Fitting: Voting and the Hough Transform

Tuesday, Sept 22
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Histograms in Matlab

- a = A(:);
  % reshapes matrix A into vector, columns first
- H = hist(A(:), 10);
  % takes a histogram from the A's values, into 10 uniformly sized bins
- H = histc(A(:), [1:N]);
  % counts values within the bins having specified edges

Last time: segmentation

- Segmentation to find object boundaries or mid-level regions, tokens.
- Bottom-up segmentation via clustering
  - General choices -- features, affinity functions, and clustering algorithms
- Grouping also useful for quantization, can create new feature summaries
  - Texton histograms for texture within local region
- Example clustering methods
  - K-means
  - Graph cuts, normalized cuts
  - Tradeoffs

Review: graph-based clustering

- Assuming we use a fully connected graph, what is the time complexity of computing the affinities for a graph cuts-based segmentation?
- Example affinity measure:

$$w_{ij} = e^{\frac{-\|\boldsymbol{F}(i)-\boldsymbol{F}(j)\|_2^2}{\sigma_I}} * \begin{cases} e^{\frac{-\|\boldsymbol{X}(i)-\boldsymbol{X}(j)\|_2^2}{\sigma_X}} & \text{if } \|\boldsymbol{X}(i)-\boldsymbol{X}(j)\|_2 < r \\ 0 & \text{otherwise,} \end{cases}$$

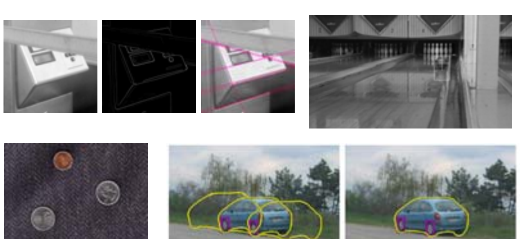
X(i) is position of node i

F(i) is a feature vector for node i based on color, texture, etc.

This affinity measure limits connections to spatially close pixels.

Now: Fitting

- Want to associate a model with observed features



[Fig from Marszalek & Schmid, 2007]

For example, the model could be a line, a circle, or an arbitrary shape.

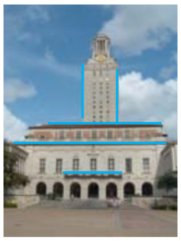
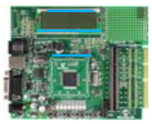
Fitting

- Choose a parametric model to represent a set of features
- Membership criterion is not local
  - Can't tell whether a point belongs to a given model just by looking at that point
- Three main questions:
  - What model represents this set of features best?
  - Which of several model instances gets which feature?
  - How many model instances are there?
- Computational complexity is important
  - It is infeasible to examine every possible set of parameters and every possible combination of features

Source: L. Lazebnik

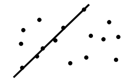
Example: Line fitting

- Why fit lines?
  Many objects characterized by presence of straight lines


- Wait, why aren't we done just by running edge detection?

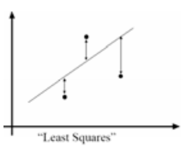
Difficulty of line fitting



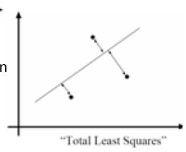
- **Extra** edge points (clutter), multiple models:
  - which points go with which line, if any?
- Only some parts of each line detected, and some parts are **missing:**
  - how to find a line that bridges missing evidence?
- **Noise** in measured edge points, orientations:
  - how to detect true underlying parameters?

Fitting lines

- Given points that belong to a line, what is the line?



Assuming all the points that belong to a line are known, can solve for line parameters that yield minimal error.



Forsyth & Ponce 15.2.1

Fitting lines

- Given points that belong to a line, what is the line?
- How many lines are there?
- Which points belong to which lines?

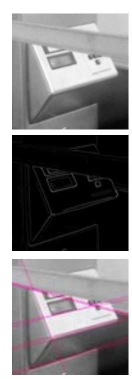
Voting

- It's not feasible to check all combinations of features by fitting a model to each possible subset.
- **Voting** is a general technique where we let the features vote for all models that are compatible with it.
  - Cycle through features, cast votes for model parameters.
  - Look for model parameters that receive a lot of votes.
- Noise & clutter features will cast votes too, *but* typically their votes should be inconsistent with the majority of "good" features.
- Ok if some features not observed, as model can span multiple fragments.

