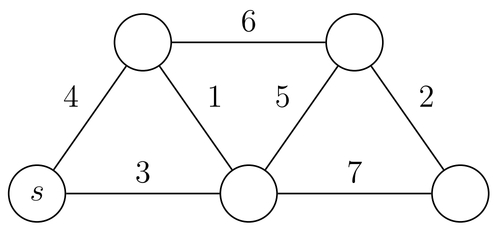
Problem Set 6

CS 331H

Due Thursday, April 1

1. Consider minimum spanning tree algorithms for the following graph:



   (a) In what order would Prim's algorithm, starting at s, add edges to the minimum spanning tree? Give the sequence of edge weights, in order.

   (b) In what order would Kruskal's algorithm add edges to the minimum spanning tree? Give the sequence of edge weights, in order.

   (c) In what order would Boruvka's algorithm add edges to the minimum spanning tree? Give the set of edge weights added in the first round, the second round, etc.

2. You are building out internet for a collection of rural houses. For each house, you need to either purchase satellite internet at that house, or connect it via a series of fiber links to a house that has purchased satellite internet.

   There are n houses, and buying satellite internet costs P dollars at any house. There are m pairs of houses that can be directly connected by fiber; this is given as a list of triples (u_i, v_i, c_i), denoting that houses u_i and v_i can be connected at a cost of c_i dollars.

   Give an $O(m \log n)$ time algorithm to determine the minimum cost of hooking everyone up to internet.

3. Consider a weighted, directed graph where all distances lie in $[1, 2)$. We would like to find an $O(E)$ time algorithm for single-source shortest paths on this graph.

   (a) Consider a variant of Dijkstra's algorithm that does not always visit the unvisited node of smallest $c(u)$, but instead arbitrarily picks one of the unvisited nodes of smallest $\lfloor c(u) \rfloor$. Show that such an algorithm still yields the correct answer.

   (b) Now give a data structure that allows this Dijkstra variant to run in $O(E)$ time. **Hint:** at any point during the execution, the set of $\lfloor c(u) \rfloor$ for unvisited nodes u can only have a small number of options.

   (c) Extend your result to $O(EC)$ time and $O(E + C)$ space for distances in $[1, C)$ for any $C \geq 1$.