time unit for M to respond.

$$z :\in M \mid timer(1) \gg N \quad \text{or} \quad z :\in M \mid N \gg timer(1, \theta)$$

- Accept the response from M if it arrives within one time unit, else accept the first response.

$$z :\in M \mid fst(N, timer(1))$$

Iteration

- Call M forever at unit time intervals until it returns a value.

  $$z :\in timer(1) * M$$

  which is

  $$z :\in M \mid timer(1) \gg (M \mid timer(1) \gg (M \mid \cdots))$$

- Same as above, but stop calling after 10 time units.

  $$z :\in timer(1) * M \mid timer(10)$$

Iteration; Contd.

- Site M returns stock price of company abc

  Site $C(x)$: returns x if $x < 20$; silent otherwise.

  $$M * C(\theta)$$

  either never returns a value (if abc never falls below 20)

  or returns a value lower than 20. Initially, $\theta \geq 20$.

- Variation: Poll M once every hour for 6-hours:

  $$timer(1) * \langle M \gg C(\theta) \rangle \mid timer(6)$$

Web search

- G is a simple search engine, returns result for query q.
- R refines a query q using q and the search result.
- S outputs its argument result when it is sufficiently refined.
- Strategy: refine the query endlessly, present each result to S for scrutiny.

$$
\begin{array}{ll}
 & G(q) \\
\gg & (\ \langle R(\theta, q) \gg G(\theta) \rangle \\
 & \ * S(\theta) \\
 &)
\end{array}
$$

Definition within Orc expression

- A machine is assembled from two parts, u and v.
- Two vendors for each part: $u1$ and $u2$ for u, and $v1$ and $v2$ for v.
- Solicit quotes from all vendors.
- Accept the first quote for each part.
- Compute the machine cost to be 20% above the sum of the part costs.

$$
\begin{array}{l}
cost :\in \quad \{\ (u+v) \times 1.2 \\
\qquad\quad \textbf{where} \\
\qquad\qquad u :\in u1 \mid u2 \\
\qquad\qquad v :\in v1 \mid v2 \\
\qquad \}
\end{array}
$$

General Orc Statements

$z :\in \{\ f(\cdots x \cdots y \cdots)$
where
$x :\in g$
$y :\in h$
$\}$

Example: M, N, R, S are sites.

$z :\in \{\ (M(x) \mid N(y)) \gg M(y) \quad \gg \{\ M(y)$
where **where**
$x :\in R(y) \mid N(y)$ $y :\in S$
$y :\in \{\ R \mid N(t)$ $\}$
where $t :\in S$
$\}$
$\}$

Syntax

statement	::=	defn
defn	::=	variable $:\in$ expr
expr	::=	term
		| expr $\mid$ expr
		| expr $\gg$ expr
		| expr $*$ expr
		| { expr **where** defn }
term	::=	site([parameter])
parameter	::=	variable | θ

[parameter] is a list of parameters, possibly empty.

Free and Bound Variables

- Variable assigned in a statement is the goal variable.
- Variables named in an expression are global or local.
- Free variables:

$$
\begin{aligned}
free(M(L)) &= \{x |\ x \in L,\ x \neq \theta\} \\
free(f\ op\ g) &= free(f) \cup free(g), \text{ where } op \in \{\ |\ ,\ \gg\ ,\ *\ \} \\
free(\{f \textbf{ where } x :\in g\}) &= (free(f) - \{x\}) \cup free(g)
\end{aligned}
$$

- In $\{\ f$ **where** $x :\in g\}$, any free occurrence of x in f is bound to the variable shown.
- $z :\in f$ is well-formed if all the free variables in f are global variables.

Flat Expression

Flat expression: without a **where** clause.
Non-flat expression:

Flat expression is a regular expression of language theory.

Terms are symbols.

Syntactic Conventions: Omit Braces; group where

$\{\{\ f$
where
$x\ :\in\ g$
$\}$
where
$y\ :\in\ h$
$\}$

is

f
where
$x\ :\in\ g$
$y\ :\in\ h$

or, $\{\ f$ **where** $x{:}\in\ g,\ y{:}\in\ h\}$

Syntactic Conventions: Nested Site Calls

- $Email(a, M(d))$ is not an expression.

  It means:

  $\{\ Email(a, u)$ **where** $u{:}\in M(d)\ \}$

- We allow $R(f, g)$ where f and g are expressions.
  It means : $\{\ R(x, y)$ **where** $x{:}\in f,\ y{:}\in g\ \}$

- $timer(f, g)$ is (after f_0 time units return g_0)

  $\{\ fst(x, y)$
  where $x{:}\in g$
  $y{:}\in \{timer(u)$ **where** $u{:}\in f\ \}$
  $\}$

Argument Evaluation in Nested Site Calls

Consider $Q(N(x), N(x), N(x))$.

For the first two arguments:
evaluate $N(x)$ once and use the value for both.

For the last argument:
reevaluate $N(x)$.

$\{\ Q(u, u, v)$
where
$u :\in N(x)$
$v :\in N(x)$
$\}$

Operational Semantics

$$z :\in \{A(x) \mid B(y) \quad \textbf{where} \quad x :\in M \mid R \quad y :\in N \} \gg \{ C(p, \theta) \quad \textbf{where} \quad p :\in N \}$$

- Execute the defns of x, y and evaluate $A(x) \mid B(y)$, all in parallel.
- Suspend evaluation of $A(x) \mid B(y)$ until x or y gets a value.
- When x gets a value, resume evaluation of $A(x)$.
- When y gets a value, resume evaluation of $B(y)$.
- Suppose $A(x)$ returns v. Evaluate $C(p, v)$. Start with $p :\in N$.

Execution Rules

- State: Variable, value pair. Value for θ in every state.

  $p.x$ is the value of x in state p.

- In the initial state only globals and θ have values.

