

$$v_i(\tilde{y}) = \log P(x_i | \tilde{y}) + \max_{\tilde{y}_{\text{prev}}} \left[\log P(\tilde{y} | \tilde{y}_{\text{prev}}) + v_{i-1}(\tilde{y}_{\text{prev}}) \right]$$

Today

- Constituency syntax: see slides
- PCFGs
- CKY algorithm

Context-free Grammars

{ N	T	S	R }
nonterminals	terminals	start symbol	rules

S, VP, NP,
etc.

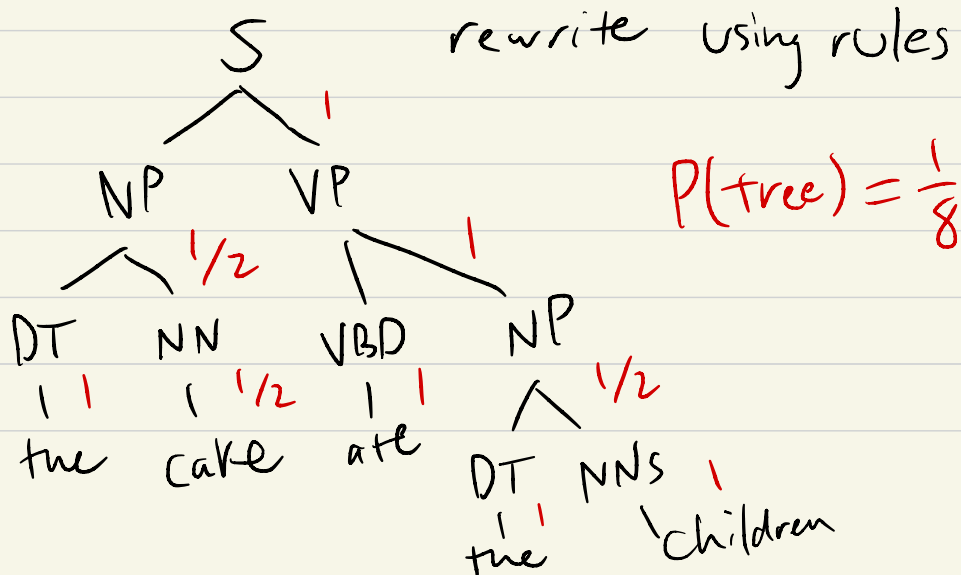
words

S

Rules

binary	probs	unary	
$S \rightarrow NP VP$	1	$DT \rightarrow the$	1
$VP \rightarrow VBD NP$	1	$NNS \rightarrow Children$	1
$NP \rightarrow DT NN$	1/2	$NN \rightarrow cake$	1/2
$NP \rightarrow DT NNS$	1/2	$NN \rightarrow spoon$	1/2
		$VBD \rightarrow ate$	1

CFGs define a set of trees



Probabilistic context-free grammars (PCFGs)

Each rule has a prob.

Probs. normalize per parent

$P(\text{rule} | \text{parent})$

Ex. $P(\text{rule} | NP) = \begin{cases} NP \rightarrow DT \ NN \ 1/2 \\ NP \rightarrow DT \ NNS \ 1/2 \end{cases}$

$P(\text{tree}) = \prod_{\text{rules} \in \text{tree}} P(\text{rule} | \text{parent})$

Building a parser

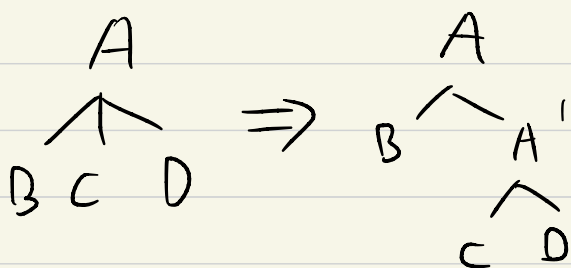
Input: treebank (sentences w/ labeled trees)

Output: grammar (PCFG)
(also need parsing algo.)

Steps

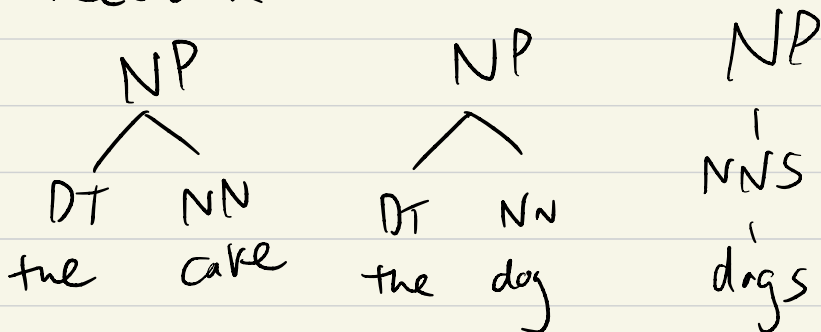
① Grammar preprocessing step

- Make trees have binary + unary rules only



② Read off grammar + compute probs. Count + normalize

Treebank:



$$P(\text{rule} | \text{NP}) = \begin{cases} \text{NP} \rightarrow \text{DT NN} & 2/3 \\ \text{NP} \rightarrow \text{NNS} & 1/3 \end{cases}$$

$$P(\text{word} | \text{NN}) = \begin{cases} \text{cake} & 1/2 \\ \text{dog} & 1/2 \end{cases}$$

