Problem Set 3

CS 331H

Due Thursday, February 11

- 1. See the Jupyter notebook on the website.
- 2. How many ways can one tile a $3 \times n$ rectangle using 2×1 tiles? For example, there are 3 ways to tile a 3×2 rectangle:



(a) [65%] Show how to compute the answer in O(n) time, assuming that the word size can represent numbers as large as the answer. You may find it useful to consider the number of ways to tile all of the following figures:



- (b) [15%] By expressing the recursion in terms of matrices, show how to compute the part (a) answer in $O(\log n)$ time, again assuming the word size can represent numbers as large as the answer.
- (c) [20%] Now suppose that the word size is $\Theta(\log n)$. How much time do your solutions take?
- (d) (Optional) Repeat part (a) for tiling a $k \times n$ rectangle using 2×1 rectangles, for any constant k. How does your complexity scale with k?