- Rules describe the bag of values computed for expression f, by structural induction on f.

expr	::=	term
		| expr | expr
		| expr $\gg$ expr
		| expr $*$ expr
		| { expr **where** defn }

Execution Rules; Starting state p

- term $M(x, y)$: call M with parameters $p.x$ and $p.y$.
  - M never responds: computation never terminates.
  - M responds with value v: Only one result state q, $q.\theta = v$ and $q.x = p.x$ for all other x.

- $f \mid g$: evaluate f and g in state p, in parallel.

  The (bag of) result states are the ones returned by both f and g.

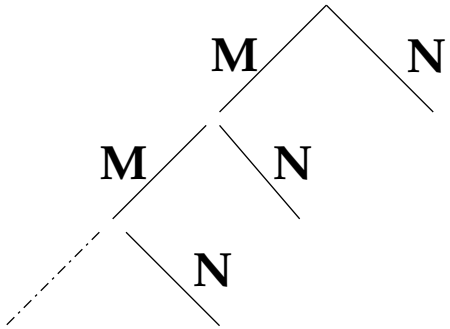
- $f \gg g$: evaluate f in p. For each result state q, evaluate g in q.

  Value computed for f is in $q.\theta$. Value of $f \gg g$ are the states returned by g.

Execution Rule for $f * g$ in state p

Evaluate both f and g in p and in any q returned by f.

The result states are the ones returned by g.



Values are returned by N.

Execution Rule for $\{ f$ **where** $x :\in g \}$ **in state** p

- Evaluate f and g in state p, in parallel.
- When g returns state q, augment p with $(x, q.\theta)$.
- In evaluating f, if we need the value of x:

  wait until the value is available (in an augmented state).
- Result states of $\{ f$ **where** $x :\in g \}$ are from f.

  Remove the tuple for x from the state because x is not defined outside this scope.

Execution Rule for $z :\in f$ in state p

- Evaluate f in p.
- Any result state, q, has the same set of variables as p, and $p.x = q.x$, for all x except θ; the result of evaluation is $q.\theta$.
- Let r be the first result state. Augment p by $(z, r.\theta)$, and return this as the result state of $z :\in f$.
- If f never responds, state p is never augmented.

Fork-Join parallelism

$z :\in fst(\mathit{true}, x)$ **where** $x :\in timer(1)$ | $fst(\mathit{false}, x)$ **where** $x :\in timer(2)$

z is assigned *true* after 1 time unit.

Angelic Nondeterminism

In $(M \mid N) \gg R$, R may be called twice. We have

$$(M \mid N) \gg R \;=\; M \gg R \;\mid\; N \gg R,$$

More generally, Right Distributivity of $\gg$ over $\mid$:

$$(f \mid g) \gg h \;=\; (f \gg h \mid g \gg h)$$

Demonic Nondeterminism; where clause

$$(N \mid R) \gg M$$

is *not* equivalent to

$$let(x) \gg M$$

where

$$x :\in N \mid R$$

Idempotence and Left Distributivity do not hold

Following laws do not hold.

(Idempotence of $\mid$) $f \mid f = f$

(Left Distributivity of $\gg$ over $\mid$) $f \gg (g \mid h) = (f \gg g) \mid (f \gg h)$

Figure 1: Schematic for $M \gg (N \mid R)$ and $M \gg N \mid M \gg R$

Parallel or

Let sites M and N return booleans. Compute their parallel or.

$$z :\in ift(x) \mid ift(y) \mid or(x, y)$$

where

$$x :\in M$$
$$y :\in N$$

Similarly, evaluate any function f of the form

$$f(x, y) = \begin{cases} p(x) & \text{if } c(x) \\ q(y) & \text{if } d(y) \\ r(x, y) & \text{otherwise} \end{cases}$$

Eight queens

- configuration: placement of queens in the last i rows.
- Represent a configuration by a list of integers j, $0 \leq j \leq 7$.
- Valid configuration: no queen captures another.
- $check(x{:}xs)$: Given xs valid, return

  $x : xs$, if it is valid

  remain silent, otherwise.

Eight queens; Contd.

$$
\begin{aligned}
&let([]) \\
&\gg \langle check(0:\theta) \mid check(1:\theta) \mid check(2:\theta)\cdots \mid check(7:\theta)\rangle \\
&\gg \langle check(0:\theta) \mid check(1:\theta) \mid check(2:\theta)\cdots \mid check(7:\theta)\rangle \\
&\gg \langle check(0:\theta) \mid check(1:\theta) \mid check(2:\theta)\cdots \mid check(7:\theta)\rangle \\
&\gg \langle check(0:\theta) \mid check(1:\theta) \mid check(2:\theta)\cdots \mid check(7:\theta)\rangle \\
&\gg \langle check(0:\theta) \mid check(1:\theta) \mid check(2:\theta)\cdots \mid check(7:\theta)\rangle \\
&\gg \langle check(0:\theta) \mid check(1:\theta) \mid check(2:\theta)\cdots \mid check(7:\theta)\rangle \\
&\gg \langle check(0:\theta) \mid check(1:\theta) \mid check(2:\theta)\cdots \mid check(7:\theta)\rangle \\
&\gg \langle check(0:\theta) \mid check(1:\theta) \mid check(2:\theta)\cdots \mid check(7:\theta)\rangle
\end{aligned}
$$

$$
\begin{aligned}
&let([]) \gg \langle \gg i : 0 \le i \le 7 : \\
&\qquad \langle \mid j : 0 \le j \le 7 : \ check(j:\theta)\rangle \\
&\rangle
\end{aligned}
$$

