A Formally Verified Symmetry Breaking Tool for SAT

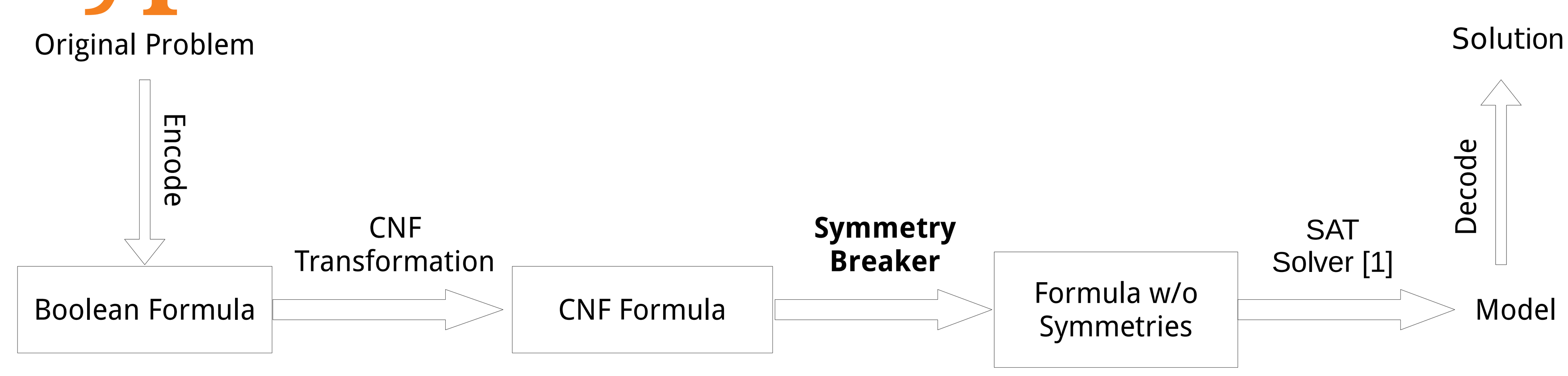
RIT

David E. Narváez <den9562@rit.edu>
Golisano College of Computing and Information Sciences
Rochester Institute of Technology

Overall Goal

Obtain a verified symmetry breaking tool for SAT by formalizing Crawford's idea for symmetry breaking [2].

Typical SAT Workflow



Motivation

- Unlike other tools in the SAT ecosystem, this requires a mix of graphs and Boolean formulas.
- Practically relevant to compare to Shatter [3], BreakID [4], etc.
- Used beyond SAT (e.g., ASP).

Preliminary Results: PVS Formalization

We have formalized Theorem 1 using PVS. This work is available at the URL below. Some of the challanges we faced:

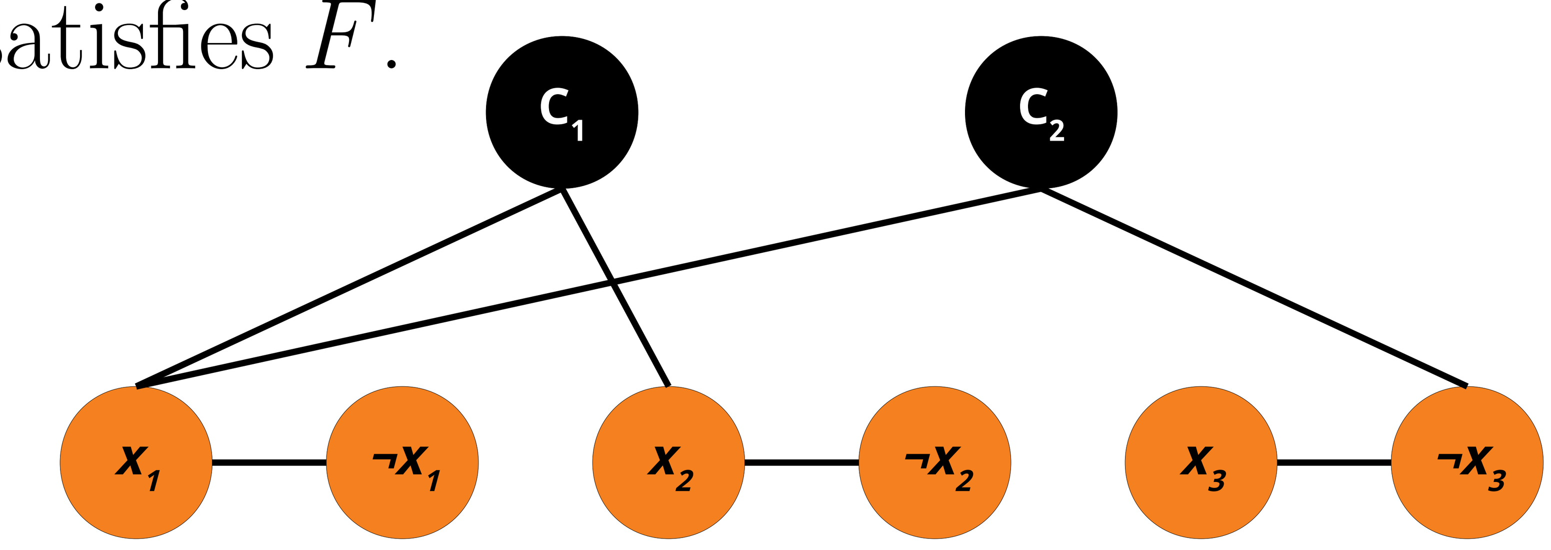
- Explicit type coercions, bloating the notation.
- Heavy case analysis over the edge datatype.

Crawford's SymmetryBreaking [2]

Theorem 1: Given a formula F and a color-preserving automorphism ϕ of its incidence graph, an assignment a satisfies F if and only if the assignment $a \circ \phi$ satisfies F.

Theorem 2: Given a CNF formula F and a color-preserving automorphism ϕ of its incidence graph, F is satisfiable if and only if $F \wedge P(\phi)$ is satisfiable, where

$$P_1(\pi) = l_1 \leq \pi(l_1)$$
$$P_i(\pi) = \left(\bigwedge_{j=1}^{i-1} l_j = \pi(l_j)\right) \to l_i \leq \pi(l_i) \text{ for } 1 < i \leq n$$
$$P(\pi) = \bigwedge_{i=1}^{n} P_i(\pi).$$



Example graph for $(x_1 \vee x_2) \wedge (x_1 \vee \neg x_3)$

Appending symmetry breaking clauses as per Theorem 2 for every color-preserving automorphism breaks the syntactic symmetries of the formula.

The Road Ahead

- Formalizing Theorem 2.
- Obtaining executable code from the formalizations.
- Adapt the formalizations to the approach in Shatter [3], BreakID [4].
- Carry out performance analysis.

References

[1] F. Marić. *Formal verification of a modern SAT solver by shallow embedding into Isabelle/HOL*. Theoretical Computer Science, 411(50):4333--4356, 2010.
[2] J. Crawford. *A theoretical analysis of reasoning by symmetry in first-order logic*. AAAI Workshop on Tractable Reasoning, pages 17--22, 1992.
[3] F. A. Aloul, K. A. Sakallah, and I. L. Markov. *Efficient symmetry breaking for boolean satisfiability*. IEEE Transactions on Computers, 55(5):549--558, 2006.
[4] J. Devriendt, B. Bogaerts, M. Bruynooghe, and M. Denecker. *Improved Static Symmetry Breaking for SAT*. SAT-16, LNCS 9710 104--122.

No need to take pictures!
You can find the poster here: