# Homework 9

CS 331H

Due Wednesday, March 29

1. You have $n$ power plants which, being fancy modern renewable energy designs, do not work all the time (they depend on sun, wind, tides, etc). Each power plant runs from a start time $s_i$ to a finish time $f_i$, and costs $c_i$ to run (you either run it the whole time or no time). You would like to find a set of power plants to run such that you have power over the entire interval $[0, T)$. What is the minimum cost achievable?

   You may suppose that the $s_i$ and $f_i$ are integers between 0 and $T = O(n)$.

   (a) Show that the answer corresponds to the shortest path on an appropriate graph, which can be solved in $O(n \log n)$ time using Dijkstra's algorithm. [Hint: have vertices corresponding to each integer time step.]

   (b) Now suppose that you can sell off extra power if you have more than one power plant running at a time. At each integer time step $[t, t+1)$, each power plant beyond the first that you run gives you value $v_t \geq 0$, decreasing your costs. Show how to modify the part (a) graph to handle this case. Your new graph is likely to have negative edge weights.

   For this part, and part (c), you may suppose that the cost $c_i$ to run a power plant is larger than the value of the electricity it produces, $\sum_{t=s_i}^{f_i-1} v_t$.

   (c) Show that Dijkstra's algorithm can still be used to find the solution to the part (b) graph in $O(n \log n)$ time, by constructing an appropriate potential function so the edge costs become nonnegative.

   (d) [Optional] Now solve the previous parts without assuming that the cost to run a power plant is larger than the value of the electricity it produces.

2. Go through the Jupyter notebook on the class website.