Local object

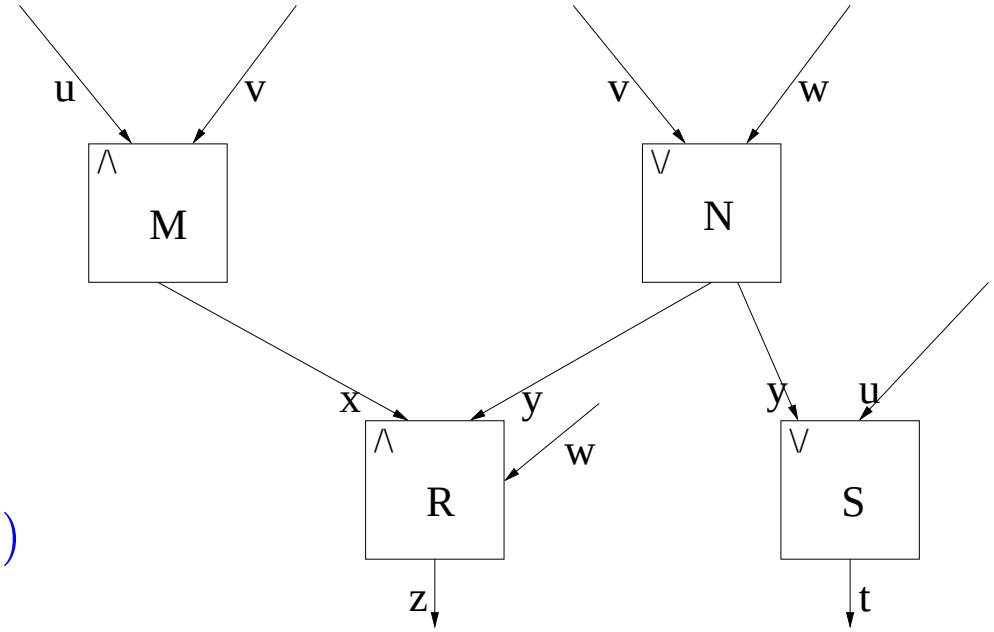
- Call sites M, N and R.
- Terminate after receiving two response.

Object $count$ with integer state. Initially, 0.

- $count.incr$ increments state;
- returns a signal if state ≥ 2, otherwise, remains silent.

$$
\begin{array}{l}
c{:}\in \\
\quad M \gg count.incr \\
\quad \mid N \gg count.incr \\
\quad \mid R \gg count.incr
\end{array}
$$

And-Or graph



$r :\in let(z, t)$

where

$z :\in R(x, y, w)$

$t :\in S(y) \mid S(u)$

where

$x :\in M(u, v)$

$y :\in N(v) \mid N(w)$

Airline

- Return any quote, from A or B, provided it is below 300.
- If neither quote is below 300, then return the cheapest quote or any quote available by time t.
- If no quote is available by t, return ∞.

Min returns the minimum of its argument values.

$threshold(x)$ returns x if x is below 300; silent otherwise.

$$z :\in threshold(x) \mid threshold(y) \mid Min(x, y)$$

where

$$x :\in A \mid timer(t, \infty)$$
$$y :\in B \mid timer(t, \infty)$$

Workflow: Visit Coordination

- $Email(p, s)$: contact p with dates s; response is date d from s.
- $Hotel(d)$: booking from hotel.
- $Airline(d)$: booking from airline.
- $Ack(p, t)$: similar to $Email$; response is an acknowledgment.
- $Confirm(t)$: confirm reservation t (for hotel or airline).
- $Room(d)$: reserve room for d. Response q: room number, time.
- $Announce(p, q)$: announce the lecture.
- $AV(q)$: contact technician with room and time information in q.

Workflow; Contd.

$z :\in let(b)$ $\gg$ $let(c, e)$ $\gg$ $let(u, v)$

where

$b :\in Ack(p, h, f)$

where

$h :\in Hotel(d)$

$f :\in Airline(d)$

where

$d :\in Email(p, s)$

where

$c :\in Confirm(h)$

$e :\in Confirm(f)$

where

$u :\in Announce(p, q)$

$v :\in AV(q)$

where

$q :\in Room(d)$

Interrupt handling

- Orc statement can not be directly interrupted.
- $Interrupt$ site: a monitor.
- $Interrupt.set$: to interrupt the Orc statement
- $Interrupt.get$: responds after $Interrupt.set$ has been called.

$z :\in f$

---

is changed to

$$z :\in f \mid Interrupt.get$$

Processing Interrupt

$z :\in \{\ f(x, y)$
where $x :\in g,\ y :\in h\ \}$

If f is interrupted, call M and N with parameters x and y, respectively, to cancel the effects of g and h.

$z :\in Normal(t) \mid Interr(t) \gg let(X, Y)$
where
$X :\in M(x)$
$Y :\in N(y)$

where
$t :\in f(x, y) \mid Interrupt.get$
where
$x :\in g$
$y :\in h$

Phase Synchronization

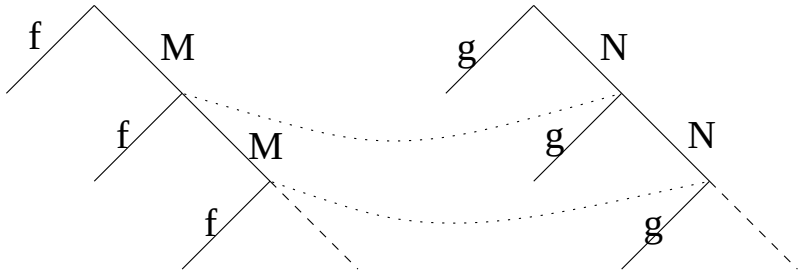
Process starts its $(k+1)^{th}$ phase only after all processes have completed their k^{th} phases.

Consider $M \gg f$ and $N \gg g$.

$$\{let(x, y)$$
where
$$x :\in M$$
$$y :\in N\}$$
$$\gg$$
$$fst(\theta) \gg f \mid snd(\theta) \gg g$$

Phase Synchronization; Contd.

Synchronize $M * f$ and $N * g$.



