# Homework 3

CS 331

Due Thursday, February 8

1. [Book exercise 3.37] You have mined a large slab of marble from a quarry. For simplicity, suppose the marble slab is a rectangle measuring $m$ inches in height and $n$ inches in width. You want to cut the slab into smaller rectangles of various sizes—some for kitchen counter tops, some for large sculpture projects, others for memorial headstones. You have a marble saw that can make either horizontal or vertical cuts across any rectangular slab. (**Note:** when the saw is used, it must cut *all the way through* the piece it is used on.) At any time, you can query the spot price $P[x, y]$ of an $x$-inch by $y$-inch marble rectangle, for any positive integers $x$ and $y$. These prices depend on customer demand, and people who buy marble counter tops are weird, so don't make any assumptions about them; in particular, larger rectangles may have significantly smaller spot prices. You may assume $P[x, y] = P[y, x]$. Given the array of spot prices and the integers $m$ and $n$ as input, describe a dynamic programming algorithm to compute how to subdivide an $m \times n$ marble slab to maximize your profit.

   Note that, to present a dynamic programming algorithm, you should give:

   - A description of the subproblems you solve, in an English sentence or two. ("$f(i)$ is 1 if $S[:i]$ can be segmented into words and 0 otherwise.")
   - A mathematical description of the recurrence involved. ("Base case: $f(0) = 0$. Recurrence: $f(i) = 1$ iff $\exists j < i$ with $f(j) = 1$ and $S[j : i]$ is a word.")
   - How to compute the final answer using this recurrence ("Answer is $f(n)$.")
   - A description of how to solve all the subproblems (for example, if you build a table, in what order do you fill it in?), and analysis of the runtime.

2. There's a Jupyter Notebook linked from the class webpage.