

Homework 12

CS 331

Due Thursday, April 25

1. Show NP-completeness for each of the following problems. Both are simple, direct reductions from one of the problems we have shown to be NP-complete in class.

- (a) **Minimum set cover.** You are given a set S , a collection of subsets $S_1, \dots, S_n \subseteq S$, and an integer k . Do there exist a set of k subsets $T \subseteq [n]$ such that

$$\bigcup_{i \in T} S_i = S?$$

Hint, encoded as ROT-13: iregrk pbire.

- (b) **Subgraph Isomorphism.** You are given two graphs, G and H . Does $G = (V_G, E_G)$ contain a subgraph isomorphic to $H = (V_H, E_H)$? That is, is there an injection $f : V_H \rightarrow V_G$ such that for every $u, v \in V_H$, $(u, v) \in E_H$ if, and only if, $(f(u), f(v)) \in E_G$?
Hint, encoded as ROT-13: znk pyvdhr be vaqrcraqrag frg.

- (c) **Partition.** You are given a set of n positive integers $x_1, \dots, x_n \in \mathbb{Z}^+$. Does there exist a subset $S \subseteq [n]$ such that

$$\sum_{i \in S} x_i = \sum_{i \in [n] \setminus S} x_i?$$

Hint, encoded as ROT-13: fhofrgfhz Extra hint: lbh jvyv jnag gb nqq n fvatyr rkgen vagrtre gb gur vachg.

2. The problem ALLORNOTHINGSAT asks, given a 3CNF boolean formula, whether there is an assignment to the variables such that each clause either has three True literals or has three False literals.

Describe a polynomial time algorithm for ALLORNOTHINGSAT.