$\{let(x, y)$

where

$x :\in M$

$y :\in N\}$

$*$

$fst(\theta) \gg f \mid snd(\theta) \gg g$

Heat transfer computation over a grid

- Value x_{ij} at point (i, j) in a phase is the average of its neighbors' values in the previous phase.
- Site $average$ returns the average of its arguments.
- Site $converge$ returns its argument value if the values have converged sufficiently, otherwise, it remains silent.

$$z :\in \{let(x)$$
where
$$\langle \forall i, j :: x_{ij} :\in average(\theta_{i-1,j}, \theta_{i+1,j}, \theta_{i,j-1}, \theta_{i,j+1}) \rangle$$
$$\}$$
$$* \ converge(\theta)$$

Environment

An environment is a set of tuples. Each tuple has:

- a name (of a variable or θ),
- a val component (its value) and
- a $clock$ component, the time at which this value was computed.

Example: p: $\langle (x, \mathit{false}, 27), (y, \mathit{true}, 12), (\theta, 13, 20) \rangle$

$p.x.val = \mathit{false}$ and $p.x.clock = 27$.

An environment is a statement about a computation:
the value of x, computed at time 27, is *false*, and the value of y, computed at time 12, $\cdots$.

Relation over Bags of Environments

Each expression and defn is a binary relation over bags of environments.

Notation: P, Q are bags of environments. $\cup$ is bag union.

Write $P\ f\ Q$ and $P\ (z{:}\in\ f)\ Q$.

Coercion rule: $$\frac{\langle \forall p : p \in P : \{p\}\ f\ Q_p \rangle}{P\ \ f\ \ \langle \cup p : p \in P : Q_p \rangle}$$

Consequently, we need only consider $\{p\}\ f\ Q$.

Note: $\{\}\ f\ \{\}$

Relation over Bags of Environments; Contd.

$\{p\}\ f\ Q$: evaluation of f started in p at time $p.\theta.clock$ yields all environments in Q in some computation.

Q may be empty: non-terminating computation.

Q may have duplicates: as in evaluating $M \mid M$.

Example

p: $\langle (x, \mathit{false}, 27), (y, \mathit{true}, 12), (\theta, 13, 20) \rangle$.

$\{p\}\ \ let(x, y)\ \ \{\langle (x, \mathit{false}, 27), (y, \mathit{true}, 12), (\theta, (\mathit{false}, \mathit{true}), 27) \rangle\}$

$\{p\}\ \ timer(2)\ \ \{\langle (x, \mathit{false}, 27), (y, \mathit{true}, 12), (\theta, SIGNAL, 22) \rangle\}$

$\{p\}\ \ u :\in let(x, y)$
$\{\langle (x, \mathit{false}, 27), (y, \mathit{true}, 12), (u, (\mathit{false}, \mathit{true}), 27), (\theta, 13, 20) \rangle\}$

$\{p\}\ let(z)\ \ \{\}$, because z is not defined in p.

$\{p\}\ \ u :\in let(z)\ \ \{p\}$

Semantics of Term

Evaluate $M(L)$ in environment p. Result is at most one environment.

- $x \in L$ and $x \notin p$: no result environment.
- Otherwise: call M with values $p.x.val$ for all x in L, at maximum of $p.\theta.clock$ and clock values of all parameters in L.
- If M responds with value v at time t, the result environment is q, where

  $q.x = p.x$, for all x in p, $x \neq \theta$
  $q.\theta.val = v$
  $q.\theta.clock = t$

Axioms about terms

Notation: $p\backslash x$: remove the tuple for x from q. $x \notin p \Rightarrow p\backslash x = p$.

- $\{p\}\ M(L)\ \{\}$, if $x \in L$ and $x \notin p$.
- Given $\{p\}\ M(L)\ \{q\}$:
  - $q.x = p.x$, for all x in p, $x \neq \theta$, and $q.\theta.clock \geq p.\theta.clock$
  - if $x \notin L$, then $\{p\backslash x\}\ M(L)\ \{q\backslash x\}$
  - if $x \in L$, let $p' = p\backslash x$ except
    $p'.\theta.clock = \max(p.\theta.clock, p.x.clock)$.
    Then, $\{p'\}\ M(L[x := p.x.val])\ \{q\backslash x\}$
- (parameters may be renamed) For $y \notin p$ and $y \notin L$,

$$\{p\}\ M(L)\ \{q\}\ \equiv\ \{p[x := y]\}\ \ M(L[x := y])\ \ \{q[x := y]\}$$

Semantics of some sites

- $\{p\}\ Fail\ \{\}$
- $\{p\}\ let(x, y)\ \{q\}$:

  $q.\theta.val = (p.x.val, p.y.val)$
  $q.\theta.clock = \max(p.\theta.clock, p.x.clock, p.y.clock)$

  In particular, $\{p\}\ let(\theta)\ \{p\}$.

- $\{p\}\ random\ \{q\}$:

  $q.\theta.val =$ a number from the specified range
  $q.\theta.clock = p.\theta.clock$

Semantics of some sites; Contd.

- $\{p\}\ fst(x, y)\ \{q\}$:

  $$q.\theta.val = p.x.val$$
  $$q.\theta.clock = \max(p.\theta.clock, p.x.clock, p.y.clock)$$

- $\{p\}\ timer(t)\ \{q\}$:

  $$q.\theta.val = SIGNAL$$
  $$q.\theta.clock = p.\theta.clock + t$$

Exercise: Properties of the timer

$x :\in timer(t) \mid timer(u) \quad \equiv \quad x :\in timer(t)$, given $t \leq u$,

$timer(t) \gg timer(u) \quad \equiv \quad timer(t + u)$

Semantics of Defn

- $$\frac{\{p\}\ f\ \{\}}{\{p\}\ (z{:}\in f)\ \{p\}}$$
- $$\frac{\{p\}\ f\ Q,\ q\ ismin\ Q}{\{p\}\ (z{:}\in f)\ \{p + (z, q.\theta)\}}$$

$q\ ismin\ Q$: $q \in Q$ and $q.\theta.clock \leq r.\theta.clock$ for every r in Q.

