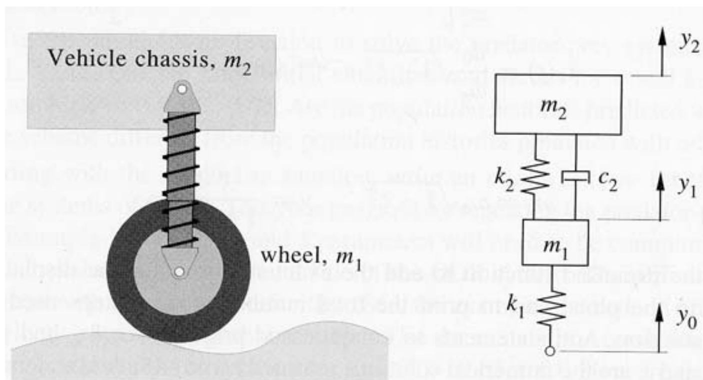
A Spring/Damper Suspension ODE Problem
Due Friday, December 6 by 12 noon

(From Recktenwald Problem 26, pp732-3)

The following is a simplified model of the suspension system of one wheel of an automobile.



The input to the system is the time-varying displacement $y_0(t)$ corresponding to changes in the terrain. The shock absorber is characterized by its spring rate k_2 and damping coefficient c_2. Damping in the tire is neglected. (There is no c_1 term.)

Applying Newton's law of motion and force balances to the wheel and vehicle chassis yields the following system of equations:

$$m_1 y_1''(t) + c_2(y_1'(t) - y_2'(t)) + k_2(y_1(t) - y_2(t)) + k_1 y_1(t) = k_1 y_0(t),$$
$$m_2 y_2''(t) - c_2(y_1'(t) - y_2'(t)) - k_2(y_1(t) - y_2(t)) = 0.$$

(a) Convert these two second-order equations into an equivalent system of first-order equations. (How many first-order equations are required?). Write a Matlab function yp = spring (t, y, m, k, c) that takes as input the time t, a column array y, and the constants m, k, and c (as arrays). Imbed the forcing function $y_0(t) = 0.05\sin(3\pi t)$.

We construct the new array $y = \begin{bmatrix} y_1^{old} \\ y_2^{old} \\ y_1'^{old} \\ y_2'^{old} \end{bmatrix}$ so

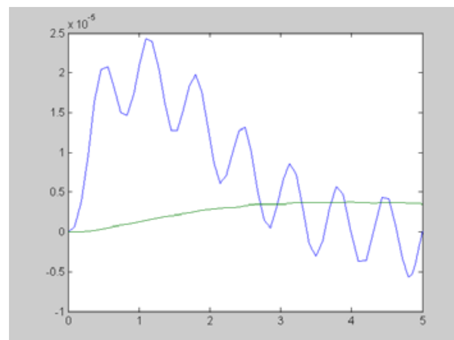
$$y' = \begin{bmatrix} y_3 \\ y_4 \\ (k_1 y_0(t) - c_2(y_1'(t) - y_2'(t)) - k_2(y_1(t) - y_2(t)) - k_1 y_1(t))/m_1 \\ (c_2(y_1'(t) - y_2'(t)) + k_2(y_1(t) - y_2(t)))/m_2 \end{bmatrix}$$ and this

is implemented in the Matlab function:

```
function yp = spring (t, y, m, k, c)
yp = zeros(4,1);
yp(1) = y(3);
yp(2) = y(4);
yp(3) = (.05*sin(3*pi*t)-c(2)*(y(3)-y(4))-k(2)*(y(1)-y(2))-k(1)*y(1))/m(1);
yp(4) = (c(2)*(y(3)-y(4))+k(2)*(y(1)-y(2)))/m(2);
```

(b) Use Matlab function ode45 integration routine to solve this system on the time interval $[0,5]$ for $m_1 = 110kg$, $k_1 = 136N/m, m_2 = 1900kg, k_2 = 16N/m$, and $c_2 = 176Ns/m$. Assume the system is at rest at $t = 0$ (i.e., $y_1(0) = 0, y_2(0) = 0, y_1'(0) = 0$, and $y_2'(0) = 0$). Produce a plot that shows both y_1 and y_2 versus t.

```
m = [110; 1900];
k = [136; 16];
c = [0; 176];
[t, y] = ode45 (@spring, [0 5], y0, [], m, k, c);
plot (t, y(:, 1), t, y(:, 2));
```



(c) Repeat the solution with c_2 reduced by a factor of 5.

```
C(2) = c(2)/5;
[t, y] = ode45 (@spring, [0 5], y0, [], m, k, c);
plot (t, y(:, 1), t, y(:, 2));
```