Overview of Resolution

Resolution is a method of proof by contradiction:

- Let F be in Conjunctive Normal Form (CNF), $F = F_1 \wedge F_2 \wedge ... \wedge F_n$ where each F_i is a disjunction (or) of literals (predicate or its negation).
- If we want to prove G, $F \rightarrow G$, then $F \rightarrow G$ is True, so $\neg(F \rightarrow G) = \neg(\neg F \vee G) = (F \wedge \neg G)$ is False.
- We add $\neg G$ to our list of clauses and try to prove that the result is False or box, $\Box$.
- If there is a new clause N such that $F \rightarrow N$, then $F \wedge N = F$. We can add N to our set of clauses without changing the result.
- The *resolution step* on clauses F_i and F_j produces a new clause N such that $F_i \wedge F_j \rightarrow N$.
- We add new clauses N to our set; if $N =$ False or box, $\Box$, then we have proved that $(F \wedge \neg G)$ is False.

Resolution Step

Suppose that we have two clauses, F_i and F_j where $F_i = A \vee L$ and $F_j = B \vee \neg L$.

The resolution step produces a new clause N by removing the complementary literals L and $\neg L$ and combining everything else from the two source clauses: $N = A \vee B$. (A and B could be composed of multiple literals.)

The result is a logical consequence of F_i and F_j, i.e. $F_i \wedge F_j \rightarrow N$.

Proof:

We want to show that $(A \vee L) \wedge (B \vee \neg L) \rightarrow (A \vee B)$.

- Case 1: $L =$ True. If $(B \vee \neg L)$ is true, then B must be true, so $(A \vee B)$ is true.
- Case 2: $L =$ False. If $(A \vee L)$ is true, then A must be true, so $(A \vee B)$ is true.