$+$ denotes expansion of an environment by a tuple.

Semantics of Expression

- $$\frac{\{p\}\ (x{:}\in g)\ \{q\},\ \{q\}\ f\ Q}{\{p\}\ \{f\ \textbf{where}\ \ x{:}\in g\}\ (Q \backslash x)}$$

- $$\frac{\{p\}\ f\ Q,\ \{p\}\ g\ R}{\{p\}\ \ (f \mid g)\ \ (Q \cup R)}$$

- $$\frac{\{p\}\ f\ Q,\ Q\ g\ R}{\{p\}\ \ (f \gg g)\ \ R}$$

- $$\frac{\{p\}\ f\ Q,\ Q\ f^*\ R}{\{p\}\ \ f^*\ \ (\{p\} \cup R)}$$

$f * g \equiv f^* \gg g$

$f^* \equiv f * \ \mathbf{1}$, where $\mathbf{1} = let(\theta)$.

Notes

- $\gg$ is relational composition.
- $\mid$ is not relational union,

  $P\ f\ Q$ does not imply $P\ (f \mid g)\ Q$.

  Under relational union M and $M \mid M$ would be identical. We treat them differently.

Kleene Algebra

(Zero and $\mid$)	$f \mid \mathbf{0} = f$
(Commutativity of $\mid$)	$f \mid g = g \mid f$
(Associativity of $\mid$)	$(f \mid g) \mid h = f \mid (g \mid h)$
(Idempotence of $\mid$)	$f \mid f = f$
(Associativity of $\gg$)	$(f \gg g) \gg h = f \gg (g \gg h)$
(Left zero of $\gg$)	$\mathbf{0} \gg f = \mathbf{0}$
(Right zero of $\gg$)	$f \gg \mathbf{0} = \mathbf{0}$
(Left unit of $\gg$)	$\mathbf{1} \gg f = f$
(Right unit of $\gg$)	$f \gg \mathbf{1} = f$
(Left Distributivity of $\gg$ over $\mid$)	$f \gg (g \mid h) = (f \gg g) \mid (f \gg h)$
(Right Distributivity of $\gg$ over $\mid$)	$(f \mid g) \gg h = (f \gg h \mid g \gg h)$
(Recursive Expansion of Kleene star)	$f^* = \mathbf{1} \mid f \gg f^*$

Corollaries

(Left Distributivity of $*$ over $|$) $\quad f * (g \mid h) = (f * g \mid f * h)$

(Regrouping $*$ over $\gg$) $\quad (f * g) \gg h = f * (g \gg h)$

Additional Properties of Non-flat Expressions

- (Narrowing the scope) Given that x is not free in g:

$$\begin{array}{lcl} \{g \textbf{ where } x{:}\in h\} & = & g \\ \{f \mid g \textbf{ where } x{:}\in h\} & = & \{f \textbf{ where } x{:}\in h\} \mid g \\ \{f \gg g \textbf{ where } x{:}\in h\} & = & \{f \textbf{ where } x{:}\in h\} \gg g \end{array}$$

- (Bound variable renaming) In the following, y is not free in f or g.

$$\{f \textbf{ where } x{:}\in g\} = \{f[x := y] \textbf{ where } y{:}\in g\}$$

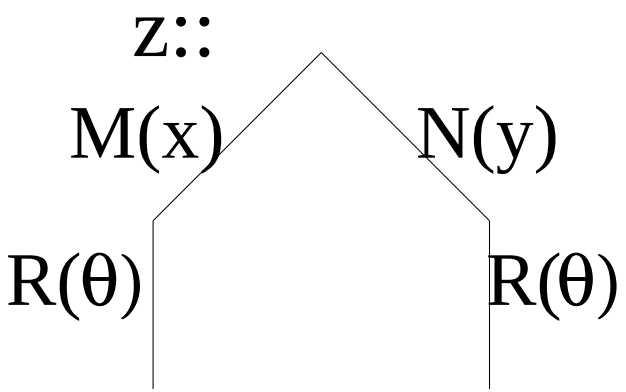
- (Independent defn) y is not free in g and x is not free in h.

$$\begin{array}{rl} & \{\{f \textbf{ where } x{:}\in g\} \textbf{ where } y{:}\in h\} \\ = & \{\{f \textbf{ where } y{:}\in h\} \textbf{ where } x{:}\in g\} \end{array}$$

Implementation

- Compile the statement into a (set of) finite state automata
- explore the automata to compute the goal variable.

For $z :\in (M(x) \mid N(y)) \gg R(\theta)$

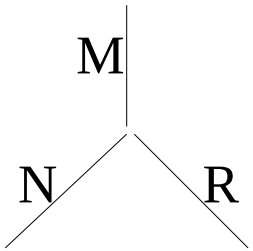


Difficulties In Automata construction

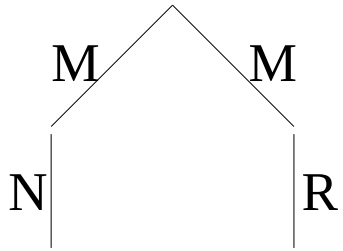
- How to compile a non-flat expression:

  $z :\in \{M(x) \mid N$ **where** $x :\in R\}$

- We cannot use standard procedures. In automata theory,
  $M \gg (N \mid R) \equiv M \gg N \mid M \gg R$



(a)



(b)

- Non-determinism: M is called twice in $M \gg N \mid M \gg R$.

Flat program

Transform $\{f$ **where** $x :\in g\}$ to

$open(x) \gg f \gg close(x)$ and add the defn $x :\in g$.