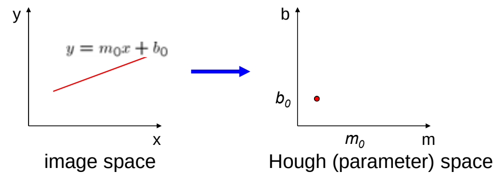
Fitting lines

- Given points that belong to a line, what is the line?
- How many lines are there?
- Which points belong to which lines?
- **Hough Transform** is a voting technique that can be used to answer all of these questions.

  Main idea:
  1. Record vote for each possible line on which each edge point lies.
  2. Look for lines that get many votes.



Finding lines in an image: Hough space

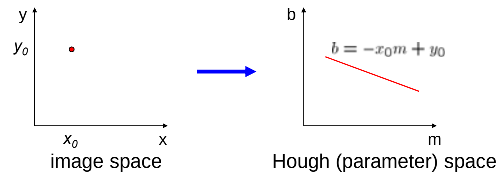


Connection between image (x,y) and Hough (m,b) spaces

- A line in the image corresponds to a point in Hough space
- To go from image space to Hough space:
  - given a set of points (x,y), find all (m,b) such that y = mx + b

Slide credit: Steve Seitz

Finding lines in an image: Hough space

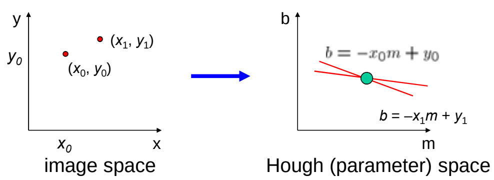


Connection between image (x,y) and Hough (m,b) spaces

- A line in the image corresponds to a point in Hough space
- To go from image space to Hough space:
  - given a set of points (x,y), find all (m,b) such that y = mx + b
- What does a point (x_0, y_0) in the image space map to?
  - Answer: the solutions of $b = -x_0 m + y_0$
  - this is a line in Hough space

Slide credit: Steve Seitz

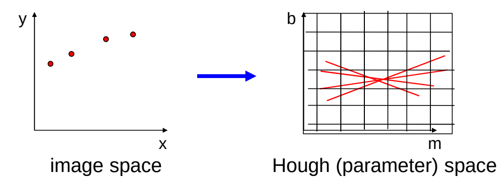
Finding lines in an image: Hough space



What are the line parameters for the line that contains both (x_0, y_0) and (x_1, y_1)?

- It is the intersection of the lines $b = -x_0 m + y_0$ and $b = -x_1 m + y_1$

Finding lines in an image: Hough algorithm

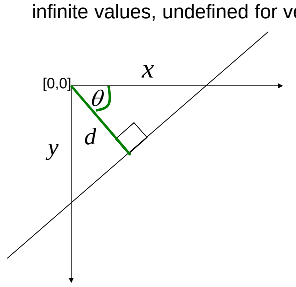


How can we use this to find the most likely parameters (m,b) for the most prominent line in the image space?

- Let each edge point in image space *vote* for a set of possible parameters in Hough space
- Accumulate votes in discrete set of bins; parameters with the most votes indicate line in image space.

Polar representation for lines

Issues with usual (*m*,*b*) parameter space: can take on infinite values, undefined for vertical lines.



d : perpendicular distance from line to origin

θ: angle the perpendicular makes with the x-axis

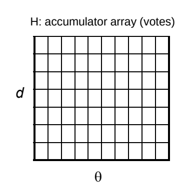
$$x\cos\theta - y\sin\theta = d$$

Point in image space → sinusoid segment in Hough space

Hough transform algorithm

Using the polar parameterization:

$$x\cos\theta - y\sin\theta = d$$



Basic Hough transform algorithm

1. Initialize H[d, θ]=0
2. for each edge point I[x,y] in the image
   for θ = 0 to 180 // some quantization
   $d = x\cos\theta - y\sin\theta$
   H[d, θ] += 1
3. Find the value(s) of (d, θ) where H[d, θ] is maximum
4. The detected line in the image is given by $d = x\cos\theta - y\sin\theta$

Hough line demo

Time complexity (in terms of number of votes per pt)?

