There are Infinitely Many Primes
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3/21/97

The following proof of the classical theorem is due to Dijkstra.

Define a *plural* to be a natural number exceeding 1. A *composite* is a plural that is a product of two plurals. A *prime* is a non-composite plural.

Lemma: A plural is a product of primes.

Proof: If the plural is a prime, then the result follows. Otherwise, the plural is a composite and, hence, a product of plurals. By induction, each plural is a product of primes.

Theorem: Given any finite set of primes there is a prime outside the set.

Proof: Let n be the product of primes in the set + 1. Note that n is a plural, because product of any set of numbers is at least 1. From the lemma, n is a product of primes. However, none of the primes in the set divide n; therefore there is a prime outside the set.

Alternative Proof: Redoing Euclid's Proof

Define a *prime* to be a plural that is divisible by no smaller prime. This definition is meaningful: 2 is a prime because there is no smaller plural; and any other plural can be cheked for primality.

Lemma : Every plural is divisible by a prime.

Proof: If the plural is a prime then it is divisible by itself. If the plural is a non-prime, from definition, it is divisible by a prime.

Theorem: For any finite set of primes there is a prime outside the set.

Proof: Let n = the product of the primes in the set + 1. Note that n is a plural, because product of any set of numbers is at least 1. From the lemma, n has a prime divisor and from the construction no prime in the given set divides n. Hence, the prime divisor of n is outside the given set.