$z :\in \{M(x)$ **where** $x :\in \{A(y)$ **where** $y :\in B\}\}$ | $\{R(u)$ **where** $u :\in B\}$

The corresponding flat program is

$z :\in open(x) \gg M(x) \gg close(x)$ | $open(u) \gg R(u) \gg close(u)$
$x :\in open(y) \gg A(y) \gg close(y)$
$y :\in B$
$u :\in B$

$open$ and $close$

- $open$ and $close$ are treated differently from usual sites.
- Calling $open(x)$ starts a new computation of x on a clone.
- A state includes the values for the global variables and θ, but reference to clone for local variable.
- $close(x)$ removes reference to x from the state.

clone

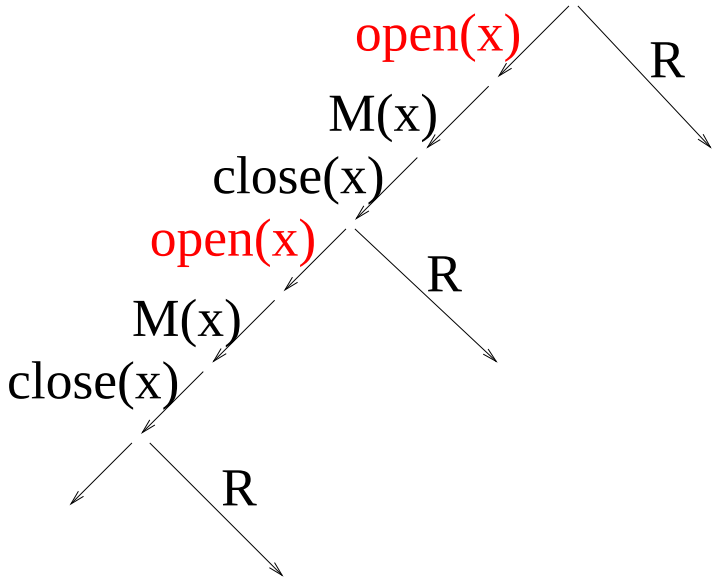
Several clones of an fsa may be simultaneously in existence.

$z :\in \{\ M(x)$ **where** $x :\in N\ \} * R$

$z :\in \langle open(x) \gg M(x) \gg close(x) \rangle * R$

$x :\in N$

has the flat program



Finite State Automata (fsa) from Flat Program

- An Orc fsa is a finite directed graph.
- Its edges are labeled with terms (including $open$ and $close$).
- A pair of nodes in the fsa may have multiple edges between them, possibly with the same label.
- Two distinguished nodes: begin and end.
- No incoming edge to begin node; no outgoing edge of end.
- For every edge, there is a path from begin to end that includes the edge.

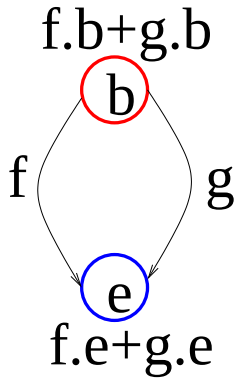
Recursive fsa Construction

merge of x, y is $x + y$: incoming, outgoing edges of x, y.

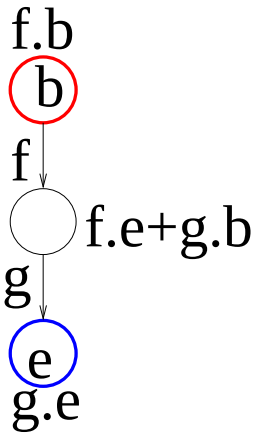
$f.b$ and $f.e$ for begin and end nodes of fsa for f.

term $M(L)$: The fsa has one edge from begin to end, labeled $M(L)$.

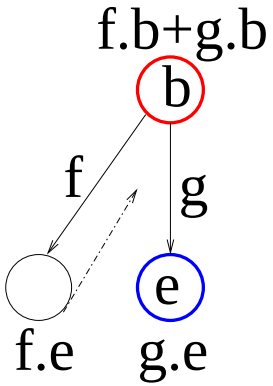
For edge from $(f * g).b$ to x with label r: make edge $(f.e, x)$ with r.



(a) f | g

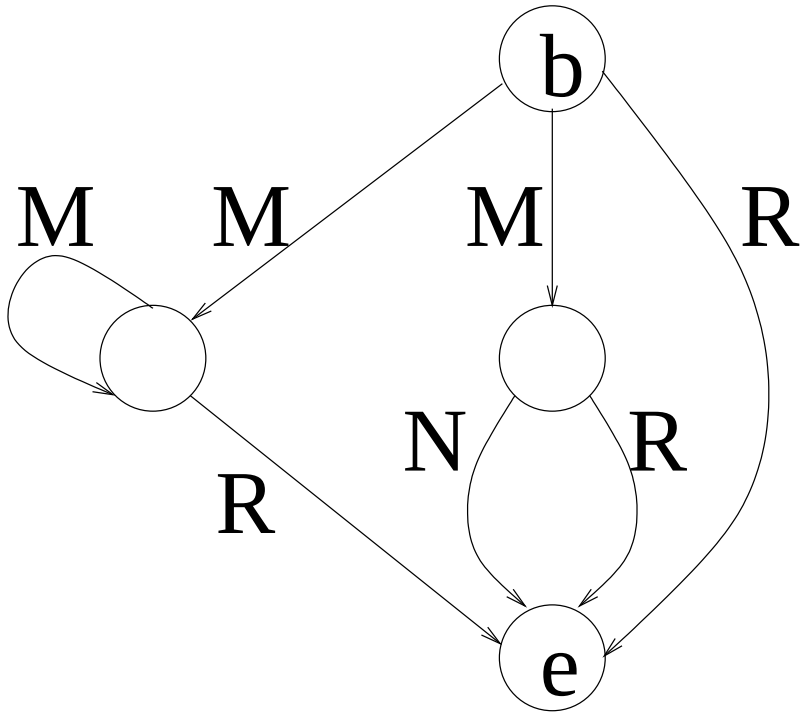


(b) f ≫ g



(c) f * g

fsa for $M \gg (N \mid R) \mid M * R$



Complexity of fsa construction

- consider
  $\underbrace{(((M * M) * M) * \cdots M)}_{n\ Ms}$
  The number of edges in the fsa is $O(n^2)$.