Source: Steve Seitz

Example: Hough transform for straight lines

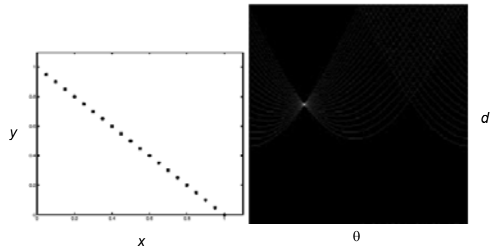


Image space edge coordinates

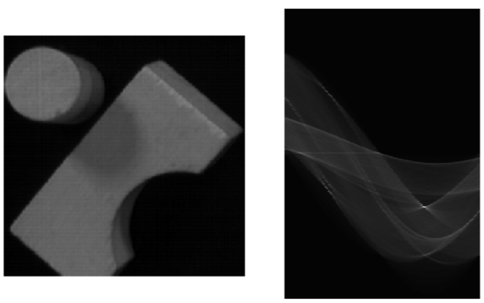
Votes

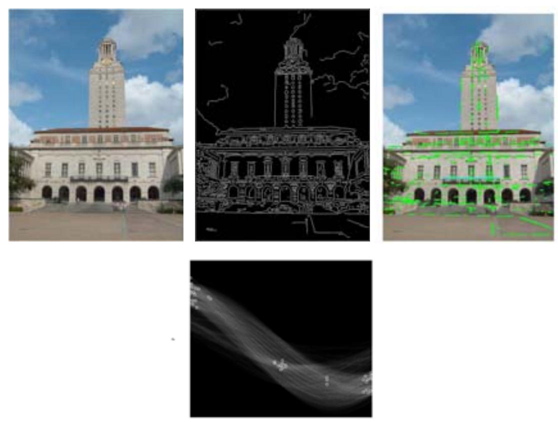
Bright value = high vote count
Black = no votes

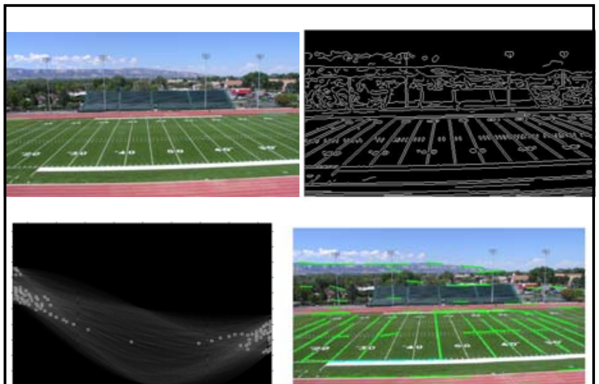
Example: Hough transform for straight lines

Square :

Example: Hough transform for straight lines







Showing longest segments found

Impact of noise on Hough

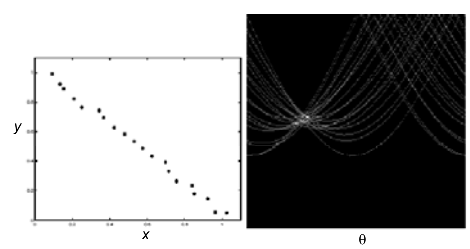


Image space edge coordinates

Votes

What difficulty does this present for an implementation?

Impact of noise on Hough

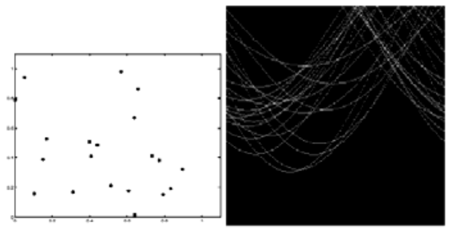


Image space edge coordinates **Votes**

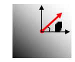
Here, everything appears to be “noise”, or random edge points, but we still see peaks in the vote space.

