Homework 3

CS 331H

Due Monday, February 6

1. [Book exercise 3.37] You have mined a large slab of marble from a quarry. For simplicity, suppose the marble slab is a rectangle measuring m inches in height and n inches in width. You want to cut the slab into smaller rectangles of various sizes—some for kitchen counter tops, some for large sculpture projects, others for memorial headstones. You have a marble saw that can make either horizontal or vertical cuts across any rectangular slab. (**Note:** when the saw is used, it must cut *all the way through* the piece it is used on.) At any time, you can query the spot price $P[x, y]$ of an x-inch by y-inch marble rectangle, for any positive integers x and y. These prices depend on customer demand, and people who buy marble counter tops are weird, so don't make any assumptions about them; in particular, larger rectangles may have significantly smaller spot prices. Given the array of spot prices and the integers m and n as input, describe a dynamic programming algorithm to compute how to subdivide an $m \times n$ marble slab to maximize your profit.

   Note that, to present a dynamic programming algorithm, you should give:

   - A description of the subproblems you solve, in an English sentence or two. ("$f(i)$ is 1 if $S[:i]$ can be segmented into words and 0 otherwise.")
   - A mathematical description of the recurrence involved. ("Base case: $f(0) = 0$. Recurrence: $f(i) = 1$ iff $\exists j < i$ with $f(j) = 1$ and $S[j : i]$ is a word.")
   - How to compute the final answer using this recurrence ("Answer is $f(n)$.")
   - A description of how to solve all the subproblems (for example, if you build a table, in what order do you fill it in?), and analysis of the runtime.

2. There's a Jupyter Notebook linked from the class webpage.

3. **Optional.** (Worth no points, but I think it's a fun problem.)

   You are given a sequence of n integers, $x_1, \ldots, x_n$, and an integer $k \in [n]$. Each integer satisfies $|x_i| \leq n^2$. Find the contiguous subset of size at least k with maximum *average*. That is, find two indices $s, t \in [n]$ with $t \geq s + k - 1$ that maximizes

$$\mu(s,t) := \frac{1}{t-s+1} \sum_{i=s}^{t} x_i.$$

   Full credit requires $O(n \log n)$ time.

   Hints (in rot-13 so you can try to solve the problem without reading them all):

   - Pna lbh svaq gur yratgu $\geq k$ pbagvthbhf fhofrg bs znkvzhz *fhz* in $O(n)$ time?
   - Pna lbh qrgrezvar *jurgure gur znkvzhz nirentr vf abaartngvir* in $O(n)$ time?
   - Pna lbh qrgrezvar vs gur znkvzhz nirentr vf ng yrnfg t, sbe n tvira t, in $O(n)$ time?
   - Gurer ner $O(n^2)$ qvssrerag pbagvthbhf fhofrgf bs x, rnpu bs juvpu unf fbzr nirentr. Fbzr bs gurfr nirentrf znl or gur fnzr, naq fbzr ner qvssrerag. Fubj gung sbe nal gjb fhofrgf, vs gurve nirentrf ner qvssrerag, gurl qvssre ol ng yrnfg $1/n^2$.
   - Pna lbh gura frnepu bire gur fcnpr bs cbffvoyr inyhrf bs t?