- Let t_f be the number of terms and s_f the number of $*$ in expression f. Then the fsa for f has

  the number of outgoing edges of $f.b \leq t_f$
  the number of edges in the fsa $\leq t_f(s_f + 1)$

- Traditional deterministic fsa construction is P-space complete.

  Orc fsa construction is quadratic.

A linear fsa construction

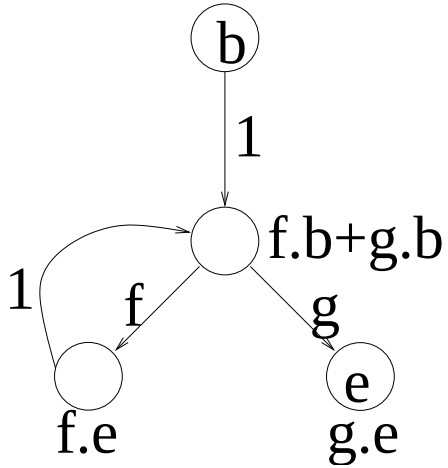


Figure 2: Alternate fsa for $f * g$

the number of outgoing edges of $f.b \leq t_f$
the number of edges in the fsa $\leq t_f + 2 \times s_f$

fsa exploration; token

- token associated with an edge of a clone of an fsa.
- Corresponds to a single step of computation.
- token has a state and a parent. parent is a token in the same clone or NIL.
- token processes label $M(x)$ on edge e:

  wait until x has a value, then call site M. On receiving v from M:

  – creates children tokens on all successor edges of e,
  – bequeaths to them the state with θ value v,
  – if e has no successor edge, reports v as the value of the computation.

fsa exploration; token; contd.

Asssume: single outgoing edge from each begin node (begin edge).

Ensure by adding an edge with label $\mathbf{1}$.

- The token processes label $open(x)$ on edge e:
  - initiate computation on a clone of the fsa for x
  - return reference to the clone as part of the state
- $close(x)$: remove the reference to the clone for x from the state.
- To initiate computation on a clone in state s:

  place a token on its begin edge with state s and NIL parent.

Site

- Any level of granularity in site,

  from simple message transmission

  to business-business transaction involving many servers

- May spawn new processes, start servers and change database contents

- May interact with peripheral devices, including displays and keyboards

Site Specification

The specification of site M is a predicate p over a triple (x, y, t):

x is the value of actual parameters,

y is the result returned by M,

t is an (absolute) time instant.

Two stage Site Operation

A site operates in two stages.

- response: Client calls. Site returns y or remains silent.

  also returns a pledge which is invisible to the Orc statement.

  pledge carries a deadline by which it should be committed.

- commit: If the caller commits the pledge at time t before the deadline,

  the site executes its commit stage,

  which establishes predicate $p(x, y, t)$.

Examples of Site Specification

- Function f:

  returns y where $y = f(x)$. The deadline is irrelevant.

  predicate: $y = f(x)$. No commitment needed.

- Site $postOffice$:

  called with description of a parcel and returns y, the cost of delivery.

  The deadline is the instant t, the time of response.

  predicate: the cost of delivery of x at time t is y.

  needs no commitment to establish this predicate.

Examples of Site Specification; Contd.

- Object $count$: has an integer value $count.v$ and two methods, $incr$ and $read$.
- Initially, $count.v = 0$.
- $incr$: $count.v := count.v + 1$;
  $read$: returns v = $count.v$.
  The moment of response, t, is the deadline.
- predicate: $count.v \geq v$ beyond t. No commitment required.
- Another spec: $count.v = v$ at the moment of commitment s, $s \leq t$.

  Implement specification by: lock $count.v$ until the moment of commitment or t, whichever comes first.

Examples of Site Specification; Contd.

- Transaction sells 80 shares of stock pqr if price is above \$25 and buys 100 shares of stock abc if it is below \$20 a share.
- Both price conditions are met before the transaction returns a signal.
- The deadline is very short.
- If the client commits within the deadline, establishes the predicate:

  client has bought and sold the requisite number of shares for the given prices at the moment of commitment.

Examples of Site Specification; Contd.

- A site call may cause state change during the response stage.
- An airline issues a price quote and changes its state during the response stage, even if no call commits.
- Its only obligation is to issue a ticket at the given price if the client commits within the deadline.

Type of pledge

- Instant pledge: site makes immediate commitment for the caller

  The pledge can only be revoked next.

  Calling site $email$ sends an email, without waiting for commitment.

  Common in concurrent computing.

- Deferred pledge: Commitment by deadline establishes the associated predicate.

Structure of pledge

A pledge has:

- An id,
- A deadline, an absolute time instant by which it has to be committed or revoked (if already committed),
- A set of pertinent arguments, the arguments of the site call which must be committed in order to commit this pledge.

$z :\in Min(x, y)$

where

$x :\in A$

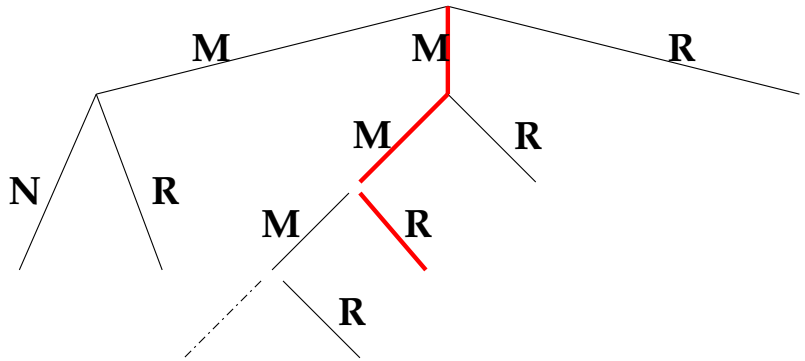
$y :\in B$

Pertinent argument from Min is the cheaper of x and y.

