A proof by Erdos

Jayadev Misra

10/1/99

A sequence, S, of numbers where S is longer than n^2 contains either an ascending or a descending subsequence longer than n.

For each position i in the sequence compute u_i, v_i where u_i is the length of the longest ascending sequence ending at S_i; similarly, v_i is the length of the longest descending sequence ending at S_i.

Claim: For distinct i, j, $(u_i, v_i) \neq (u_j, v_j)$.

Proof: Let $i < j$. $S_i < S_j \Rightarrow u_i < u_j$, because S_j can be appended to any sequence ending at S_i to form alonger sequence. Similarly, $S_i > S_j \Rightarrow v_i < v_j$.

There are n^2 distinct pairs of the form u, v where $1 \leq u \leq n,\ 1 \leq v \leq n$. Therefore, there is at least one position that has an associated pair one of whose components exceeds n.