$$P(\text{word} | \text{NNS}) = \{\text{dogs} \quad 1\}$$

Similar to HMM emissions

$$P(\text{word} | \text{tag})$$

③ CKY Inputs: PCFG, sentence \bar{x}

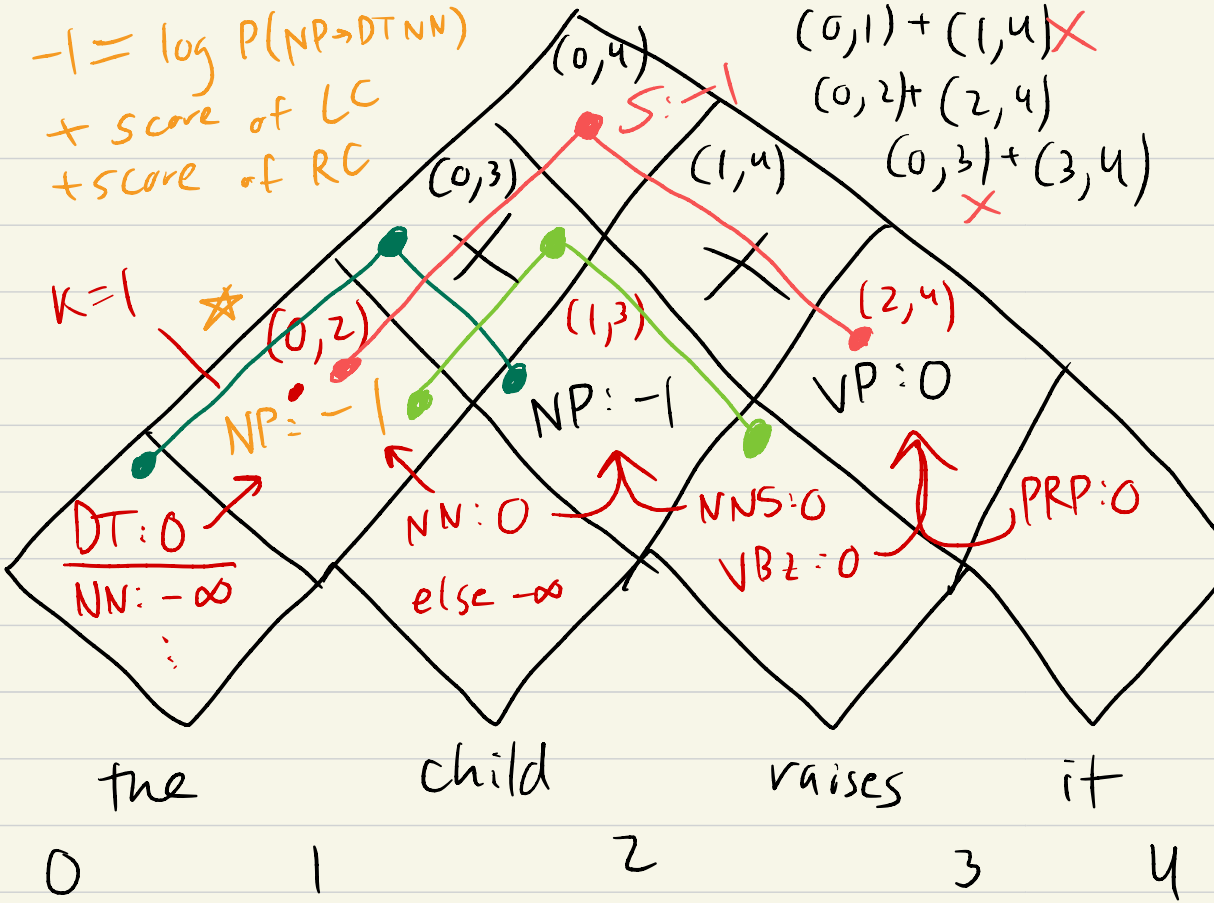
$$\text{Output: } \underset{T}{\text{argmax}} P(T | \bar{x})$$

most likely tree T for that
sentence

Dynamic program: track the best score for building a nonterminal over each span of the sentence ($\approx v_i(\tilde{y})$)

$T(i, j, X)$ = score (log prob) of the best way to build constituent X over span (ij) (word i up through word j)

We compute $\underset{T}{\text{argmax}} P(T, \bar{x})$



Grammar

- DT → the |
 - NN → child |
 - NNS → raises |
 - VBZ → raises |
 - PRP → it |
 - $S \rightarrow NP VP$ |
 - ★ $NP \rightarrow DT NN$ $1/2$
 - $NP \rightarrow NN NNS$ $1/2$
 - $VP \rightarrow VBZ PRP$ |
- Assume $\log(1/2) = -1$

CKY:

runtime: $O(n^3)$
 $O(n^3 \log n)$

Base case: $T(i, i+1, X) = \log P(w_i | X)$
(loop over all X)

Recursive case

$T(i, j, X)$ loop over all $i_s + j_s + X_s$
loop over split points k

$$= \max_{k: i < k < j} \max_{X \rightarrow X_1, X_2} \left[\log P(X \rightarrow X_1, X_2) \right. \\ \left. + T(i, k, X_1) + T(k, j, X_2) \right]$$

"transition"

Iterate upwards in terms of span length