Critical Path

Response from a terminal edge, e, is assigned as the variable value.

Edge e defines the Critical Path.



Commit all pledges and their pertinent pledges along the critical path. Transitive closure needed.

Revoke all other instant pledges.

Definitions

For any clone c:

$child(c)$ = $\{d |$ d is a clone spawned by an $open$ in $c\}$

$pert(c)$ = $\{d |$ d is not a global variable or θ, and
d is a pertinent argument in a deferred pledge
along c's critical path$\}$

$instant(c)$ = instant pledge ids received in c

$deferred(c)$ = deferred pledge ids received in c

$instant^{+}(c)$ = instant pledge ids received along the critical path in c

$deferred^{+}(c)$ = deferred pledge ids received along the critical path in c

Transitive closure definitions

For any clone c:

$all\_instant(c)$ instant pledge ids received in c and its descendants.

$all\_deferred(c)$ deferred pledge ids received in c and its descendants.

$all\_instant^{+}(c)$ and $all\_deferred^{+}(c)$, limited to the critical paths.

$$
\begin{aligned}
all\_instant(c) &= instant(c) \\
&\quad \cup \langle \cup d : d \in child(c) : all\_instant(d) \rangle \\
all\_deferred(c) &= deferred(c) \\
&\quad \cup \langle \cup d : d \in child(c) : all\_deferred(d) \rangle \\
all\_instant^{+}(c) &= instant^{+}(c) \\
&\quad \cup \langle \cup d : d \in pert(c) : all\_instant^{+}(d) \rangle \\
all\_deferred^{+}(c) &= deferred^{+}(c) \\
&\quad \cup \langle \cup d : d \in pert(c) : all\_deferred^{+}(d) \rangle
\end{aligned}
$$

Pledges to be Committed, Revoked

For any clone c:

$all^+(c)$: pledges necessary and sufficient to commit c (received by c and its descendants).

$$all^+(c) = all\_instant^+(c) \cup all\_deferred^+(c)$$

Instant pledges in $all^+(c)$ are already committed.

$pos(c)$: pledges which remain to be committed in $all^+(c)$; i.e.,

$$pos(c) = all\_deferred^+(c)$$

$neg(c)$: pledges which need to be revoked, i.e., already committed and not part of $all^+(c)$,

$$neg(c) = all\_instant(c) - all^+(c), \text{ i.e.,}$$
$$neg(c) = all\_instant(c) - all\_instant^+(c)$$

Commitment and Revocation Algorithm

- Client: sends to the appropriate sites,

  $commit_{init}$ for pledges in $pos(c)$,

  $revoke_{init}$ for pledges in $neg(c)$

- Site: responds with

  ack_{init} if it is ready to commit/revoke the pledge or

  $nack_{init}$ if it can not.

  Treat failure to respond (timely) as $nack_{init}$.

Commitment and Revocation Algorithm; Contd.

- Client:
  - If all responses are ack_{init}, sends $commit_{final}$ to sites corresponding to $pos(c)$ and $revoke_{final}$ to sites corresponding to $neg(c)$.
  - Otherwise, sends $abort_{final}$ to all sites.
- Site:

  commit after receiving $commit_{final}$

  revoke after receiving $revoke_{final}$,

  recovery computation after receiving $abort_{final}$.

Explicit Commit and Revoke

To handle transactions of differing deadlines.

Commit and Revoke sites.

A successful call to $Commit(x, y)$:

returns *true* and guarantees that x and y are committed with their pertinent variables. Similarly, $Revoke$.

Make explicit the commit in $z :\in f$

$$z :\in \{\ Commit(y) \gg let(y)$$
where
$$y :\in f$$
$$\}$$

Example

Reserve a hotel room and an airline ticket.

The hotel responds after a long delay but gives a long deadline.

The airline usually responds quickly but gives a short deadline.

Strategy

- Contact the hotel. After it responds, contact the airline.
- Airline responds before the hotel's deadline:

  if its quote is excessively high: cancel vacation plan and assign ∞ to the goal variable.

  Otherwise: commit to both and return the sum of the quotes as the goal variable value.

Example; Contd.

- Airline does not respond before the hotel's deadline: commit to the hotel; and wait 1 unit for the airline response.
  - Airline responds before the new deadline:
    if its quote is excessive: revoke the hotel commitment and cancel vacation plans.
    Otherwise: commit to the airline and return the sum of the quotes as the goal variable value.
  - Airline does not respond before the new deadline: revoke the hotel commitment and cancel vacation plans.

Example; Contd.

$$
\begin{array}{l}
z :\in \\
\quad \{ \quad exc(a) \quad \gg let(\infty) \\
\quad \mid \neg exc(a) \quad \gg Commit(a,h) \gg Plus(a,h) \\
\quad \mid let(t0) \quad \gg Commit(h) \quad \gg \\
\quad\quad (\ (exc(a) \mid let(t1)) \gg Revoke(h) \gg let(\infty) \\
\quad\quad\quad \mid \neg exc(a) \qquad\quad \gg Commit(a) \gg Plus(a,h) \\
\quad\quad)
\end{array}
$$

where

$$
\begin{array}{ll}
(h,d) & :\in Hotel \\
a & :\in let(h) \gg Airline \\
t0 & :\in timer(d) \\
t1 & :\in let(t0) \gg timer(1)
\end{array}
$$

$$\}$$