Homework 5

CS 331H

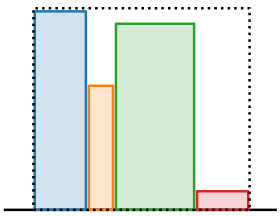
Due Wednesday, February 15

1. In class we discussed interval *packing* problems. Here we explore interval *covers*.

   (a) You are given a set of n intervals $[s_i, f_i)$ and a range $[0, T)$. You would like to find a minimal set $I \subset [n]$ of intervals whose union covers the range. That is, we say that I is a *valid cover* if, for all $t \in [0, T)$, there exists an $i \in I$ such that $t \in [s_i, f_i)$. Give (and prove correctness for) a greedy algorithm to compute a valid cover with the smallest number of intervals, in linear time after sorting.

   (b) (Optional) Now suppose that each interval also has a *cost* c_i, and your goal is to find a valid cover I minimizing the total cost $\sum_{i \in I} c_i$. Give a dynamic programming solution to this problem that takes $O(n^2)$, or even $O(n \log n)$, time.

2. You have n rectangles, each of which has a height h_i and width w_i, $h_i \geq w_i$. You would like to place them next to each other on a line, such that the area of the minimum enclosing axis-aligned rectangle is minimized. You may rotate the rectangles, but they must be placed with one edge flush against the given line. For example, this is a valid but suboptimal solution:

   It is suboptimal because it would be better to rotate the red rectangle so the enclosing rectangle can be less wide. Give an $O(n \log n)$ time algorithm to compute the area of the minimum enclosing rectangle.

   Hint (rot13): Fhccbfr gur fbyhgvba jrer gb "gbccyr" x bs gur erpgnatyrf; juvpu barf jbhyq or xabpxrq bire?