

# Homework 4

CS 331

Due Thursday, February 15

1. In class we discussed interval *packing* problems. Here we explore interval *covers*.
  - (a) You are given a set of  $n$  intervals  $[s_i, f_i)$  and a range  $[0, T)$ . You would like to find a minimal set  $I \subset [n]$  of intervals whose union covers the range. That is, we say that  $I$  is a *valid cover* if, for all  $t \in [0, T)$ , there exists an  $i \in I$  such that  $t \in [s_i, f_i)$ . Give (and prove correctness for) a greedy algorithm to compute a valid cover with the smallest number of intervals, in linear time after sorting.
  - (b) (Optional) Now suppose that each interval also has a *cost*  $c_i$ , and your goal is to find a valid cover  $I$  minimizing the total cost  $\sum_{i \in I} c_i$ . Give a dynamic programming solution to this problem that takes  $O(n^2)$ , or even  $O(n \log n)$ , time.
2. There's a Jupyter Notebook linked from the class webpage.