Homework 13

CS 331H

Due Monday, April 24

1. Show NP-completeness for each of the following problems. They are all simple, direct reductions from one of the problems we have shown to be NP-complete in class.

   (a) **Minimum set cover**. You are given a set S, a collection of subsets $S_1, \ldots, S_n \subseteq S$, and an integer k. Does there exist a set of k subsets $T \subseteq [n]$ such that

   $$\bigcup_{i \in T} S_i = S?$$

   Hint (ROT-13): `iregrk pbire`.

   (b) **Subgraph Isomorphism**. You are given two graphs, G and H. Does $G = (V_G, E_G)$ contain a subgraph isomorphic to $H = (V_H, E_H)$? That is, is there an injection $f : V_H \to V_G$ such that for every $u, v \in V_H$, $(u, v) \in E_H$ if, and only if, $(f(u), f(v)) \in E_G$?

   Hint (ROT-13): `Znk pyvdhr be vaqrcraqrag frg`.

   (c) **Partition**. You are given a set of n positive integers $x_1, \ldots, x_n \in \mathbb{Z}^+$. Does there exist a subset $S \subseteq [n]$ such that

   $$\sum_{i \in S} x_i = \sum_{i \in [n] \setminus S} x_i?$$

   Hint (ROT-13): `fhofrg fhz`. Extra hint: `Lbh jvyy jnag gb nqq n fvatyr rkgen vagrtre gb gur vachg`.

2. The problem AllOrNothingSat asks, given a 3CNF boolean formula, whether there is an assignment to the variables such that each clause either has three True literals or has three False literals.

   Describe a polynomial time algorithm for AllOrNothingSat.