A Counting Problem Communicated by Dijkstra

Jayadev Misra

4/9/96

Problem: Given that there are 25 boys and 25 girls. A *party* has 12 *tables*, each of which seats 2 boys and 2 girls. Thus, a party is attended by 24 boys and 24 girls. A boy *sees* 2 girls at his table in a party, and so do the girls. A set of parties, P, is *feasible* if each boy/girl sees different girls/boys in different parties in P. What is the size of the largest feasible set?

Observation: A boy attends at most 12 parties in any feasible set.
Proof: Since a boy sees 2 different girls in each party, he can attend at most 12 parties before seeing a previously-seen girl.

Lemma 1: A feasible set of parties has size ≤ 12.
Proof: Consider the total number of attendances in all parties in the set by the boys. Each party has 24 boys attending; so for a feasible set of size s, the number of attendances $= 24 \times s$. Now, each boy attends at most 12 parties in this set; therefore, number of attendances $\leq 25 \times 12$. From the inequality, $24 \times s \leq 25 \times 12$, and that s is integer, we conclude that $s \leq 12$.

Lemma 2: There is a feasible set of size 12.
Proof: Consider any party, and imagine that the tables are arranged circularly. Create another party by having the boys stay at their tables and each girl move from her current table to its right table. This step can be repeated 11 more times, each yielding a party in which each boy sees 2 different girls (and each girl sees 2 different boys).
Thanks: E.W.Dijkstra communicated this problem to me.