

CS 378 Computer Vision

Oct 22, 2009

Outline: Stereopsis and calibration

I. Computing correspondences for stereo

A. Epipolar geometry gives hard geometric constraint, but only reduces match for a point to be on a line. Other “soft” constraints are needed to assign corresponding points:

- Similarity – how well do the pixels match in a local region by the point?
 - o Normalized cross correlation
 - o Dense vs. sparse correspondences
 - o Effect of window size
- Uniqueness—up to one match for every point
- Disparity gradient—smooth surfaces would lead to smooth disparities
- Ordering—points on same surface imaged in order
 - o Enforcing ordering constraint with scanline stereo + dynamic programming

(Aside from point-based matching, or order-constrained DP, graph cuts can be used to minimize energy function expressing preference for well-matched local windows and smooth disparity labels.)

Sources of error when computing correspondences for stereo

B. Examples of applications leveraging stereo

- Segmentation with depth and spatial gradients
- Body tracking with fitting and depth
- Camera+microphone stereo system
- Virtual viewpoint video

II. Camera calibration

A. Estimating projection matrix

- Intrinsic and extrinsic parameters; we can relate them to image pixel coordinates and world point coordinates via perspective projection.
- Use a calibration object to collect correspondences.
- Set up equation to solve for projection matrix when we know the correspondences.

B. Weak calibration

- When all we have are corresponding image points (and no camera parameters), can solve for the *fundamental matrix*. This gives epipolar constraint, but unlike essential matrix does not require knowing camera parameters.
- Stereo pipeline with weak calibration: must estimate both fundamental matrix and correspondences. Start from correspondences, estimate geometry, refine.

Stereo matching Calibration

Thursday, Oct 22

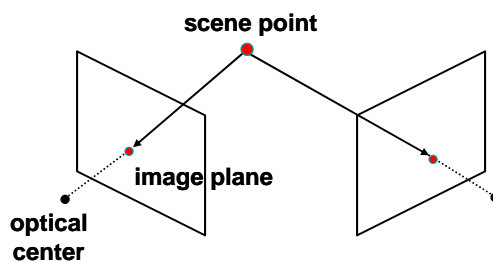
Kristen Grauman
UT-Austin

Today

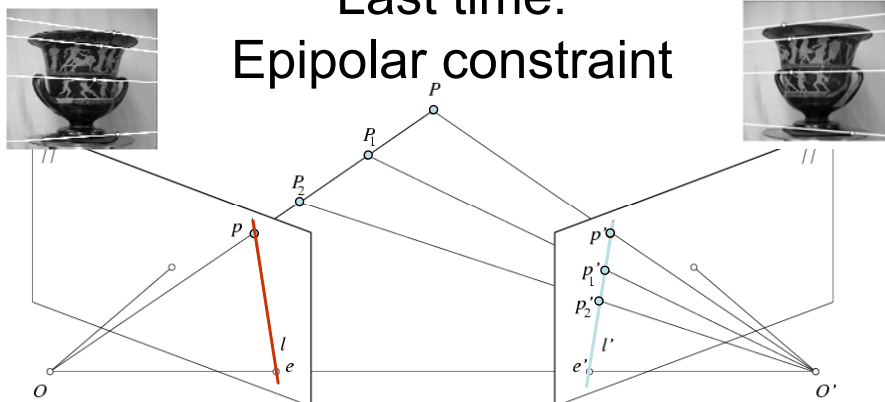
- Correspondences, matching for stereo
 - A few stereo applications
- Camera calibration

Last time: Estimating depth with stereo

- **Stereo**: shape from “motion” between two views
- We need to consider:
 - Info on camera pose (“calibration”)
 - Image point correspondences



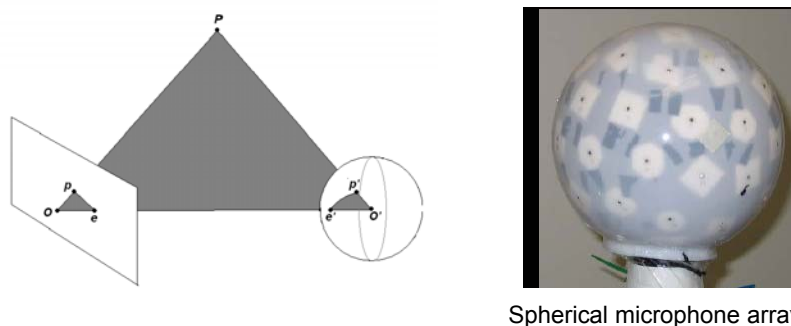
Last time: Epipolar constraint



- Potential matches for p have to lie on the corresponding epipolar line l' .
- Potential matches for p' have to lie on the corresponding epipolar line l .

Slide credit: M. Pollefeys

An audio camera & epipolar geometry



Adam O' Donovan, [Ramani Duraiswami](#) and [Jan Neumann](#)
 Microphone Arrays as Generalized Cameras for Integrated Audio
 Visual Processing, IEEE Conference on Computer Vision and
 Pattern Recognition (CVPR), Minneapolis, 2007

