Homework 2

CS 331

Due Thursday, February 1

1. [Book exercise 2.3] An addition chain for an integer n is an increasing sequence of integers that starts with 1 and ends with n, such that each entry after the first is the sum of two earlier entries. More formally, the integer sequence $x_0 < x_1 < x_2 < \cdots < x_\ell$ is an addition chain for n if and only if:

   - $x_0 = 1$

   - $x_\ell = n$

   - For every index $k > 0$, there are indices $i \leq j < k$ such that $x_k = x_i + x_j$.

   The length of an addition chain is the number of elements minus 1; we don't bother to count the first entry. For example,

   $$(1, 2, 3, 5, 10, 20, 23, 46, 92, 184, 187, 374)$$

   is an addition chain for 374 of length 11.

   (a) Describe a recursive backtracking algorithm to compute a minimum-length addition chain for a given positive integer n. Show correctness, but don't analyze or optimize your algorithm's running time except to satisfy your own curiosity. A correct algorithm whose running time is exponential in n is sufficient for full credit.

   (b) (Optional) Describe a recursive backtracking algorithm to compute a minimum-length addition chain for a given positive integer n in time that is sub-exponential in n. [Hint: it doesn't have to be polynomial. How long is the answer?]

2. There's a Jupyter Notebook linked from the class webpage. Run through it, then answer the questions at the end. Don't wait till the last day to do this: setting up the required libraries may take some time.