Extensions

Extension 1: Use the image gradient

1. same
2. for each edge point I[x,y] in the image
   θ = gradient at (x,y)
   $d = x\cos\theta - y\sin\theta$
   H[d, θ] += 1
3. same
4. same

(Reduces degrees of freedom)



$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$

Extensions

Extension 1: Use the image gradient

1. same
2. for each edge point I[x,y] in the image
   compute unique (d, θ) based on image gradient at (x,y)
   H[d, θ] += 1
3. same
4. same

(Reduces degrees of freedom)

Extension 2
- give more votes for stronger edges (use magnitude of gradient)

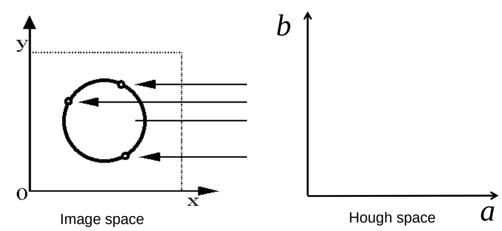
Extension 3
- change the sampling of (d, θ) to give more/less resolution

Extension 4
- The same procedure can be used with circles, squares, or any other shape…

Source: Steve Seitz

Hough transform for circles

- Circle: center (a,b) and radius r
  $$(x_i - a)^2 + (y_i - b)^2 = r^2$$
- For a fixed radius r, unknown gradient direction



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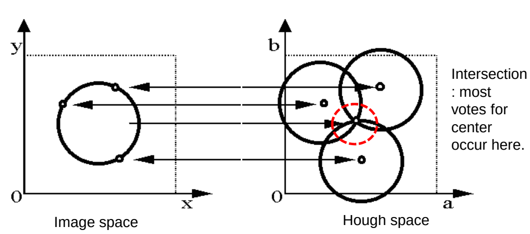

Hough transform for circles

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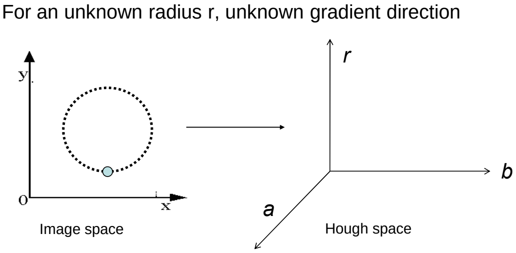

Hough transform for circles

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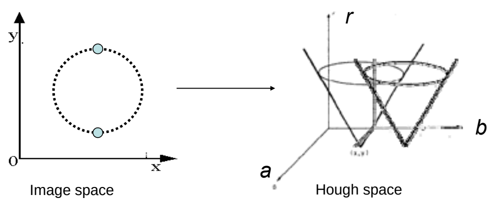
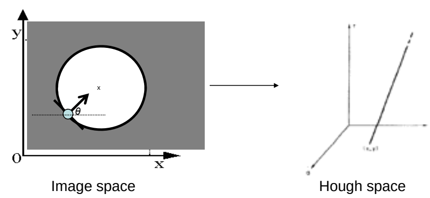


Hough transform for circles

- Circle: center (a,b) and radius r

$$(x_i - a)^2 + (y_i - b)^2 = r^2$$

- For an unknown radius r, **known** gradient direction



Hough transform for circles

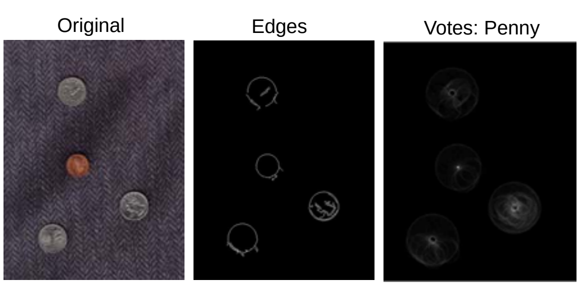
For every edge pixel (*x*,*y*) :
  For each possible radius value *r*:
    For each possible gradient direction *θ: // or use estimated gradient*
      $a = x - r\cos(\theta)$
      $b = y + r\sin(\theta)$
      H[*a*,*b*,*r*] += 1
  end
end

- Check out online demo : http://www.markschulze.net/java/hough/

Example: detecting circles with Hough

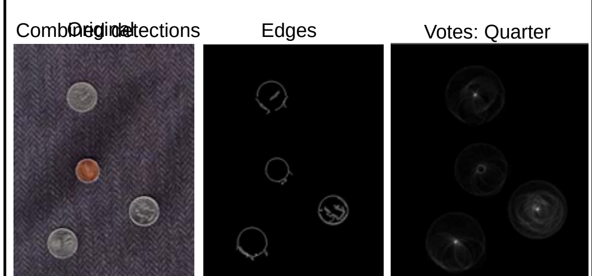
Crosshair indicates results of Hough transform, bounding box found via motion differencing.