An audio camera & epipolar geometry

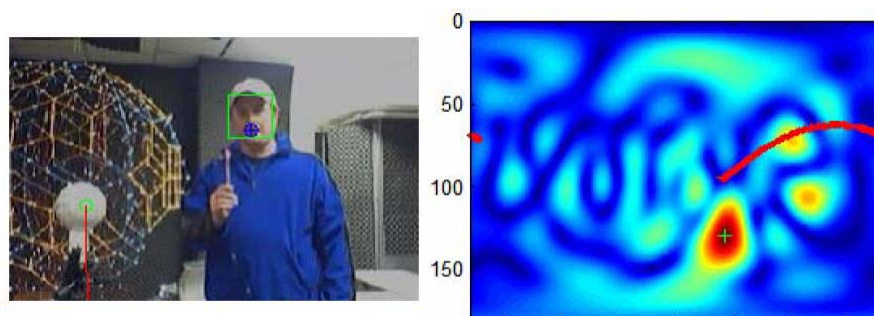
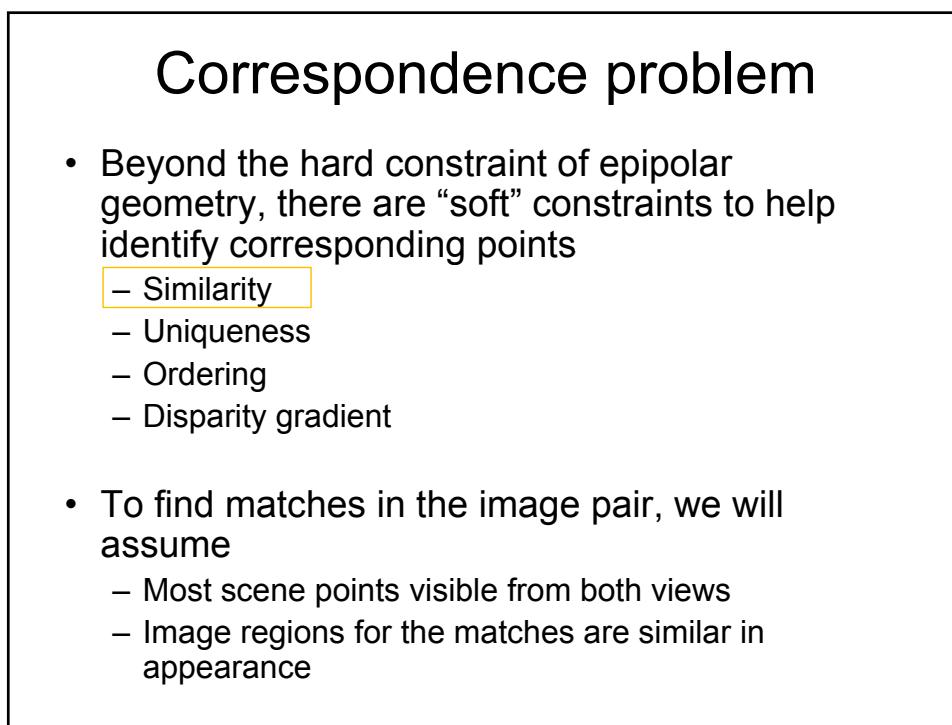
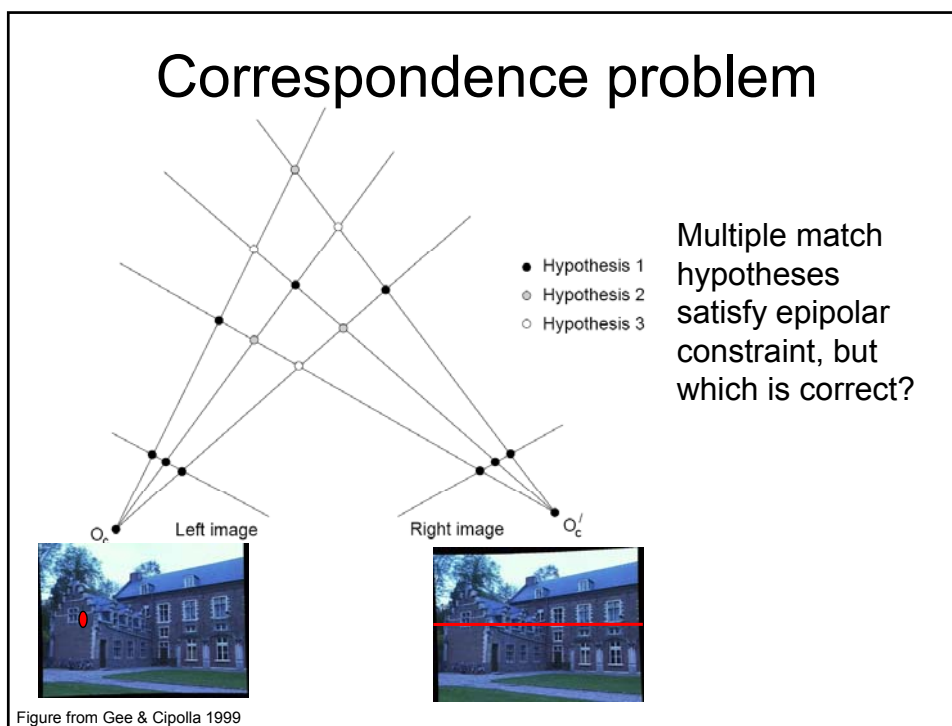


Figure 4. An example of the use of the system in speaker tracking with noise suppression. The bright red spot on the sound image (marked with a +) corresponds to the dominant source. The less dominant source however lies on the epipolar line in the sound image induced by the location of the mouth in the camera image, and this source is beamformed.



Correspondence problem



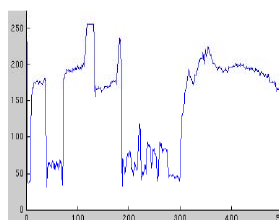
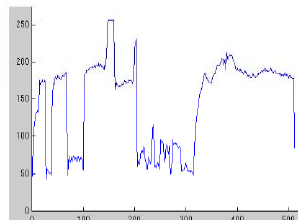
Parallel camera example: epipolar lines are corresponding image scanlines

Source: Andrew Zisserman

Correspondence problem



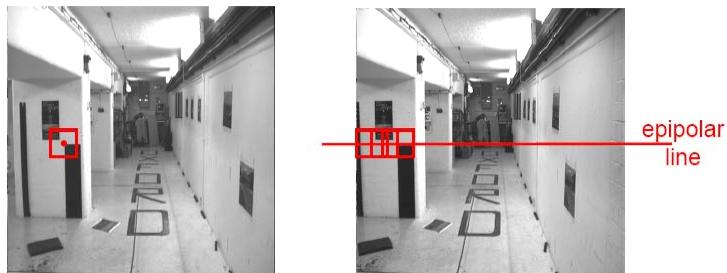
Intensity profiles



- Clear correspondence between intensities, but also noise and ambiguity

Source: Andrew Zisserman

Correspondence problem



Neighborhoods of corresponding points are similar in intensity patterns.

Source: Andrew Zisserman

Normalized cross correlation

subtract mean: $A \leftarrow A - \langle A \rangle, B \leftarrow B - \langle B \rangle$

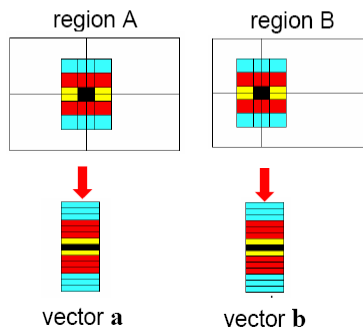
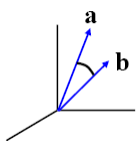
$$\text{NCC} = \frac{\sum_i \sum_j A(i, j) B(i, j)}{\sqrt{\sum_i \sum_j A(i, j)^2} \sqrt{\sum_i \sum_j B(i, j)^2}}$$

Write regions as vectors

$A \rightarrow \mathbf{a}, B \rightarrow \mathbf{b}$

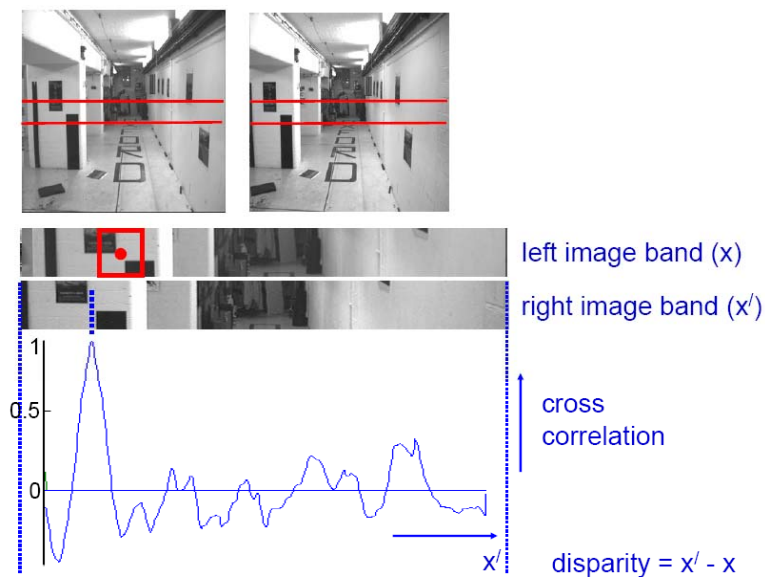
$$\text{NCC} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|}$$

$$-1 \leq \text{NCC} \leq 1$$



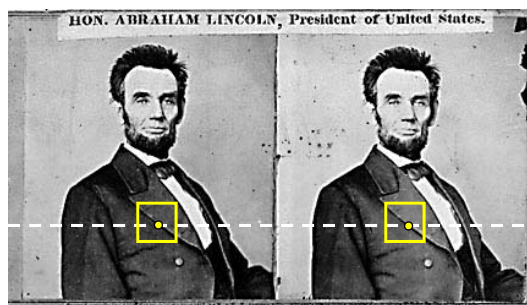
Source: Andrew Zisserman

Correlation-based window matching



Source: Andrew Zisserman

Dense correspondence search



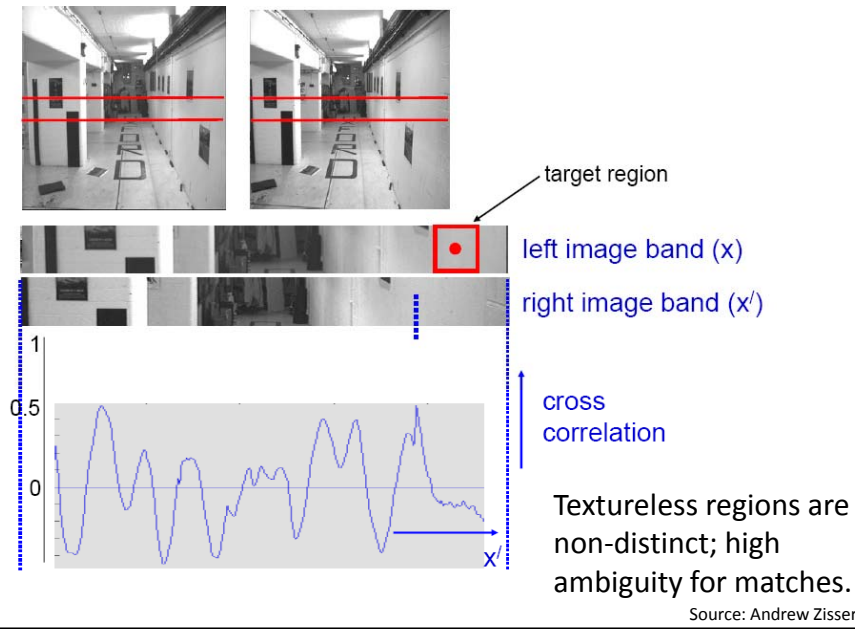
For each epipolar line

For each pixel / window in the left image

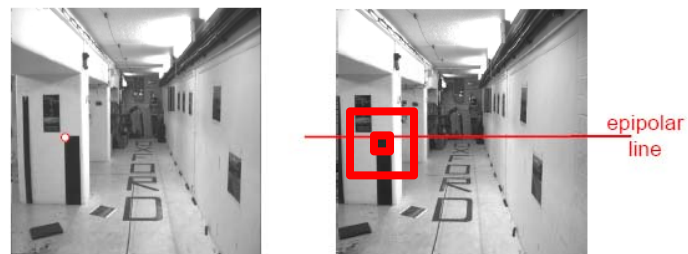
- compare with every pixel / window on same epipolar line in right image
- pick position with minimum match cost (e.g., SSD, correlation)

Adapted from Li Zhang

Textureless regions



Effect of window size



Source: Andrew Zisserman

Effect of window size



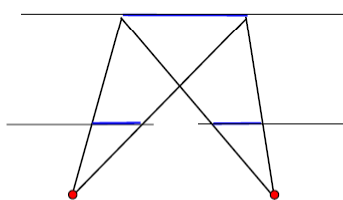
$W = 3$

$W = 20$

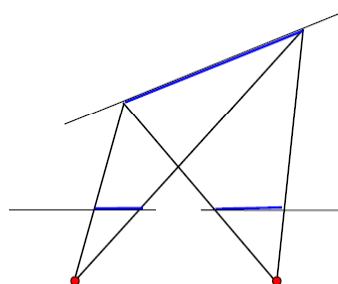
Want window large enough to have sufficient intensity variation, yet small enough to contain only pixels with about the same disparity.

Figures from Li Zhang

Foreshortening effects



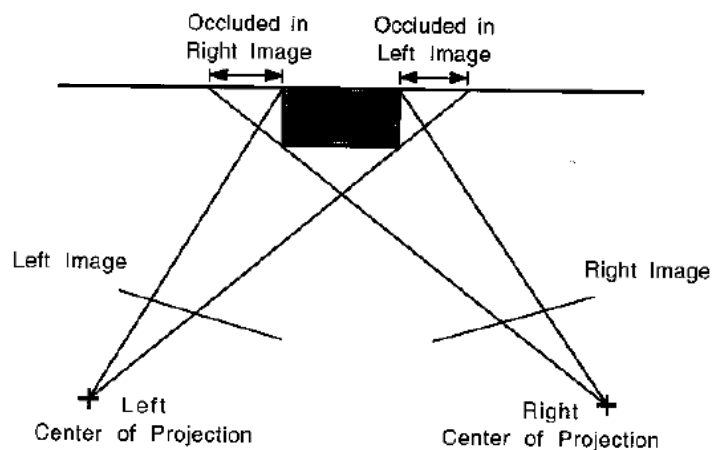
fronto-parallel surface
imaged length the same



slanting surface
imaged lengths differ

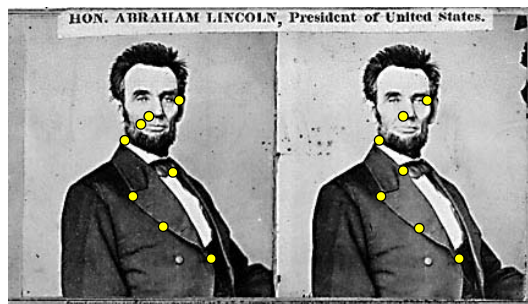
Source: Andrew Zisserman

Occlusion



Slide credit: David Kriegman

Sparse correspondence search



- Restrict search to sparse set of detected features
- Rather than pixel values (or lists of pixel values) use *feature descriptor* and an associated *feature distance*
- Still narrow search further by epipolar geometry

Correspondence problem

- Beyond the hard constraint of epipolar geometry, there are “soft” constraints to help identify corresponding points
 - Similarity
 - Uniqueness
 - Disparity gradient
 - Ordering

Uniqueness constraint

- Up to one match in right image for every point in left image

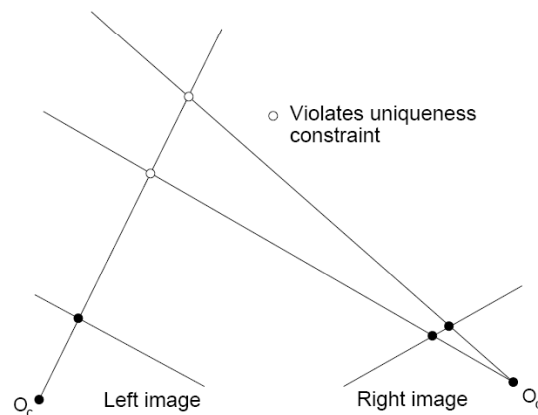
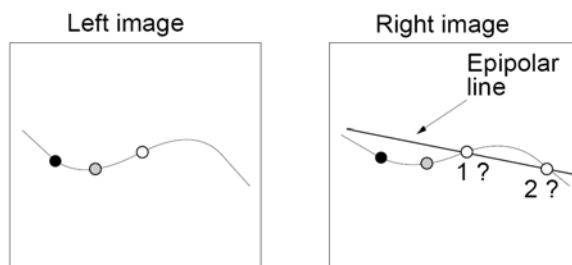


Figure from Gee & Cipolla 1999

Disparity gradient constraint

- Assume piecewise continuous surface, so want disparity estimates to be locally smooth



Given matches ● and ○, point ○ in the left image must match point 1 in the right image. Point 2 would exceed the disparity gradient limit.

Figure from Gee & Cipolla 1999

Ordering constraint

- Points on **same surface** (opaque object) will be in same order in both views

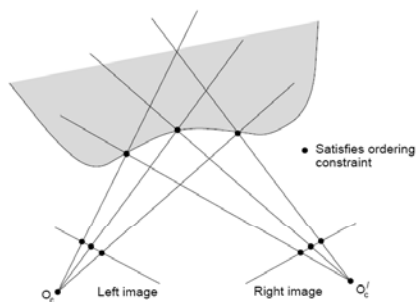
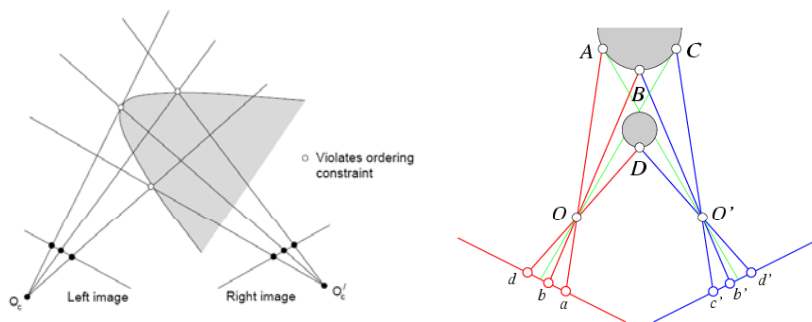


Figure from Gee & Cipolla 1999

Ordering constraint

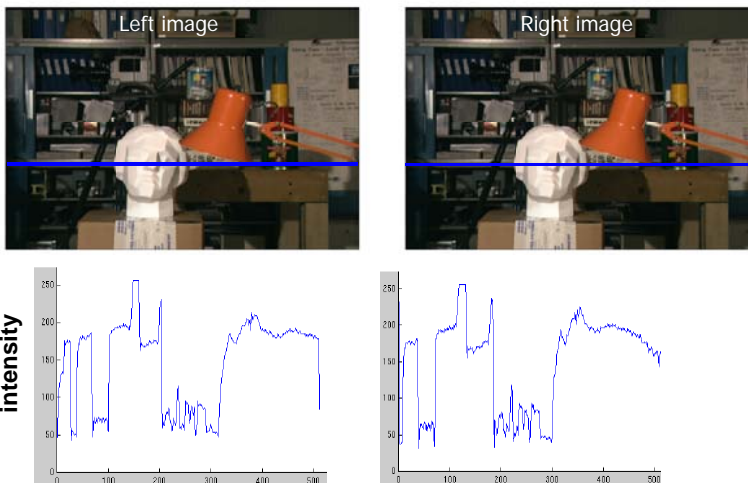
- Won't always hold, e.g. consider transparent object, or an occluding surface



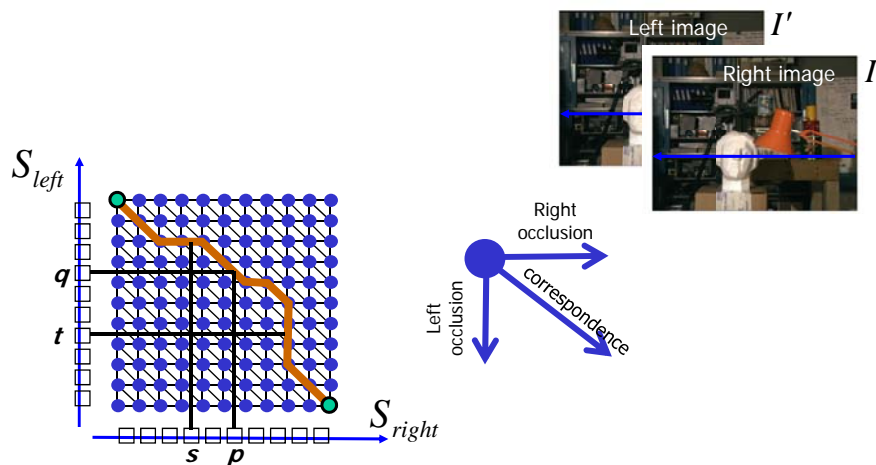
Figures from Forsyth & Ponce

Scanline stereo

- Try to coherently match pixels on the entire scanline
- Different scanlines are still optimized independently



“Shortest paths” for scan-line stereo

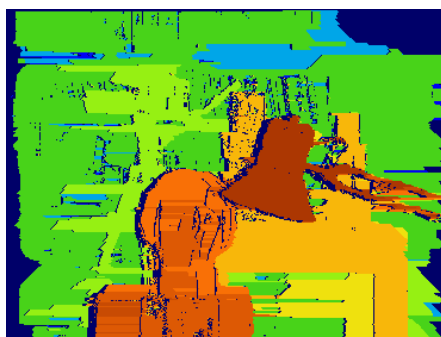


Can be implemented with dynamic programming
Ohta & Kanade '85, Cox et al. '96

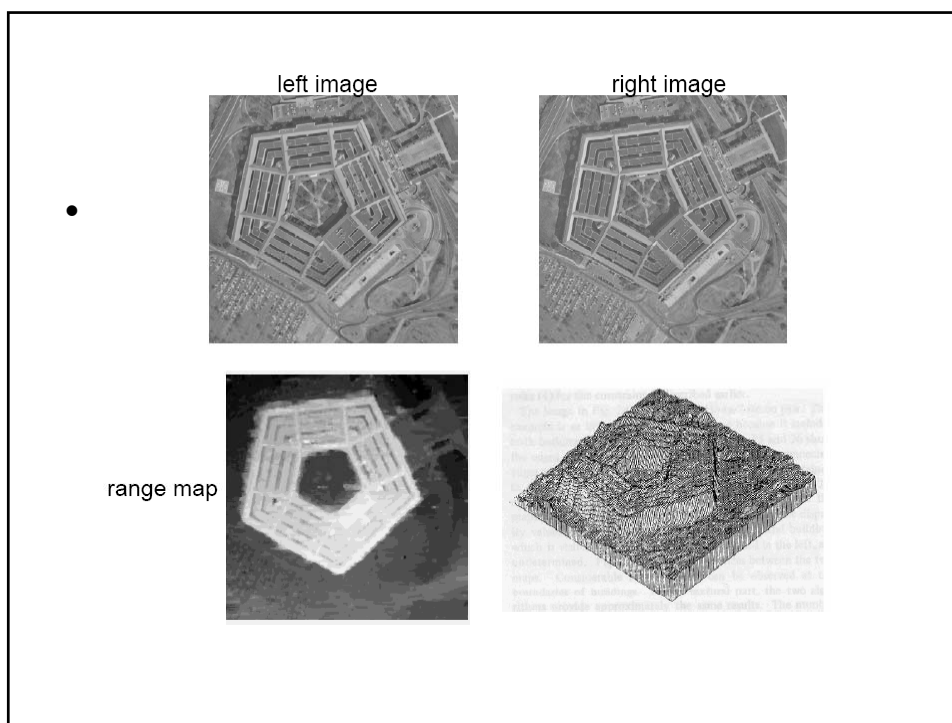
Slide credit: Y. Boykov

Coherent stereo on 2D grid

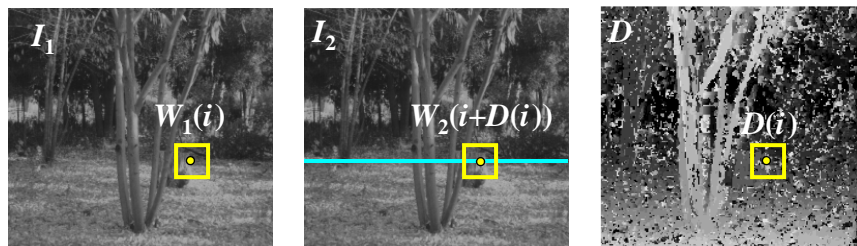
- Scanline stereo generates streaking artifacts



- Can't use dynamic programming to find spatially coherent disparities/ correspondences on a 2D grid



Stereo matching as energy minimization



$$E = \alpha E_{\text{data}}(I_1, I_2, D) + \beta E_{\text{smooth}}(D)$$

$$E_{\text{data}} = \sum_i (W_1(i) - W_2(i + D(i)))^2$$

$$E_{\text{smooth}} = \sum_{\text{neighbors } i, j} \rho(D(i) - D(j))$$

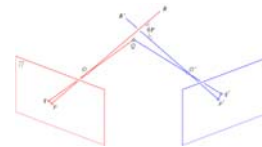
- Energy functions of this form can be minimized using *graph cuts*

Y. Boykov, O. Veksler, and R. Zabih, [Fast Approximate Energy Minimization via Graph Cuts](#), PAMI 2001

Source: Steve Seitz

Recap: stereo with calibrated cameras

- Image pair
- Detect some features
- Compute \mathbf{E} from given \mathbf{R} and \mathbf{T}
- Match features using the epipolar and other constraints
- Triangulate for 3d structure



Error sources

- Low-contrast ; textureless image regions
- Occlusions
- Camera calibration errors
- Violations of *brightness constancy* (e.g., specular reflections)
- Large motions

Today

- Correspondences, matching for stereo
 - A few stereo applications
- Camera calibration

Depth for segmentation

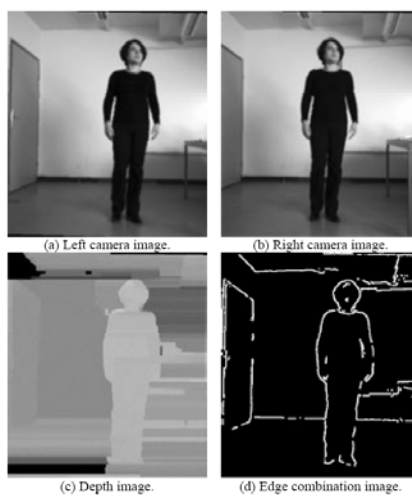
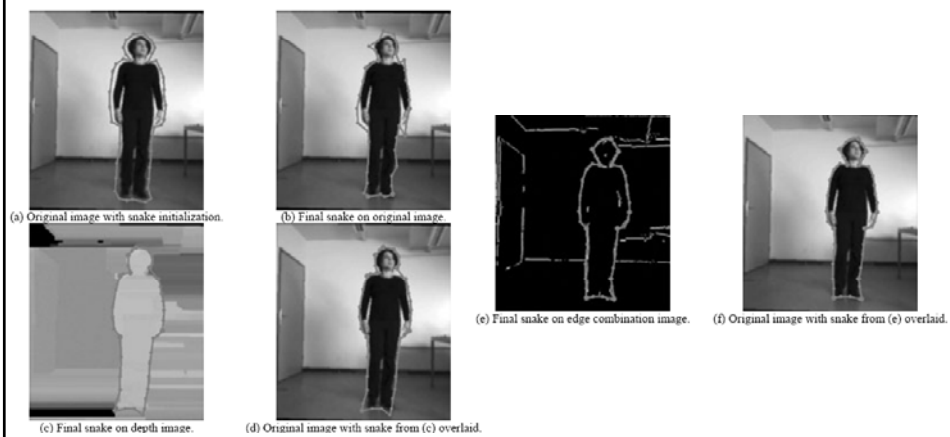


Figure 3 Stereo video frames with computed depth map and edge combination result.

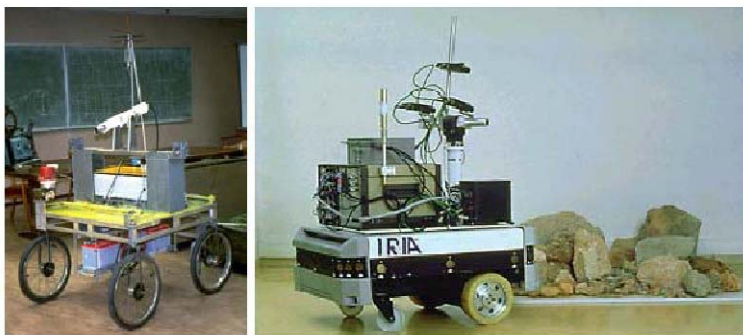
Danjela Markovic and Margrit Gelautz, Interactive Media Systems Group, Vienna University of Technology

Depth for segmentation



Danijela Markovic and Margrit Gelautz, Interactive Media Systems Group, Vienna University of Technology

Stereo in machine vision systems



Left : The Stanford cart sports a single camera moving in discrete increments along a straight line and providing multiple snapshots of outdoor scenes

Right : The INRIA mobile robot uses three cameras to map its environment

Forsyth & Ponce

Model-based body tracking, stereo input

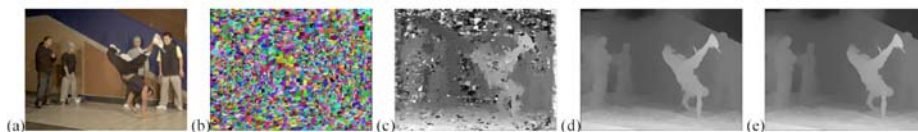


David Demirdjian, MIT Vision Interface Group
<http://people.csail.mit.edu/demirdji/movie/artic-tracker/turn-around.m1v>

First without beamforming

- Adam O' Donovan, [Ramani Duraiswami](#) and [Jan Neumann](#).
Microphone Arrays as Generalized Cameras for Integrated Audio
Visual Processing, IEEE Conference on Computer Vision and
Pattern Recognition (CVPR), Minneapolis, 2007

Virtual viewpoint video



(a) Figure 6: Sample results from stereo reconstruction stage: (a) input color image; (b) color-based segmentation; (c) initial disparity estimates $\hat{d}_{i,j}$; (d) refined disparity estimates; (e) smoothed disparity estimates $d_s(x)$.
 (d) A depth-matted object from earlier in the sequence is inserted into the video.

C. Zitnick et al, High-quality video view interpolation using a layered representation, SIGGRAPH 2004.

Virtual viewpoint video



<http://research.microsoft.com/IVM/VVV/>

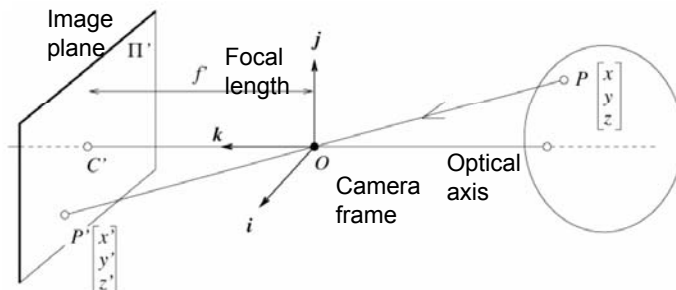
Uncalibrated case

- What if we don't know the camera parameters?

Today

- Correspondences, matching for stereo
 - A few stereo applications
- Camera calibration

Perspective projection



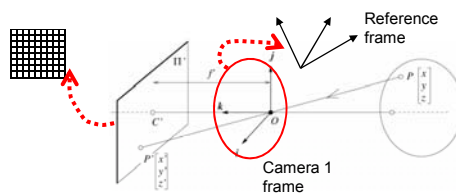
$$(x, y, z) \rightarrow \left(f \frac{x}{z}, f \frac{y}{z}\right)$$

Scene point \rightarrow Image coordinates

Thus far, in camera's reference frame only.

Camera parameters

- **Extrinsic:** location and orientation of camera frame with respect to reference frame
- **Intrinsic:** how to map pixel coordinates to image plane coordinates



Extrinsic camera parameters

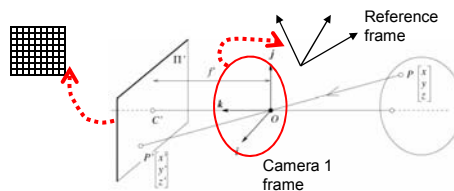
$$\mathbf{P}_c = \mathbf{R}(\mathbf{P}_w - \mathbf{T})$$

↑ Camera reference frame ↑ World reference frame

$$\mathbf{P}_c = (X, Y, Z)^T$$

Camera parameters

- Extrinsic: location and orientation of camera frame with respect to reference frame
- **Intrinsic: how to map pixel coordinates to image plane coordinates**

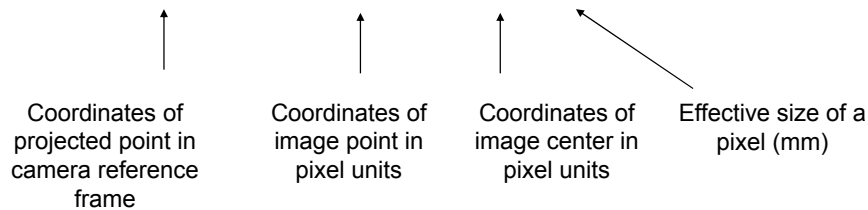


Intrinsic camera parameters

- Ignoring any geometric distortions from optics, we can describe them by:

$$x = -(x_{im} - o_x) s_x$$

$$y = -(y_{im} - o_y) s_y$$



Camera parameters

- We know that in terms of camera reference frame:

$$x = f \frac{X}{Z} \quad y = f \frac{Y}{Z} \quad \text{and} \quad \begin{matrix} \mathbf{P}_c = \mathbf{R}(\mathbf{P}_w - \mathbf{T}) \\ \mathbf{P}_c = (X, Y, Z)^T \end{matrix}$$

- Substituting previous eqns describing intrinsic and extrinsic parameters, can relate *pixels coordinates* to *world points*:

$$-(x_{im} - o_x) s_x = f \frac{\mathbf{R}_1 \cdot (\mathbf{P}_w - \mathbf{T})}{\mathbf{R}_3 \cdot (\mathbf{P}_w - \mathbf{T})}$$

$$-(y_{im} - o_y) s_y = f \frac{\mathbf{R}_2 \cdot (\mathbf{P}_w - \mathbf{T})}{\mathbf{R}_3 \cdot (\mathbf{P}_w - \mathbf{T})}$$

\mathbf{R}_i = Row i of rotation matrix

Projection matrix

- This can be rewritten as a matrix product using homogeneous coordinates:

$$\begin{bmatrix} wx_{im} \\ wy_{im} \\ w \end{bmatrix} = \underbrace{\mathbf{M}_{int} \mathbf{M}_{ext}}_{\mathbf{M}} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

where:

$$\mathbf{M}_{int} = \begin{bmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{M}_{ext} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & -\mathbf{R}_1^T \mathbf{T} \\ r_{21} & r_{22} & r_{23} & -\mathbf{R}_2^T \mathbf{T} \\ r_{31} & r_{32} & r_{33} & -\mathbf{R}_3^T \mathbf{T} \end{bmatrix}$$

Projection matrix

- This can be rewritten as a matrix product using homogeneous coordinates:

$$\begin{bmatrix} wx_{im} \\ wy_{im} \\ w \end{bmatrix} = \mathbf{M} \mathbf{P}_w$$

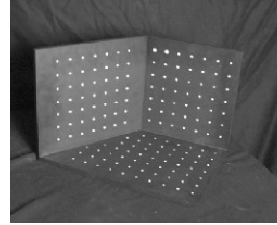
where:

$$\mathbf{M}_{int} = \begin{bmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{M}_{ext} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & -\mathbf{R}_1^T \mathbf{T} \\ r_{21} & r_{22} & r_{23} & -\mathbf{R}_2^T \mathbf{T} \\ r_{31} & r_{32} & r_{33} & -\mathbf{R}_3^T \mathbf{T} \end{bmatrix}$$

Calibrating a camera

- Compute intrinsic and extrinsic parameters using observed camera data



Main idea

- Place “calibration object” with known geometry in the scene
- Get correspondences
- Solve for mapping from scene to image: estimate $\mathbf{M} = \mathbf{M}_{\text{int}} \mathbf{M}_{\text{ext}}$



The Opti-CAL Calibration Target Image

Estimating the projection matrix

$$\begin{bmatrix} wx_{im} \\ wy_{im} \\ w \end{bmatrix} = \mathbf{M} \mathbf{P}_w$$

For a given feature point

$$x_{im} = \frac{\mathbf{M}_1 \cdot \mathbf{P}_w}{\mathbf{M}_3 \cdot \mathbf{P}_w} \longrightarrow x_{im} (\mathbf{M}_3 \cdot \mathbf{P}_w) = \mathbf{M}_1 \cdot \mathbf{P}_w$$

$$y_{im} = \frac{\mathbf{M}_2 \cdot \mathbf{P}_w}{\mathbf{M}_3 \cdot \mathbf{P}_w}$$

Estimating the projection matrix

$$\begin{bmatrix} wx_{im} \\ wy_{im} \\ w \end{bmatrix} = \mathbf{M} \mathbf{P}_w$$

For a given feature point

$$x_{im} = \frac{\mathbf{M}_1 \cdot \mathbf{P}_w}{\mathbf{M}_3 \cdot \mathbf{P}_w} \longrightarrow 0 = \mathbf{M}_1 \cdot \mathbf{P}_w - x_{im} (\mathbf{M}_3 \cdot \mathbf{P}_w)$$

$$y_{im} = \frac{\mathbf{M}_2 \cdot \mathbf{P}_w}{\mathbf{M}_3 \cdot \mathbf{P}_w}$$

Estimating the projection matrix

$$\begin{bmatrix} wx_{im} \\ wy_{im} \\ w \end{bmatrix} = \mathbf{M} \mathbf{P}_w$$

For a given feature point

$$x_{im} = \frac{\mathbf{M}_1 \cdot \mathbf{P}_w}{\mathbf{M}_3 \cdot \mathbf{P}_w} \longrightarrow 0 = (\mathbf{M}_1 - x_{im} \mathbf{M}_3) \cdot \mathbf{P}_w$$

$$y_{im} = \frac{\mathbf{M}_2 \cdot \mathbf{P}_w}{\mathbf{M}_3 \cdot \mathbf{P}_w} \longrightarrow 0 = (\mathbf{M}_2 - y_{im} \mathbf{M}_3) \cdot \mathbf{P}_w$$

Estimating the projection matrix

$$0 = (\mathbf{M}_1 - x_{im} \mathbf{M}_3) \cdot \mathbf{P}_w$$

$$0 = (\mathbf{M}_2 - y_{im} \mathbf{M}_3) \cdot \mathbf{P}_w$$

$$\begin{pmatrix} X_w & Y_w & Z_w & 1 & 0 & 0 & 0 & 0 & -x_{im} X_w & -x_{im} Y_w & -x_{im} Z_w & -x_{im} \\ 0 & 0 & 0 & 0 & X_w & Y_w & Z_w & 1 & -y_{im} X_w & -y_{im} Y_w & -y_{im} Z_w & -y_{im} \end{pmatrix} \begin{pmatrix} m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{24} \\ m_{31} \\ m_{32} \\ m_{33} \\ m_{34} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Estimating the projection matrix

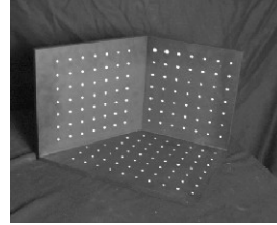
This is true for every feature point, so we can stack up n observed image features and their associated 3d points in single equation:

$$\begin{pmatrix} X_w^{(1)} & Y_w^{(1)} & Z_w^{(1)} & 1 & 0 & 0 & 0 & 0 & -x_{im}^{(1)} X_w^{(1)} & -x_{im}^{(1)} Y_w^{(1)} & -x_{im}^{(1)} Z_w^{(1)} & -x_{im}^{(1)} \\ 0 & 0 & 0 & 0 & X_w^{(1)} & Y_w^{(1)} & Z_w^{(1)} & 1 & -y_{im}^{(1)} X_w^{(1)} & -y_{im}^{(1)} Y_w^{(1)} & -y_{im}^{(1)} Z_w^{(1)} & -y_{im}^{(1)} \end{pmatrix} \begin{pmatrix} m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{24} \\ m_{31} \\ m_{32} \\ m_{33} \\ m_{34} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Solve for m_{ij} 's (the calibration information)
[F&P Section 3.1]

Calibrating a camera

- Compute intrinsic and extrinsic parameters using observed camera data



Main idea

- Place “calibration object” with known geometry in the scene
- Get correspondences
- Solve for mapping from scene to image: estimate $\mathbf{M} = \mathbf{M}_{\text{int}} \mathbf{M}_{\text{ext}}$



The Opti-CAL Calibration Target Image

When would we calibrate this way?

- Makes sense when geometry of system is not going to change over time
- ...When would it change?

Weak calibration

- Want to estimate world geometry without requiring calibrated cameras
 - Archival videos
 - Photos from multiple unrelated users
 - Dynamic camera system
- Main idea:
 - Estimate epipolar geometry from a (redundant) set of point correspondences between two uncalibrated cameras

Uncalibrated case

For a given camera: $\bar{\mathbf{p}} = \mathbf{M}_{\text{int}} \mathbf{p}$ ← Camera coordinates

So, for two cameras (left and right):

$$\begin{array}{l}
 \text{Camera coordinates} \swarrow \searrow \\
 \mathbf{p}_{(left)} = \mathbf{M}_{left,int}^{-1} \bar{\mathbf{p}}_{(left)} \quad \leftarrow \text{Image pixel coordinates} \\
 \mathbf{p}_{(right)} = \mathbf{M}_{right,int}^{-1} \bar{\mathbf{p}}_{(right)} \quad \leftarrow \text{Image pixel coordinates} \\
 \underbrace{\hspace{10em}} \\
 \text{Internal calibration matrices, one per camera}
 \end{array}$$

$$\mathbf{p}_{(left)} = \mathbf{M}_{left,int}^{-1} \bar{\mathbf{p}}_{(left)}$$

$$\mathbf{p}_{(right)} = \mathbf{M}_{right,int}^{-1} \bar{\mathbf{p}}_{(right)}$$

Uncalibrated case:
fundamental matrix

$$\mathbf{p}_{(right)}^T \mathbf{E} \mathbf{p}_{(left)} = 0$$

From before, the
essential matrix \mathbf{E} .

$$\left(\mathbf{M}_{right,int}^{-1} \bar{\mathbf{p}}_{right} \right)^T \mathbf{E} \left(\mathbf{M}_{left,int}^{-1} \bar{\mathbf{p}}_{left} \right) = 0$$

$$\bar{\mathbf{p}}_{right}^T \left(\mathbf{M}_{right,int}^{-T} \mathbf{E} \mathbf{M}_{left,int}^{-1} \right) \bar{\mathbf{p}}_{left} = 0$$

$$\bar{\mathbf{p}}_{right}^T \mathbf{F} \bar{\mathbf{p}}_{left} = 0$$

↑
Fundamental matrix

Fundamental matrix

- Relates pixel coordinates in the two views
- More general form than essential matrix: we remove need to know intrinsic parameters
- If we estimate fundamental matrix from correspondences in *pixel coordinates*, can reconstruct epipolar geometry without intrinsic or extrinsic parameters

Computing F from correspondences

$$\mathbf{F} = \left(\mathbf{M}_{right,int}^{-T} \mathbf{E} \mathbf{M}_{left,int}^{-1} \right)$$

$$\bar{\mathbf{p}}_{right}^T \mathbf{F} \bar{\mathbf{p}}_{left} = 0$$

- Cameras are uncalibrated: we don't know \mathbf{E} or left or right \mathbf{M}_{int} matrices
- Estimate \mathbf{F} from 8+ point correspondences.

