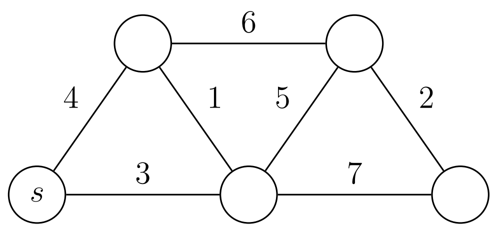
Homework 7

CS 331

Due Thursday, March 21

1. The following problem is short answer only. Consider minimum spanning tree algorithms for the following graph:



   (a) In what order would Prim's algorithm, starting at s, add edges to the minimum spanning tree? Give the sequence of edge weights, in order.

   (b) In what order would Kruskal's algorithm add edges to the minimum spanning tree? Give the sequence of edge weights, in order.

   (c) In what order would Boruvka's algorithm add edges to the minimum spanning tree? Give the set of edge weights added in the first round, the second round, etc.

2. (Problem 21 of book chapter 6) Kris is a professional rock climber who is competing in the U.S. climbing nationals. The competition requires Kris to use as many holds on the climbing wall as possible, using only transitions that have been explicitly allowed by the route-setter. The climbing wall has n holds. Kris is given a list of m pairs (x, y) of holds, each indicating that moving directly from hold x to hold y is allowed; however, moving directly from y to x is not allowed unless the list also includes the pair (y, x). Kris needs to figure out a sequence of allowed transitions that uses as many holds as possible, since each new hold increases his score by one point. The rules allow Kris to choose the first and last hold in his climbing route. The rules also allow him to use each hold as many times as he likes; however, only the first use of each hold increases Kris's score.

(a) Define the natural graph representing the input. Describe and analyze an algorithm to solve Kris's climbing problem if you are guaranteed that the input graph is a dag.

(b) Describe and analyze an algorithm to solve Kris's climbing problem with no restrictions on the input graph. Both of your algorithms should output the maximum possible score that Kris can earn.

3. You are building out internet for a collection of rural houses. For each house, you need to either purchase satellite internet at that house, or connect it via a series of fiber links to a house that has purchased satellite internet.

   There are n houses, and buying satellite internet costs P dollars at any house. There are m pairs of houses that can be directly connected by fiber; this is given as a list of triples (u_i, v_i, c_i), denoting that houses u_i and v_i can be connected at a cost of c_i dollars.

   Give an $O(m \log n)$ time algorithm to determine the minimum cost of hooking everyone up to internet.