Example: detecting circles with Hough



Note: a different Hough transform (with separate accumulators) was used for each circle radius (quarters vs. penny).

Example: detecting circles with Hough



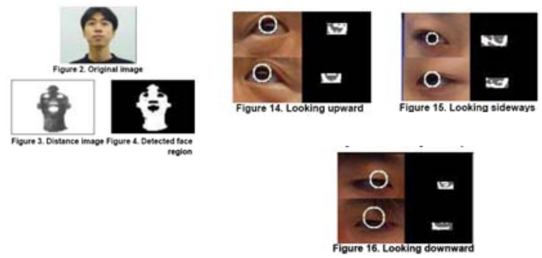
Coin finding sample images from: Vivek Kwatra

Example: iris detection

Gradient+threshold | Hough space (fixed radius) | Max detections

- Hemerson Pistori and Eduardo Rocha Costa
http://rsbweb.nih.gov/ij/plugins/hough-circles.html

Example: iris detection



- An Iris Detection Method Using the Hough Transform and Its Evaluation for Facial and Eye Movement, by Hideki Kashima, Hitoshi Hongo, Kunihito Kato, Kazuhiko Yamamoto, ACCV 2002.

Voting: practical tips

- Minimize irrelevant tokens first (take edge points with significant gradient magnitude)
- Choose a good grid / discretization

Too fine ← ? → Too coarse

- Vote for neighbors, also (smoothing in accumulator array)
- Utilize direction of edge to reduce free parameters by 1
- To read back which points voted for "winning" peaks, keep tags on the votes.

Hough transform: pros and cons

Pros
- All points are processed independently, so can cope with occlusion
- Some robustness to noise: noise points unlikely to contribute consistently to any single bin
- Can detect multiple instances of a model in a single pass

Cons
- Complexity of search time increases exponentially with the number of model parameters
- Non-target shapes can produce spurious peaks in parameter space
- Quantization: hard to pick a good grid size

Generalized Hough transform

- What if want to detect arbitrary shapes defined by boundary points and a reference point?

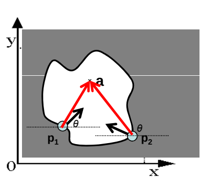


Image space

At each boundary point, compute displacement vector: $\mathbf{r} = \mathbf{a} - \mathbf{p}_i$.

For a given model shape: store these vectors in a table indexed by gradient orientation θ.

[Dana H. Ballard, Generalizing the Hough Transform to Detect Arbitrary Shapes, 1980]

Generalized Hough transform

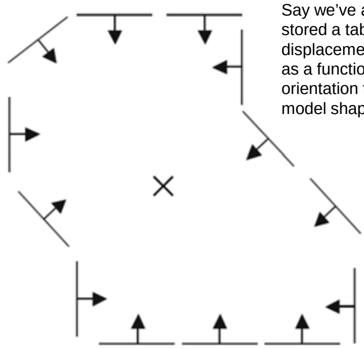
To *detect* the model shape in a new image:

- For each edge point
  - Index into table with its gradient orientation θ
  - Use retrieved **r** vectors to vote for position of reference point
- Peak in this Hough space is reference point with most supporting edges

Assuming translation is the only transformation here, i.e., orientation and scale are fixed.

Example

Say we've already stored a table of displacement vectors as a function of edge orientation for this model shape.

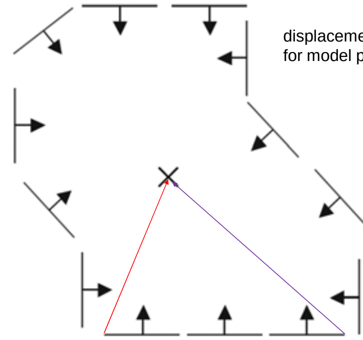


model shape

Adapted from Lana Lazebnik

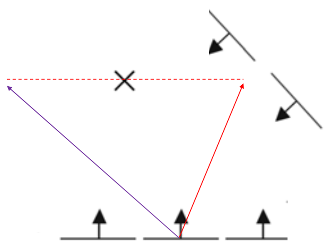
Example

displacement vectors for model points



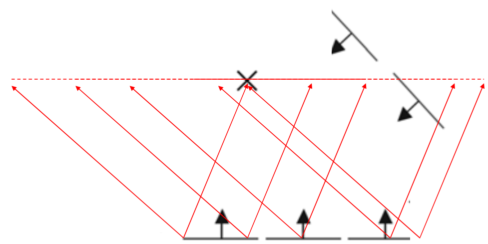
Example

Now we want to look at some edge points detected in a *new* image, and vote on the position of that shape.



range of voting locations for test point

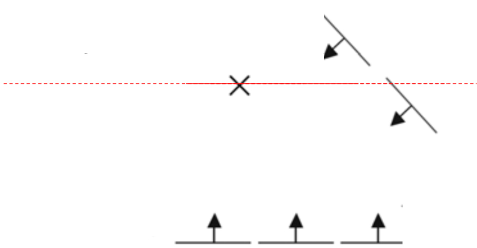
Example



range of voting locations for test point

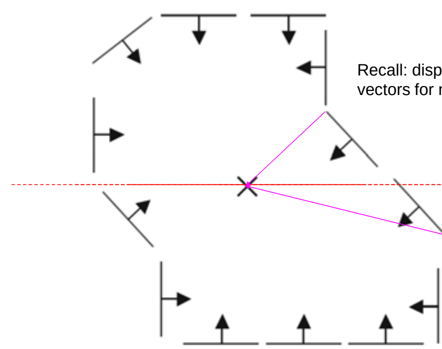
Example

votes for points with $\theta = \uparrow$

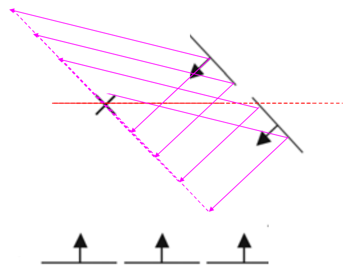


Example

Recall: displacement vectors for model points

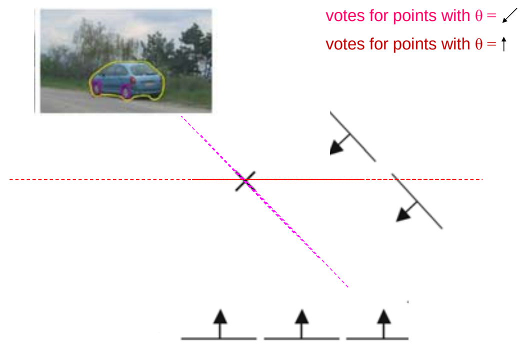


Example



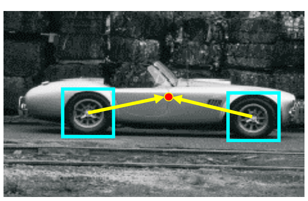
range of voting locations for test point

Example



Application in recognition

- Instead of indexing displacements by gradient orientation, index by "visual codeword"



training image

visual codeword with displacement vectors

B. Leibe, A. Leonardis, and B. Schiele, Combined Object Categorization and Segmentation with an Implicit Shape Model, ECCV Workshop on Statistical Learning in Computer Vision 2004

Source: L. Lazebnik

Application in recognition

- Instead of indexing displacements by gradient orientation, index by "visual codeword"

test image

B. Leibe, A. Leonardis, and B. Schiele, Combined Object Categorization and Segmentation with an Implicit Shape Model, ECCV Workshop on Statistical Learning in Computer Vision 2004

Source: L. Lazebnik

Summary

- **Grouping/segmentation** useful to make a compact representation and merge similar features
  - associate features based on defined similarity measure and clustering objective
- **Fitting** problems require finding any supporting evidence for a model, even within clutter and missing features.
  - associate features with an explicit model
- **Voting** approaches, such as the **Hough transform**, make it possible to find likely model parameters without searching all combinations of features.
  - Hough transform approach for lines, circles, …, arbitrary shapes defined by a set of boundary points, recognition from patches.

Next

- Thursday 9/24: Deformable contours
- Pset 2: texture + clustering
  - Out today, due 10/6