Computing F from correspondences

Each point
correspondence
generates one
constraint on \mathbf{F}

$$\bar{\mathbf{p}}_{right}^T \mathbf{F} \bar{\mathbf{p}}_{left} = 0$$

$$\begin{bmatrix} u' & v' & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = 0$$

Collect n of these
constraints

$$\begin{bmatrix} u'_1 u_1 & u'_1 v_1 & u'_1 & v'_1 u_1 & v'_1 v_1 & v'_1 & u_1 & v_1 & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = \mathbf{0}$$

Solve for \mathbf{f} , vector of parameters.

Stereo pipeline with weak calibration

- So, where to start with uncalibrated cameras?
 - Need to find fundamental matrix F **and** the correspondences (pairs of points $(u',v') \leftrightarrow (u,v)$).

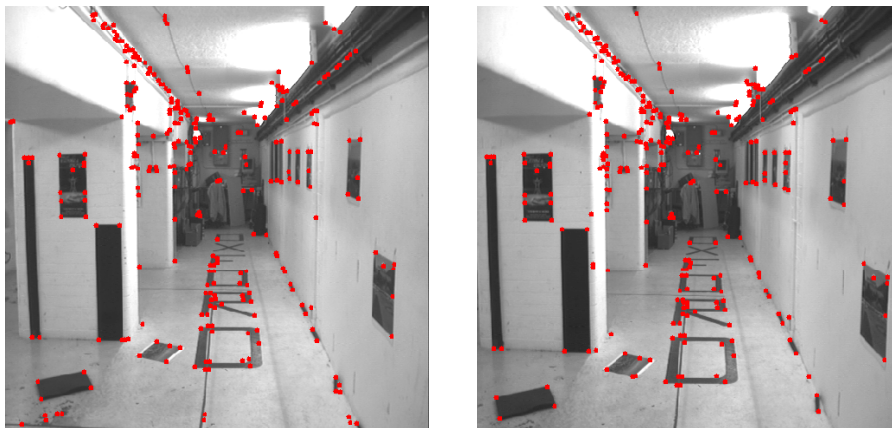


- 1) Find interest points in image (more on this later)
- 2) Compute correspondences
- 3) Compute epipolar geometry
- 4) Refine

Example from Andrew Zisserman

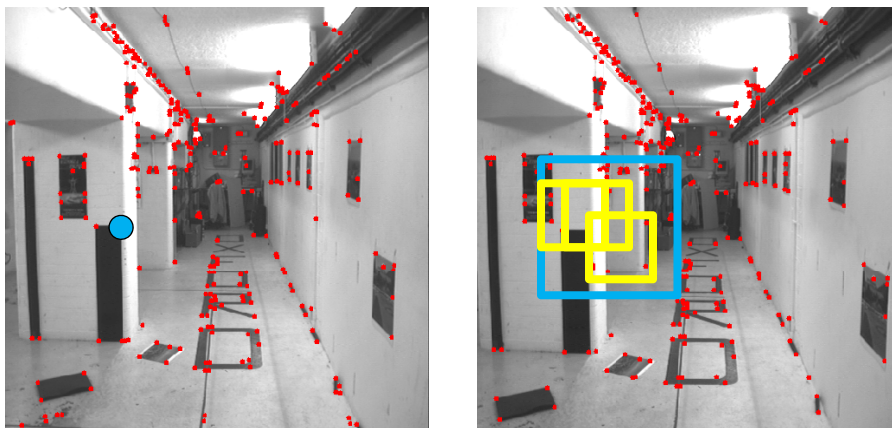
Stereo pipeline with weak calibration

- 1) Find interest points (next week)

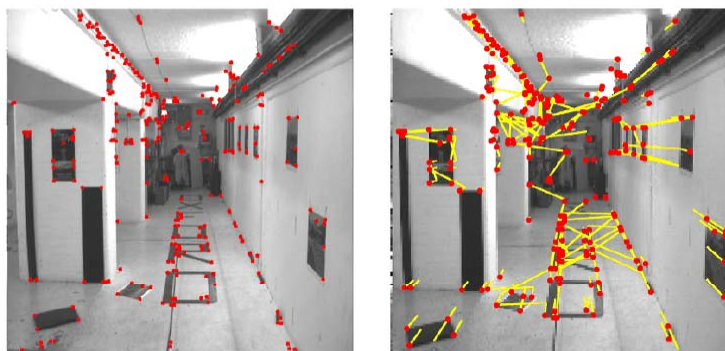


Stereo pipeline with weak calibration

2) Match points only using proximity



Putative matches based on correlation search



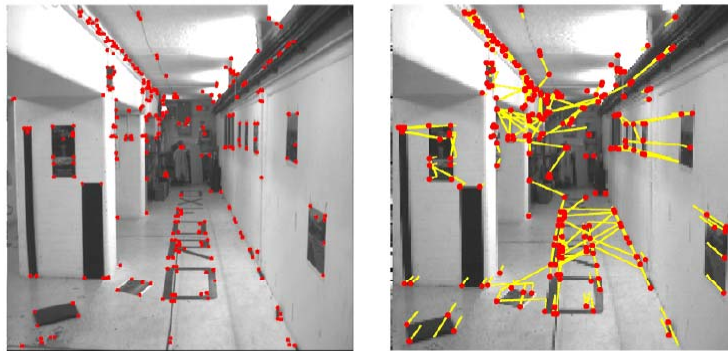
- Many wrong matches (10-50%), but enough to compute F

RANSAC for robust estimation of the fundamental matrix

- Select random sample of correspondences
- Compute F using them
 - This determines epipolar constraint
- Evaluate amount of support – inliers within threshold distance of epipolar line
- Choose F with most support (inliers)



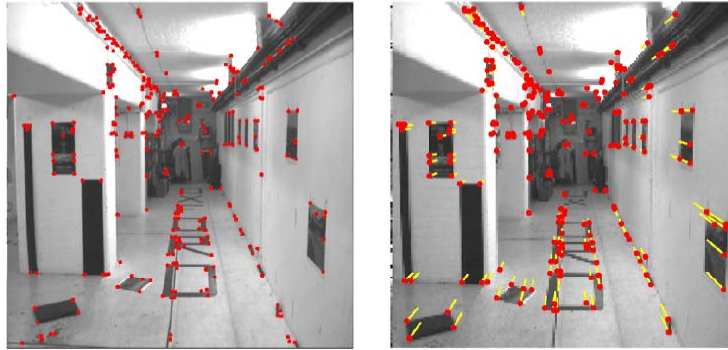
Putative matches based on correlation search



- Many wrong matches (10-50%), but enough to compute F

Pruned matches

- Correspondences consistent with epipolar geometry



- Resulting epipolar geometry



Next:

- Tuesday: local invariant features
 - How to find interest points?
 - How to describe local neighborhoods more robustly than with a list of pixel intensities?

